Flavour physics – Lecture 2

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VIIIth SERC school on EHEP

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Outline

- CKM again and its predictions
  - Charm, bottom and top discovery
- Mixing
- Types of CP violation
- Kaon physics
- Charm physics

CKM matrix

- Extension of Cabibbo's two by two mixing matrix
- Unitary complex matrix
  - 18 parameters
- Unitary constraints ($VV^{+}=I$)
  - 9 parameters
- Quark fields can absorb five unobservable phases
  - 4 parameters
  - 3 mixing angle and 1 phase

$$\begin{pmatrix} u & c & t \end{pmatrix} \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Responsible for CP violation

CKM matrix parameterisations

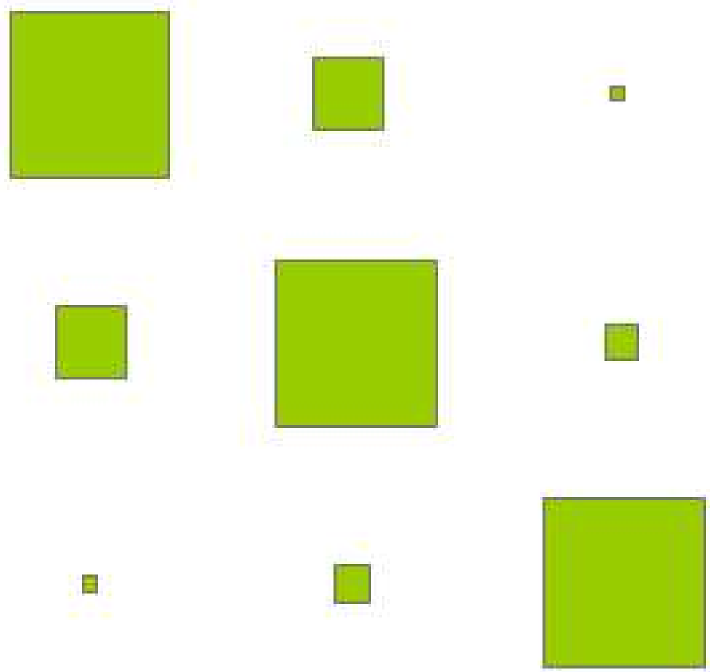
- 3 mixing angles (θ_{12}, θ_{13} ,θ_{23})+ one phase (δ)
  - PDG (Chau and Keung) – s_{ij}=sin θ_{ij} and c_{ij}=cos θ_{ij}

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

  - s_{12}~0.2 s_{23}~0.04 s_{13}~0.004
- However, Wolfenstein parameterisation that exploits this hierarchy and expands matrix in terms of $\lambda = s_{12}$
  - then $s_{23} = A\lambda^2$ and $s_{13}e^{i\delta} = A\lambda^3(\rho + i\eta)$

Wolfenstein parameterisation

$$V = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}\left(\lambda^4\right)$$



Why the strong ranking?

Phase convention indep.

$$\bar{\rho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$

$$= \rho + i\eta + O\left(\lambda^5\right)$$

Unitarity

$$V^{\dagger}V = VV^{\dagger} = I$$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \qquad V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0$$

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1 \qquad V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$$|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 = 1 \qquad V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0$$

$$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1 \qquad V_{ud}V_{cd}^* + V_{us}V_{cs}^* + V_{ub}V_{cb}^* = 0$$

$$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1 \qquad V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* = 0$$

$$|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1 \qquad V_{cd}V_{td}^* + V_{cs}V_{ts}^* + V_{cb}V_{tb}^* = 0$$

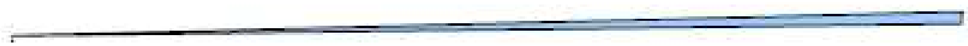
Unitarity triangles

Null relations can be expressed as triangles in complex plane

Not to scale

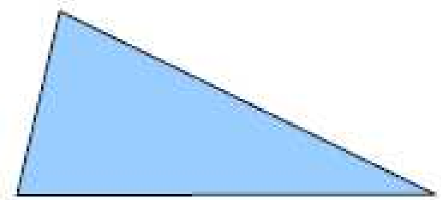
$$V_{ud}V_{us}^{*} + V_{cd}V_{cs}^{*} + V_{td}V_{ts}^{*} = 0$$

$$\lambda \qquad \lambda \qquad \lambda^5$$



$$V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0$$

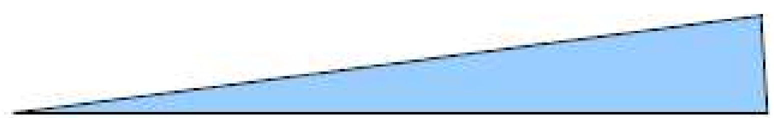
$$\lambda^3 \qquad \lambda^3 \qquad \lambda^3$$



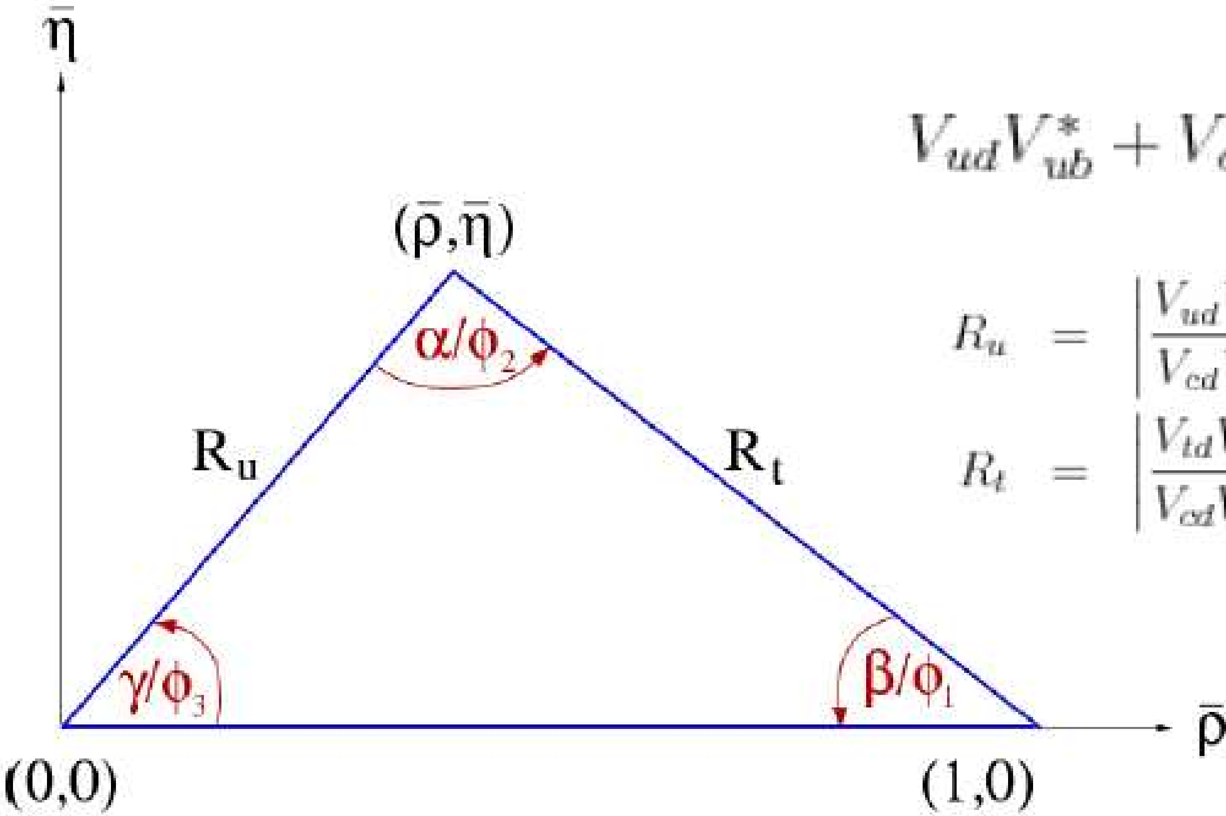
Area is the same

$$V_{us}V_{ub}^{*} + V_{cs}V_{cb}^{*} + V_{ts}V_{tb}^{*} = 0$$

$$\lambda^4 \qquad \lambda^2 \qquad \lambda^2$$



The unitarity triangle



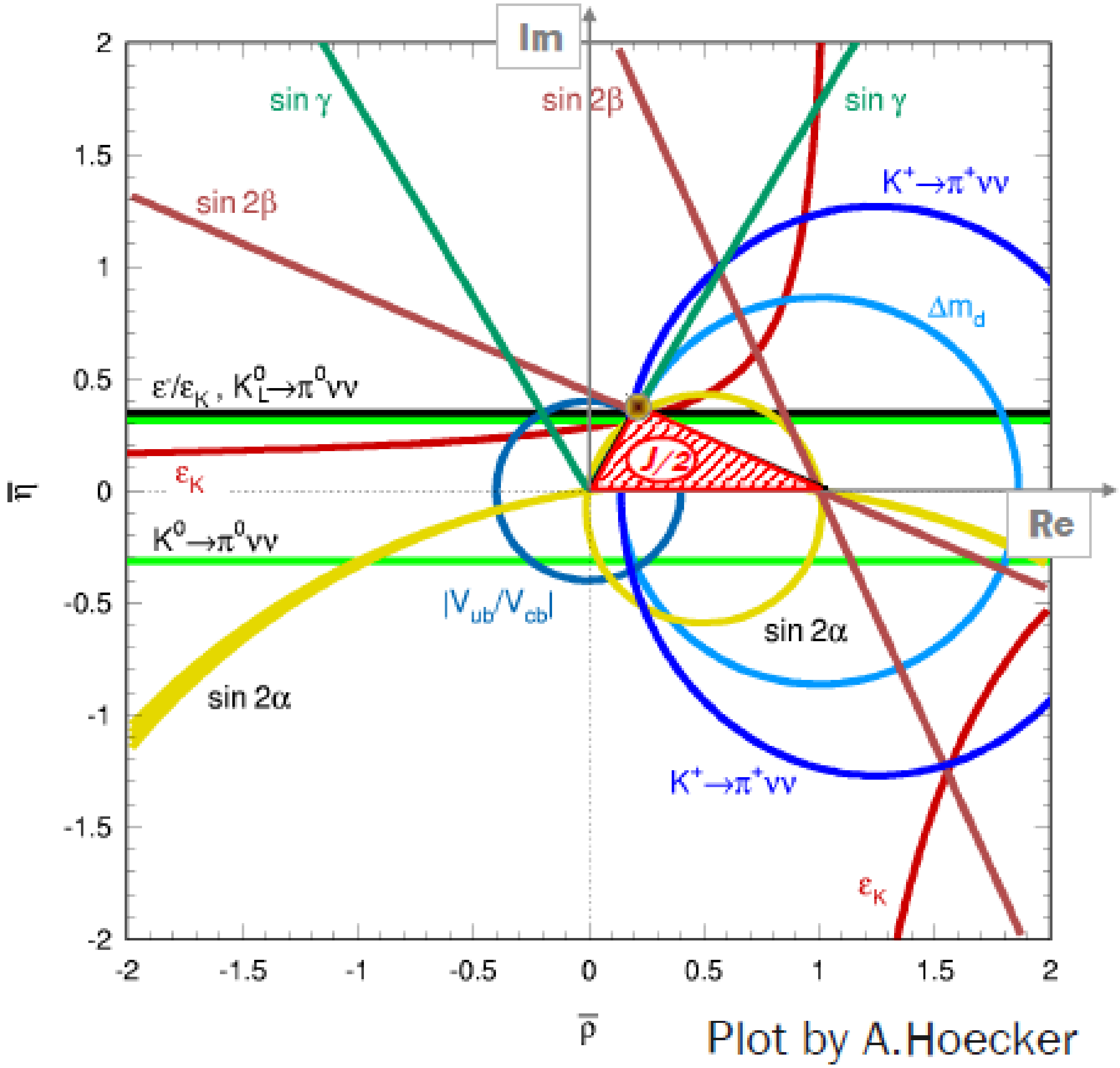
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0,$$

$$R_u = \left|\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right| = \sqrt{\bar{\rho}^2 + \bar{\eta}^2},$$

$$R_t = \left|\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}\right| = \sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2}.$$

Predictive nature of KM mechanism

All measurements must agree



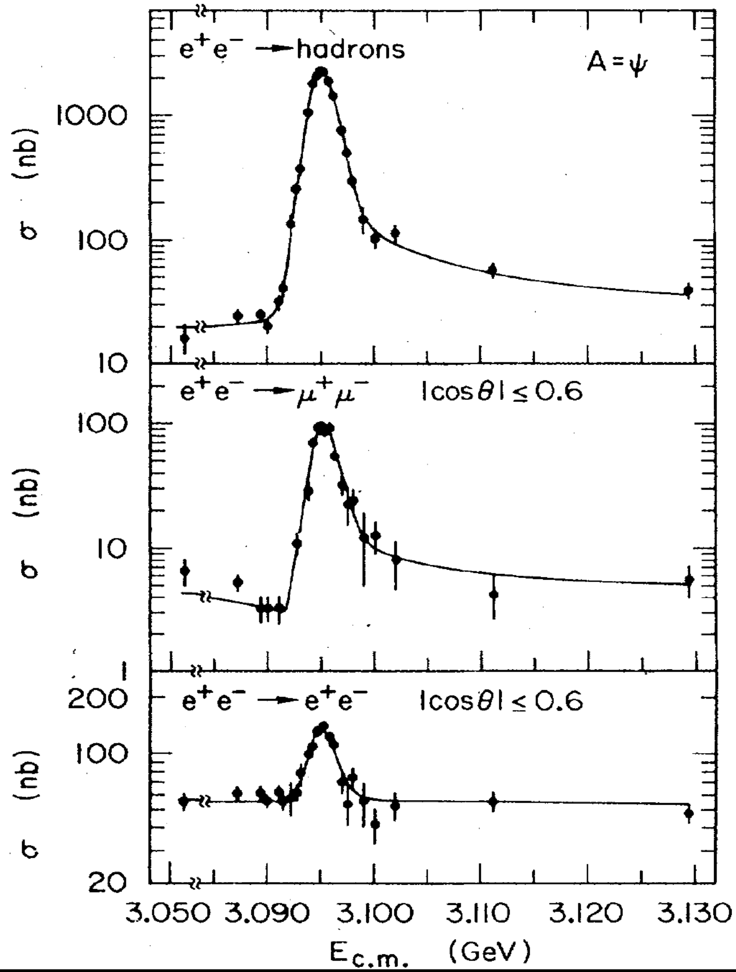
But the first KM (and GIM) predictions

- More quarks
- Discovery of charm then beauty (and finally top)
- Worth spending a little time as it highlights a couple of features of experiments
- Today principle interest now in onia is QCD
  - Exotic X, Y, Z states found at Babar and Belle
    - New states D-molecules, tetraquarks etc
  - Some states only just observed
    - h_b

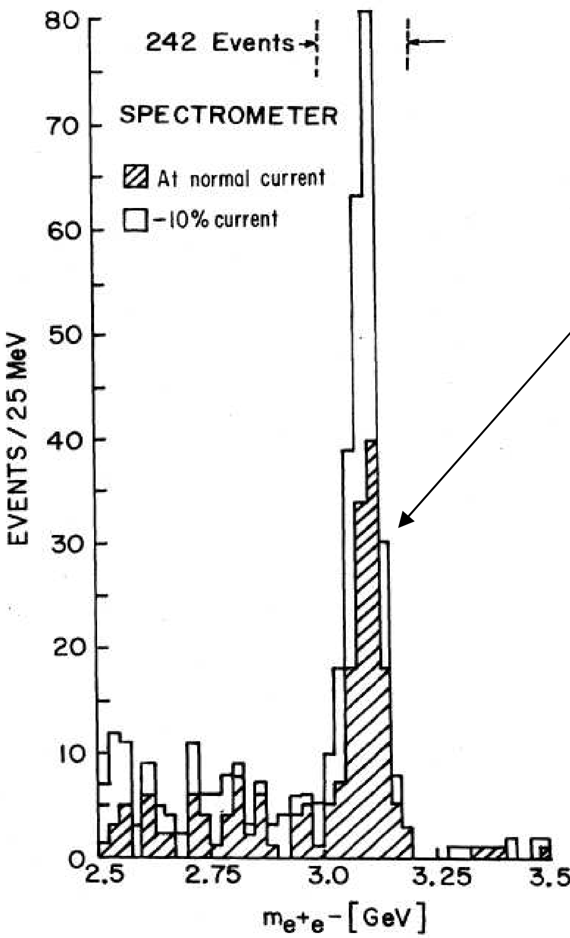
Discovery of Charmonium

- The charm quark had been proposed in 1970 by Glashow, Illiopoulos and Maiani to explain the suppression of transitions involving flavour changing neutral currents such as $K^0 \rightarrow \mu\mu$ which $\Delta S=1$
  - Flavour changing charged currents had been observed since β decay which is $u \rightarrow de^+ v_e$
- The first direct evidence for the charm quark came in 1974 from two different experiments:
  - e^+e^- collisions at a centre of mass energy ~3.0 GeV/c^2
    - SPEAR collider at Stanford Linear Accelerator Center
  - p+Be fixed target experiment with 28 GeV protons
    - Brookhaven alternating-gradient synchrotron

Discovery of the *J/ψ*



Cross-section to different final states in e^+e^- collisions at $E_{c..m}$ ~ 3.1 GeV



Invariant mass of e^+e^- pairs

$$p + \mathrm{Be} \rightarrow \mathrm{J}/\psi + \text{anything}$$

followed by $\mathrm{J}/\psi \rightarrow e^+e^-$

More charmonium

- An additional resonance was soon observed at SLAC with a mass of 3686 MeV and was called the ψ' or $\psi(2S)$
- Similar arguments to those used for the J/ψ led to the same assignment of quantum numbers: $J^{PC}=1^{--}$, I=0
- The ψ' is narrow Γ=0.28 MeV but there were new decay modes observed:
  - $\psi' \rightarrow J/\psi + 2\pi$ (BR=50%)
  - $\psi' \rightarrow \chi + \gamma$ (BR=24%)

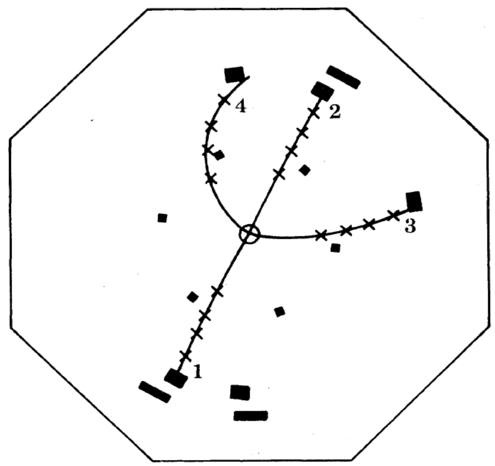


FIG. 3. An example of the decay $\psi(3684) \rightarrow \pi^+ + \pi^- + \psi(3095)$, where $\psi(3095) \rightarrow e^+ + e^-$, from an off-line reconstruction of the data. The event is seen in the x–y projection where z is the beam (and magnetic field) direction. Also shown are the trigger and shower counters which detected the tracks. Tracks 3 and 4 are the slow pions and tracks 1 and 2 are the two leptons from $\psi(3095)$ decay.

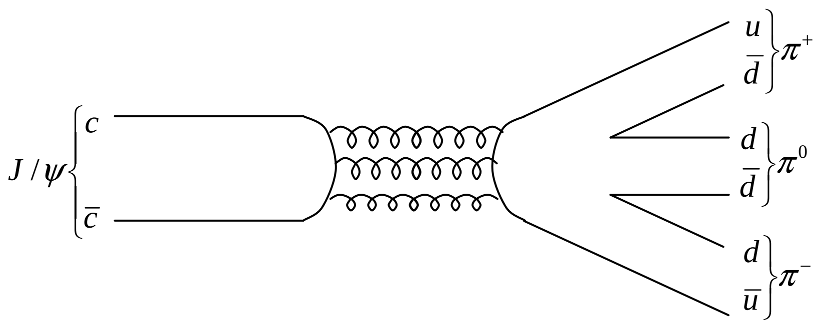
- There were a further 4 states observed labelled $\psi(3S)$, $\psi(4S)$ and $\psi(5S)$ – but their widths were much greater between 24 and 78 MeV

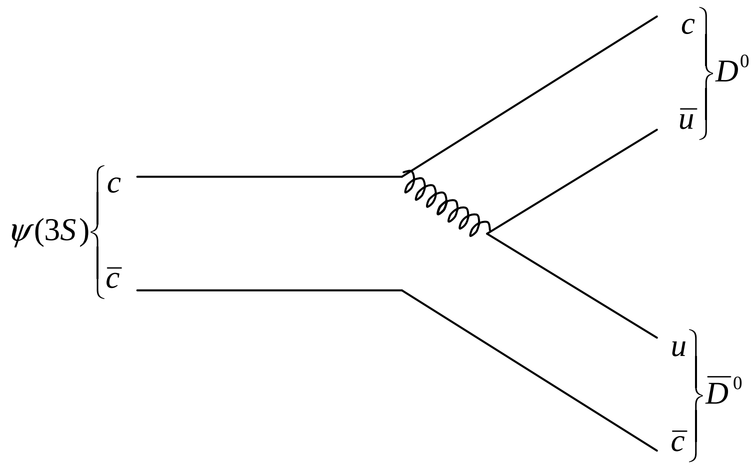
Decays of the charmonium

- Allowed hadronic final states governed by kinematics

$$m(J/\psi) < m(\psi') < 2m(D) < m(\psi(3S))$$

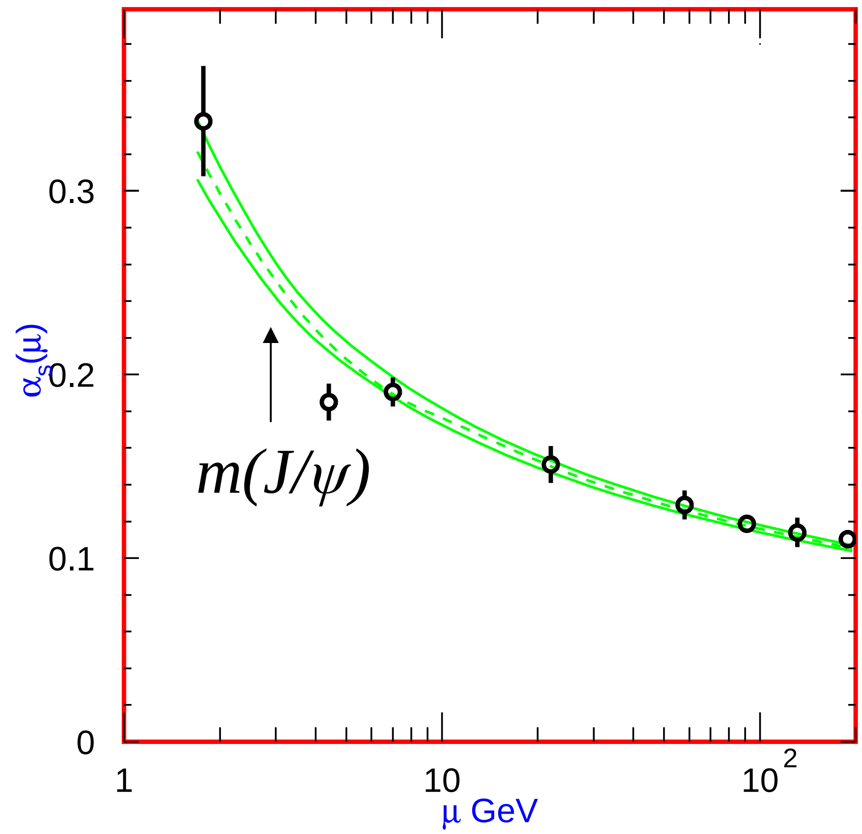
- Strong decay of the J/ψ is via the annihilation of the charm and anti-charm quarks to 3 gluons
  - An odd number of gluons is required to conserve C
    - Gluon and J/ψ both have C=−1
  - A single gluon is not colourless therefore is not colour charge conserving
  - α_s^6 dependence
- Strong decay of $\psi(3S)$ is a single gluon radiated to form D meson pair
  - α_s^2 dependence





Running of the strong coupling

- Additional factor comes the strength of the strong force varying rapidly as a function of the momentum transfer of the interaction μ
- The coupling is much stronger for a $\mu \sim m(u/d)$ than for $\mu \sim m(J/\psi)$

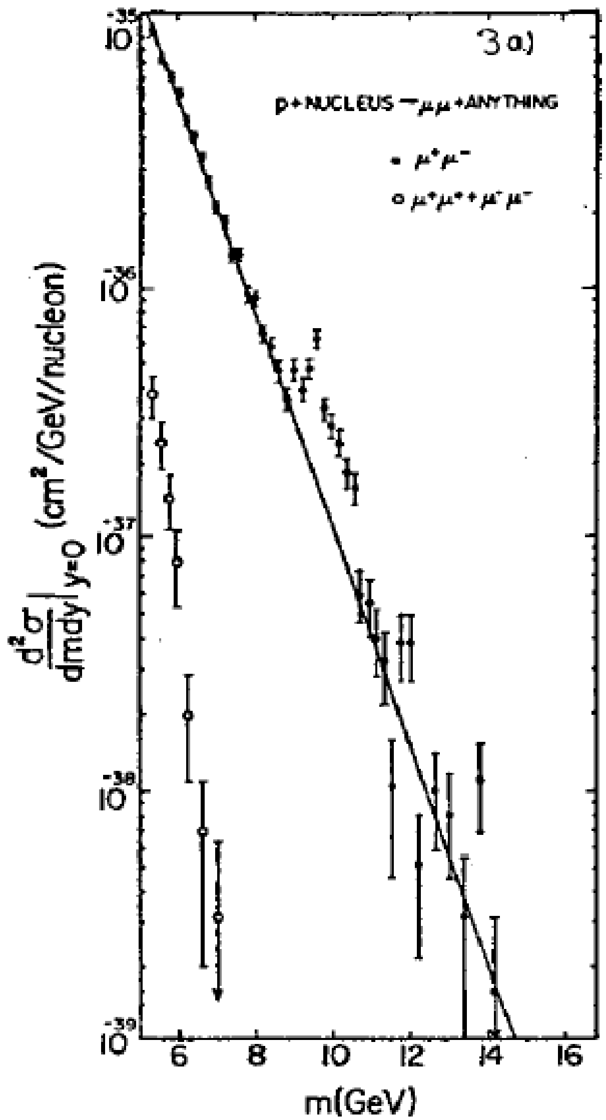


Bottomonium

- In 1977 there were a similar set of resonances observed in the centre of mass region 9.5-10.5 GeV
- Observed in interactions of proton beam on fixed targets:

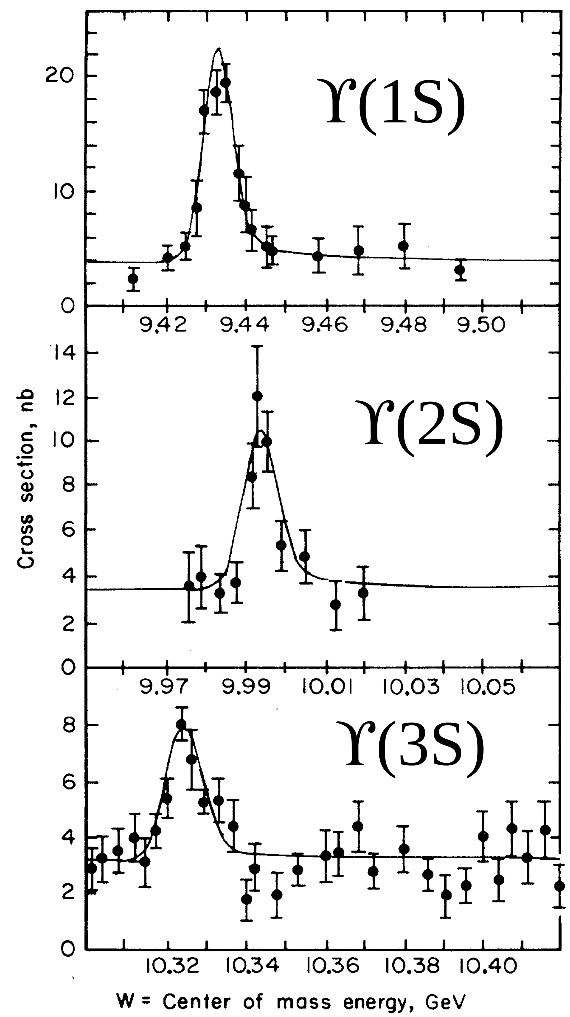
$$p + \mathrm{Be, Cu, Pt} \rightarrow \mu^{+} + \mu^{-} + \mathrm{anything}$$

- Resolution of 500 MeV but width of resonance 1.2 GeV suggested it might be several resonances



Bottomonium

- Resonances were then observed in e^+e^- collisions and they were resolved into three narrow states $\Upsilon(1S)$, $\Upsilon(2S)$ and $\Upsilon(3S)$
- A broader resonance $\Upsilon(4S)$ that decays into B meson pairs was also found
- Associated with bound states of b quarks and anti-quarks with $\Upsilon(4S)$ the first resonance to be heavier than 2 B mesons



Mixing

- For neutral particles with opposite flavour quantum numbers (i.e. K0 and K0bar) CPT theorem requires:
  - Equal masses
  - Equal lifetimes
- However, mixed states K_S and K_L can have different masses and widths

Mixing formalism

Arbitrary mixture of B^0 and $\overline{\mathrm{B}}^0$

$$p\left|B^0\right\rangle + q\left|\overline{B}^0\right\rangle$$

Evolution governed by time-dependent Schrodinger equation

$$\mathrm{i}\frac{\partial}{\partial \mathrm{t}}\begin{pmatrix} p \\ q \end{pmatrix} = H\begin{pmatrix} p \\ q \end{pmatrix} = \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix} p \\ q \end{pmatrix}$$

H is Hamiltonian

M and Γ are 2×2 Hermitian matrices

Encapsulate the perturbative weak effects that lead to mixing

CPT requires the diagonal elements of M and Γ are equal

Mixing formalism

Heavy and light mass eigenstates

$$|B_L\rangle = p|B\rangle + q|\overline{B}\rangle$$

$$|B_H\rangle = p|B\rangle - q|\overline{B}\rangle,$$

Eigenvalue equation gives

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}},$$

$$\lambda_{H,L} = m_{H,L} - \frac{i}{2}\Gamma_{H,L},$$

Where

$$m_H, m_L = M \pm Re\sqrt{|M_{12}|^2 - \frac{|\Gamma_{12}|^2}{4} - iRe(M_{12}\Gamma_{12}^*)} \equiv M \pm \Delta m/2,$$

$$\Gamma_H, \Gamma_L = \Gamma \pm 2Im\sqrt{|M_{12}|^2 - \frac{|\Gamma_{12}|^2}{4} - iRe(M_{12}\Gamma_{12}^*)} \equiv \Gamma \pm \Delta\Gamma/2,$$

Mixing formalism

Mass and width difference related
To off diagonal elements

$$\Delta m^2 - \frac{\Delta\Gamma^2}{4} = 4\left(|M_{12}|^2 - \frac{|\Gamma_{12}|^2}{4}\right)$$

$$\Delta m \Delta\Gamma = 4Re(M_{12}\Gamma_{12}^*).$$

Can express time-evolution of flavour state in terms of mass and width parameters

$$|B(t)\rangle = \frac{1}{2p}(|B_L(t)\rangle + |B_H(t)\rangle)$$

$$= \frac{1}{2}e^{-iMt}e^{-\frac{\Gamma}{2}t}\Bigg(\left(e^{\frac{\Delta\Gamma}{4}t}e^{i\frac{\Delta m}{2}t} + e^{-\frac{\Delta\Gamma}{4}t}e^{-i\frac{\Delta m}{2}t}\right)|B\rangle$$

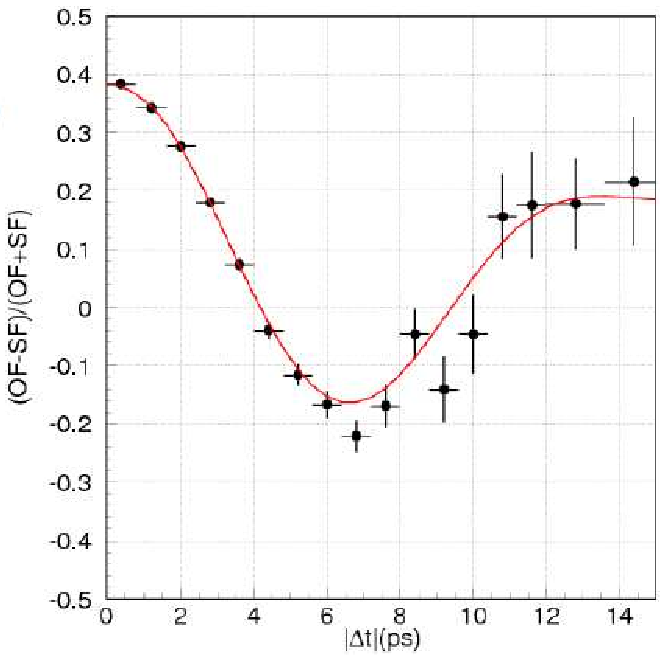
$$+\frac{q}{p}\left(e^{\frac{\Delta\Gamma}{4}t}e^{i\frac{\Delta m}{2}t} - e^{-\frac{\Delta\Gamma}{4}t}e^{-i\frac{\Delta m}{2}t}\right)|\overline{B}\rangle\Bigg).$$

Similar for Bbar

Mixing formalism

- Probability to mix

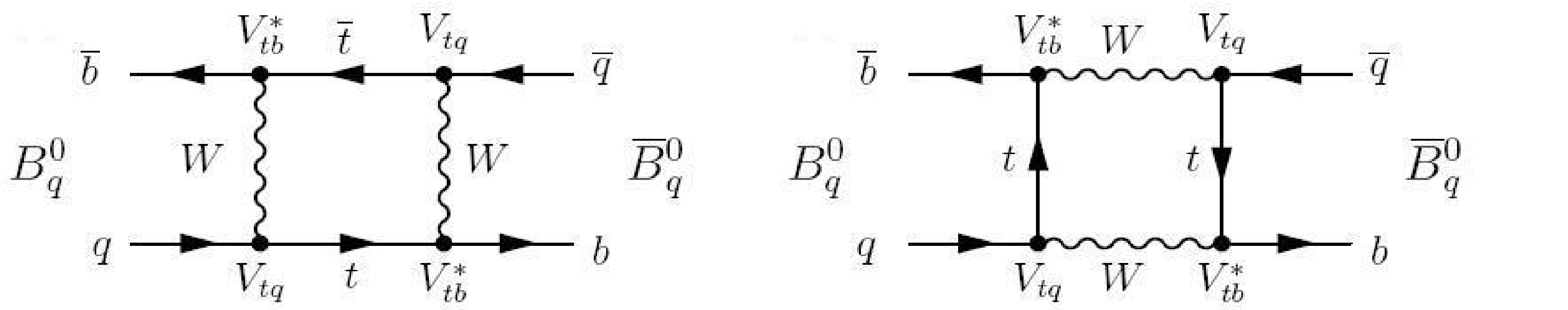
$$P_m^B(t) = \frac{|q/p|^2 e^{-\Gamma t}}{\Gamma\left(\frac{1+|q/p|^2}{\Gamma^2-\Delta\Gamma^2/4} + \frac{1-|q/p|^2}{\Gamma^2+\Delta m^2}\right)}\left(\cosh\frac{\Delta\Gamma}{2}t - \cos\Delta m t\right)$$



Techniques next Lecture but plot From Belle clearly Shows mixing

$\Delta\Gamma \sim 0$ $\Delta M = 0.5\ ps^{-1}$

Mixing



Feynman box diagrams for B mixing.

Weak box diagrams drive mixing

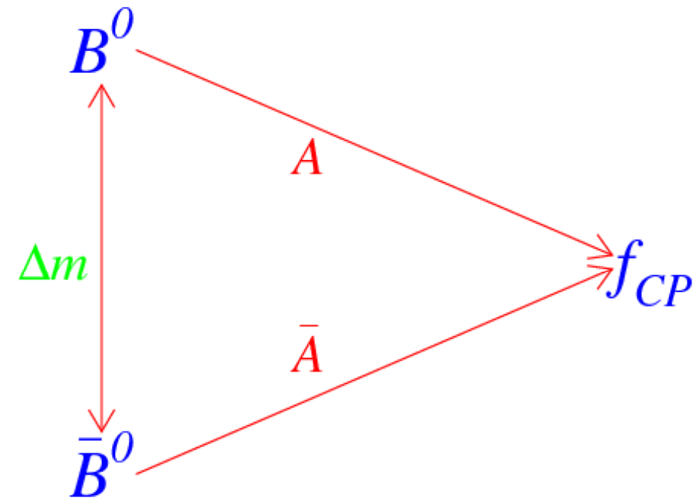
Sensitive to CKM elements Vtd (Vts) for B (Bs) mixing

More later on relating to unitarity triangle

Types of CP violation

- Consider decay of neutral particle to a CP eigenstate

$$\lambda_{CP} = \frac{q}{p}\frac{\bar{A}}{A}$$



$$|\frac{q}{p}| \neq 1$$

CP violation in mixing

$$|\frac{\bar{A}}{A}| \neq 1$$

CP violation in decay (direct CPV)

$$\Im\left(\frac{q}{p}\frac{\bar{A}}{A}\right) \neq 0$$

CP violation in interference between mixing and decay

CP violation kaons

- CP violation in neutral kaon system due to |p/q|=(1+ε)/(1−ε)≠1 and possibility of in decay (ε′)
- Observables studied

$$A_L = \frac{\Gamma(K_L^0 \to \pi^-\ell^+\nu) - \Gamma(K_L^0 \to \pi^+\ell^-\nu)}{\Gamma(K_L^0 \to \pi^-\ell^+\nu) + \Gamma(K_L^0 \to \pi^+\ell^-\nu)} = 2\mathrm{Re}\ \epsilon/(1+|\epsilon|^2) \approx 2\mathrm{Re}\ \epsilon$$

$$\eta_{+-} = A(K_L^0 \to \pi^+\pi^-)/A(K_S^0 \to \pi^+\pi^-)$$

$$= |\eta_{+-}|\ e^{i\phi_{+-}} = \epsilon + \epsilon'$$

$$\eta_{00} = A(K_L^0 \to \pi^0\pi^0)/A(K_S^0 \to \pi^0\pi^0)$$

$$= |\eta_{00}|\ e^{i\phi_{00}} = \epsilon - 2\epsilon'$$

$$|\epsilon| = (2.228 \pm 0.011) \times 10^{-3}\ ,$$

PDG values

$$\epsilon'/\epsilon = (1.65 \pm 0.26) \times 10^{-3}$$

Direct CP violation

- New superweak force not KM gives CP (Wolfenstein)
- There would only be CPV in K mixing and not in other system
- Long search in kaon physics to sort this out
  - NA48 and KTeV to measure

– if CPV in $K_L \to \pi^+\pi^- \neq$ CPV in $K_L \to \pi^0\pi^0 \leftrightarrow$ CPV cannot be in mixing only

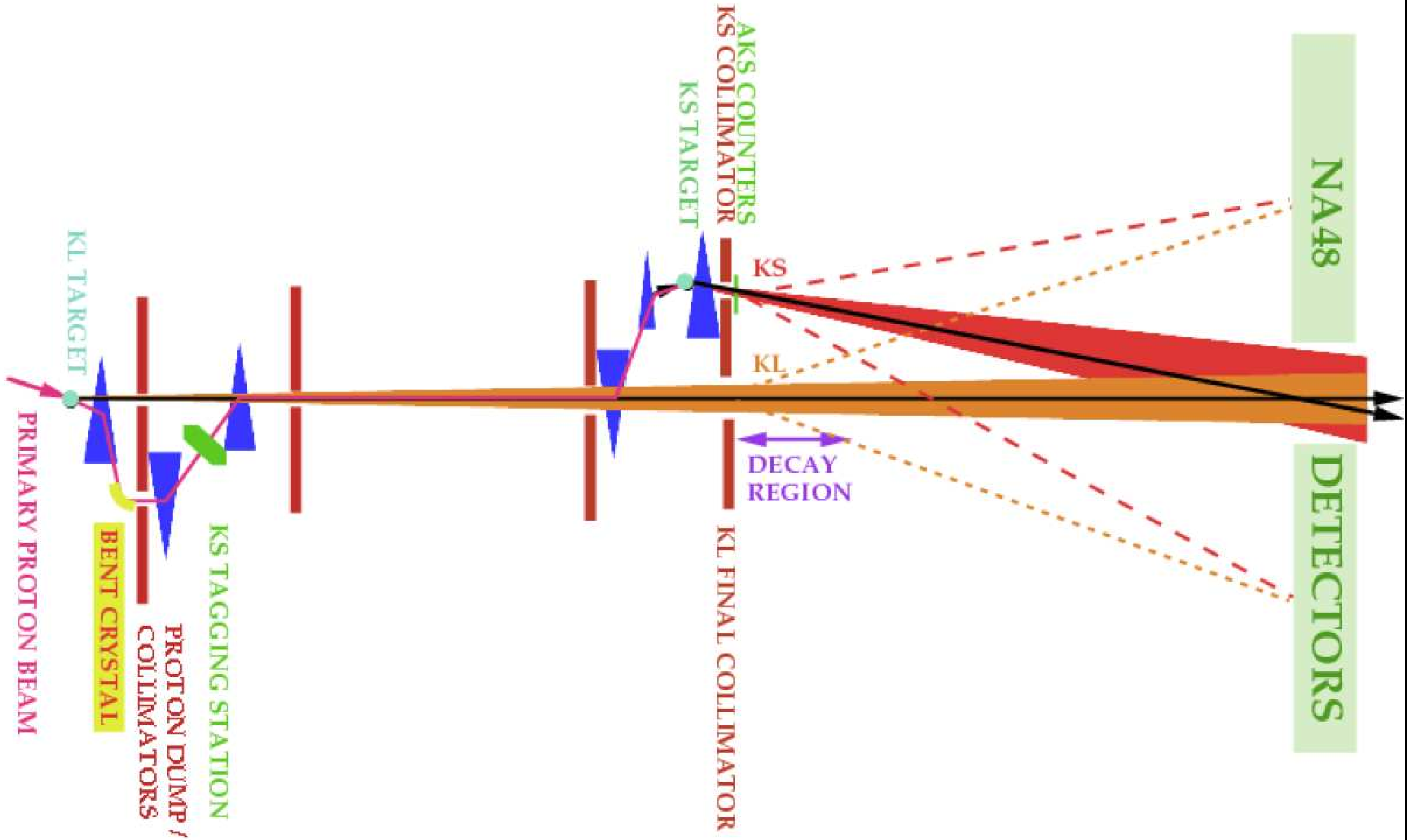
$R = (K_L \to \pi^0\pi^0)/(K_S \to \pi^0\pi^0)/(K_L \to \pi^+\pi^-)/(K_S \to \pi^+\pi^-)$

$R = 1 - 6\,\mathrm{Re}(\epsilon'/\epsilon)$

NA48 technique

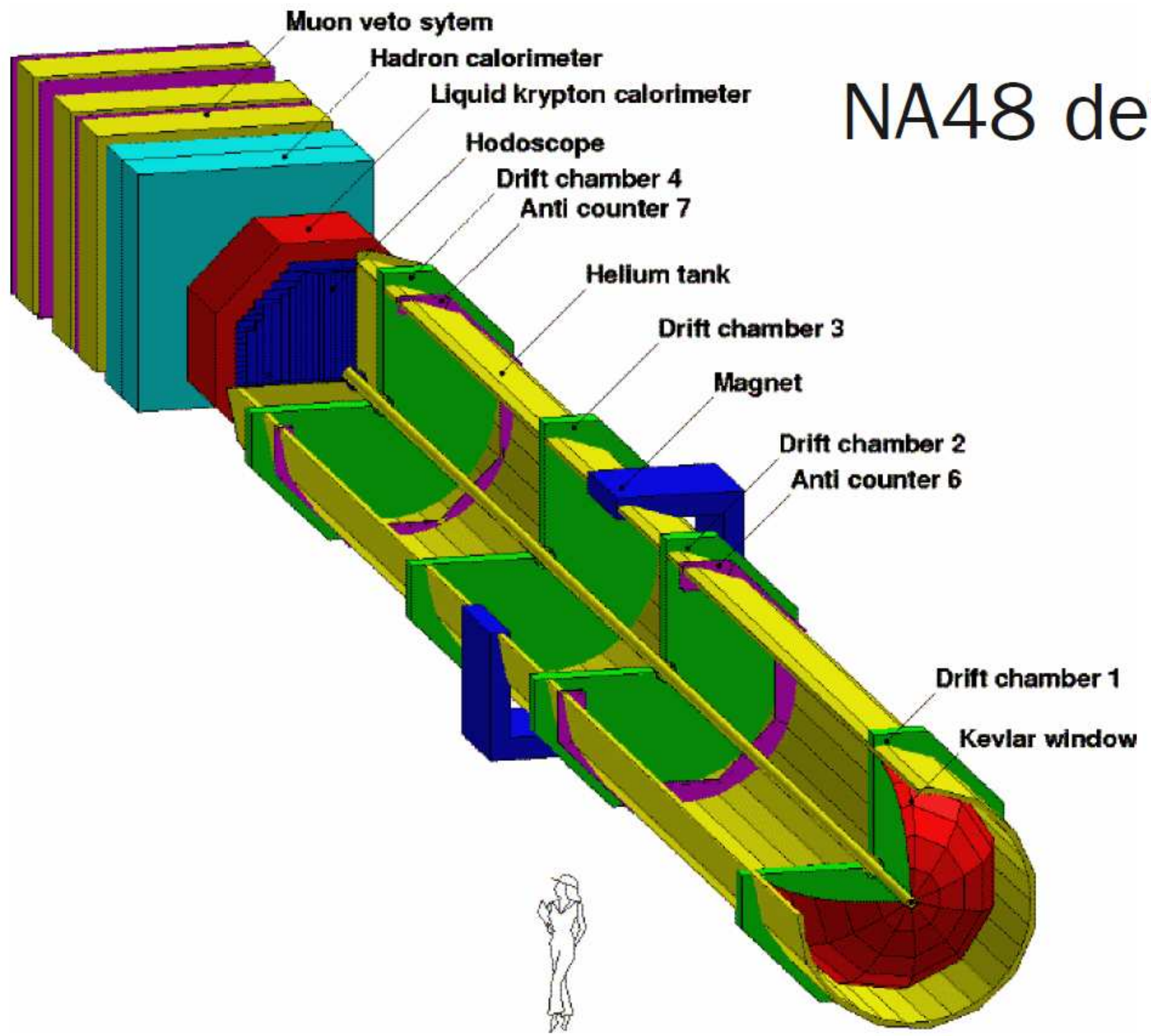
- Precise measurement – need control of systematic errors
  - use simultaneous K_L and K_S beams
  - take data in all four modes together
  - make acceptance as similar as possible
  - perform analysis in bins on kaon energy
    - correct for differences in K_L and K_S energy spectra
  - weight K_L events according to K_S decay distribution

NA48 beams

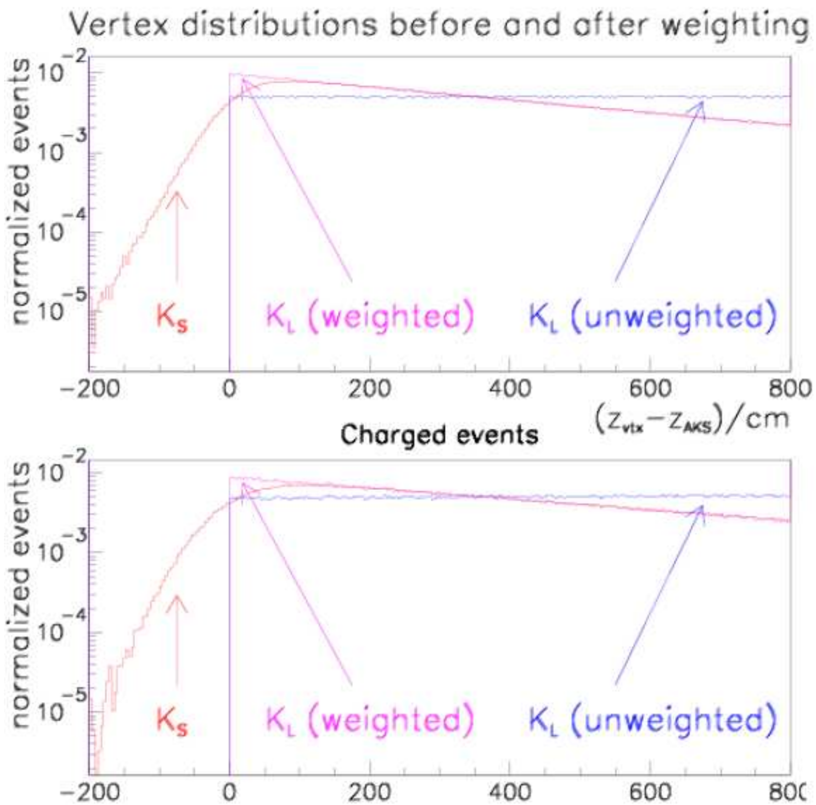


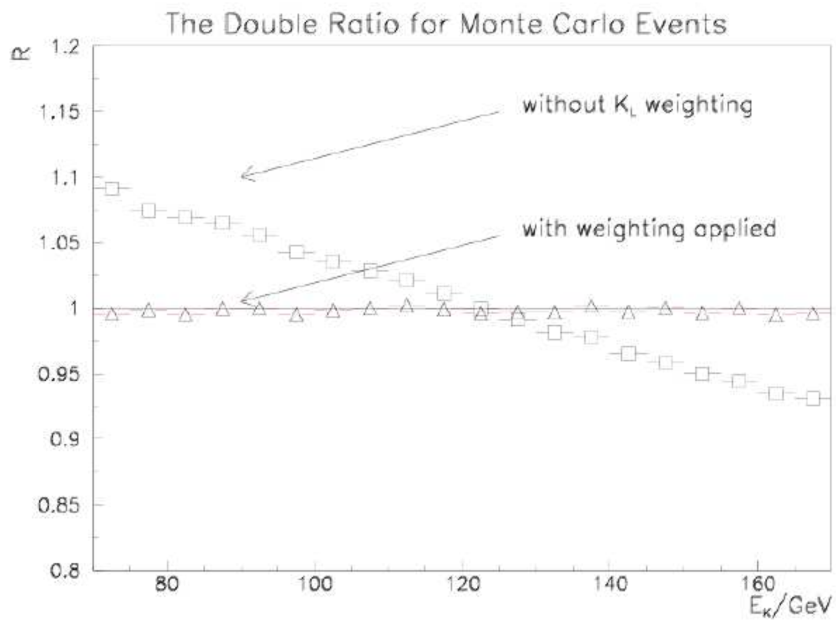
KL TARGET
PRIMARY PROTON BEAM
BENT CRYSTAL
PROTON DUMP / COLLIMATORS
KS TAGGING STATION
KS TARGET
KS COLLIMATOR
AKS COUNTERS
KS
KL
DECAY REGION
KL FINAL COLLIMATOR
NA48
DETECTORS

NA48 detector



NA48 – K_L weighting



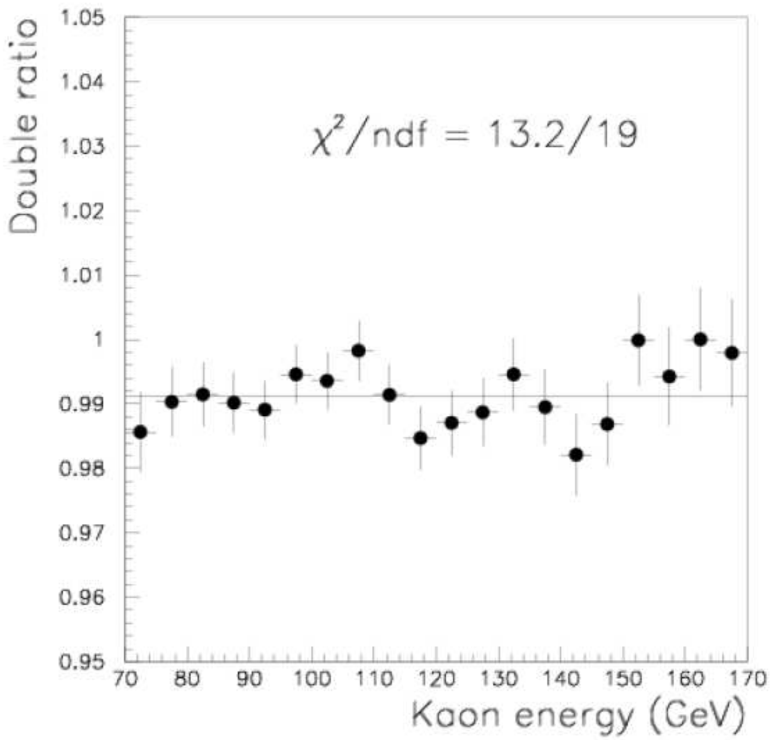


Increase in statistical error ~ 40%

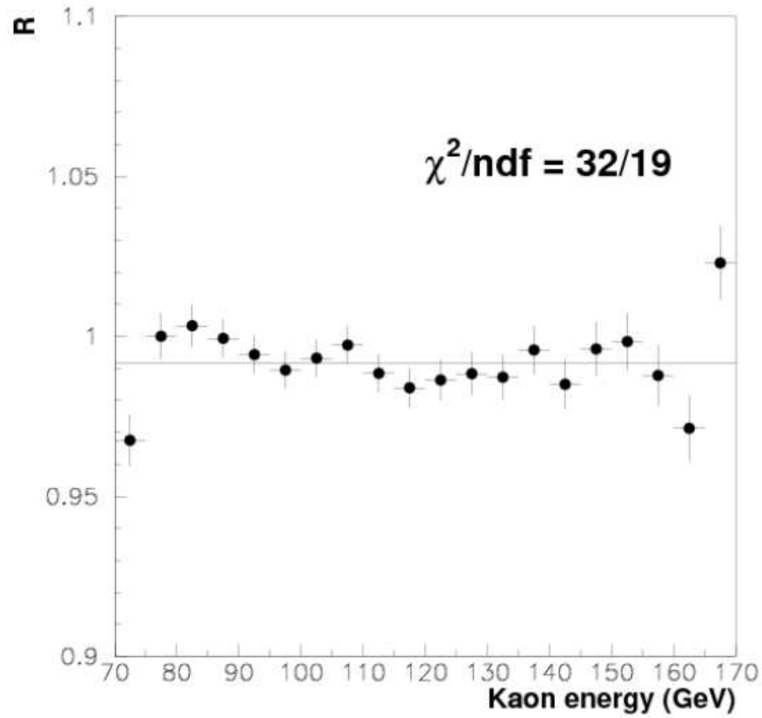
NA48 result

$\mathrm{Re}(\epsilon'/\epsilon) = (14.7 \pm 2.2) \times 10^{-4}$

1997-1999 data set

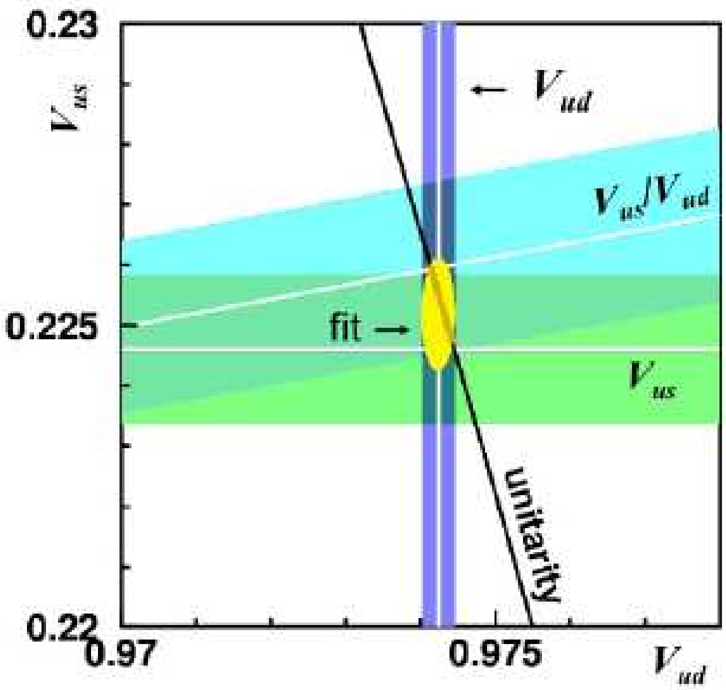


2001 data set



Current and future kaon physics

- Precision measurements of $|V_{us}|$



We use $f_+(0) = 0.9644(49)$, $f_K/f_\pi = 1.189(7)$

from RBC/UKQCD '07

M.Palutan at Kaon'09
see also
http://ific.uv.es/flavianet/

Now can fit:

- V_{us} from $Kl3$ ← KLOE, KTeV, BNL E685, ISTRA+, NA48
- V_{us}/V_{ud} from $K_{\mu 2}/\pi_{\mu 2}$
- V_{ud} from β decay ← New survey of superallowed $0^+ \rightarrow 0^+$ β decays J.Hardy & I.Towner, PRC 79 (2009) 055502

V_{ud} = 0.97424(22)
V_{us} = 0.2252(9)
χ^2/ndf = 0.52/1 (47%)

$$V_{ud}^2 + V_{us}^2 - 1 = -0.0001(6)$$

Future kaon physics

- "Holy grail" of kaon physics : rare decays $K \to \pi \nu \underline{\nu}$
  - Allow precise measurements of CKM matrix parameters
  - Sensitive to new physics effects
  - Extraordinarily challenging experimentally
  - $K^+ \to \pi^+ \nu \underline{\nu}$
    - Current measurement (1.7 ± 1.1) 10^{-10} BNL-E949 (expectation 0.9 10^{-10})
    - NA62 experiment (CERN) will observe O(100) SM events
  - $K^0 \to \pi^0 \nu \underline{\nu}$
    - Current UL < 6.7 10^{-8} KEK-E391 (expectation 2.5 10^{-11})
    - KOTO experiment (JPARC) will reach SM sensitivity

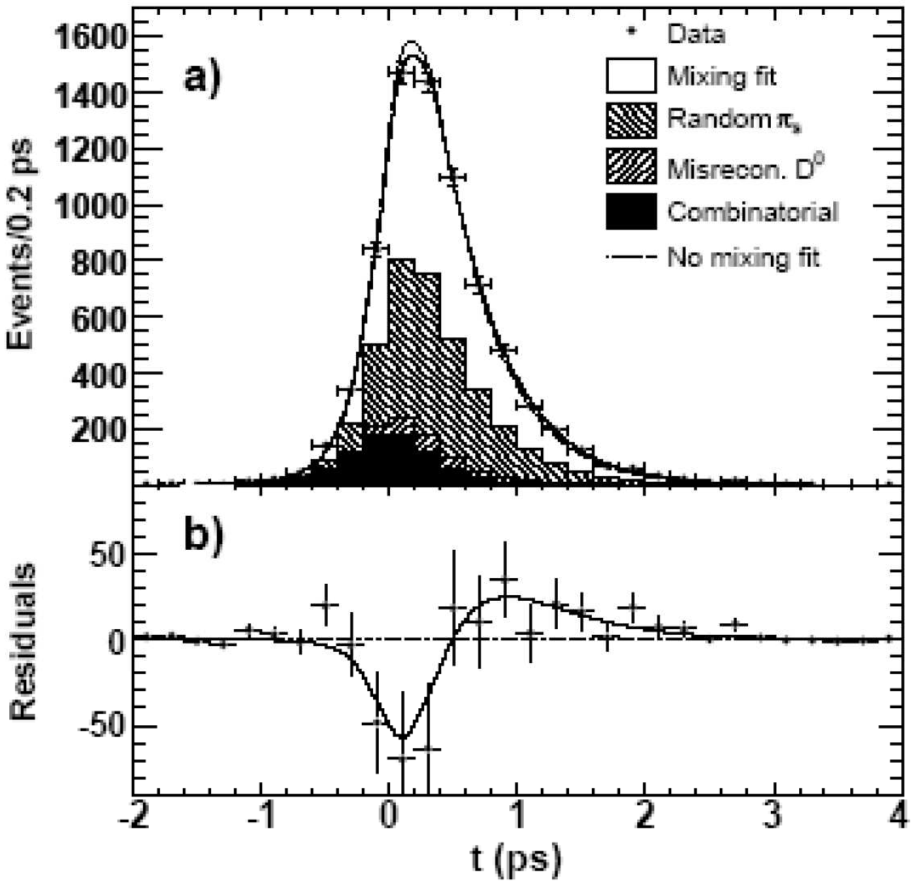
Rate prop. to. η
- Height of UT

Charm physics

- Mixing has down type quarks in the box
- Only place this can be studied as top quark does not hadronise
- Also, SM CPV very small
  - Any CPV is new physics
- B-factories have produced lots of Ds and observed D-mixing
  - May yet see CPV
  - Big target of flavour physics at the LHC

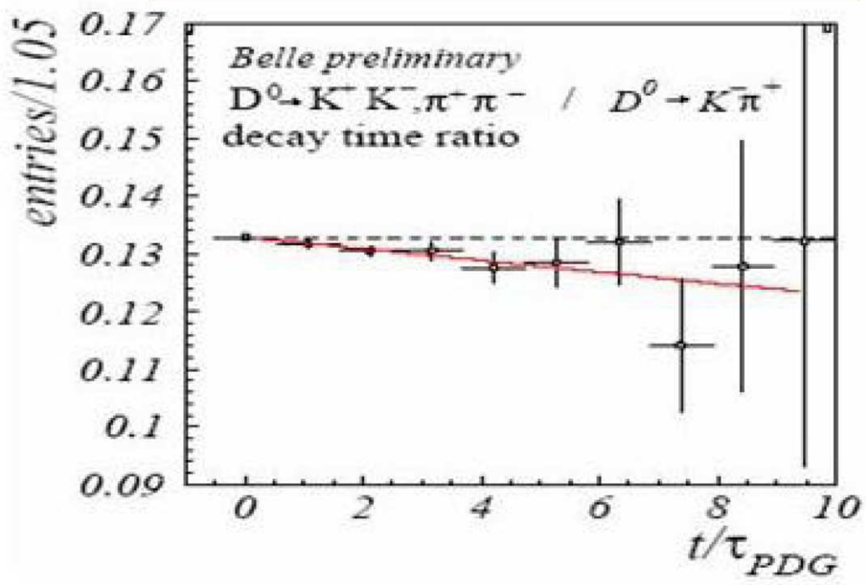
Measurements

BABAR $D^0 \to K\pi$
PRL 98 (2007) 211802



3.9σ significance

BELLE $D^0 \to KK$
PRL 98 (2007) 211803



3.2σ significance

Also:

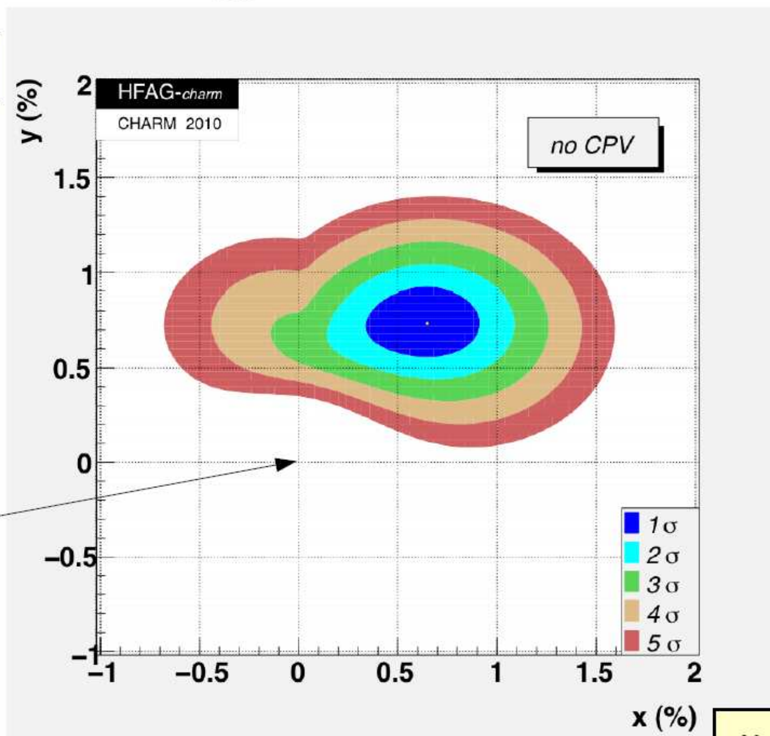
BELLE $D^0 \to K_S\pi\pi$ PRL 99 (2007) 131803

CDF $D^0 \to K\pi$ PRL100 (2008) 121802

and others ...

Combination of measurements

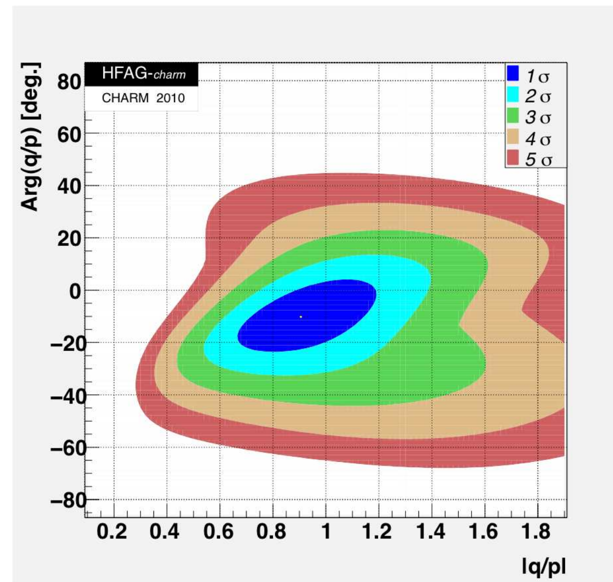
$y = \Delta\Gamma/2\Gamma$



No mixing

$x = \Delta m/\Gamma$

Still no single measurement with >5 sigma significance of mixing ...



Mixing confirmed but no CPV in mixing

Lecture 2 - summary

- CKM matrix and the KM mechanism
- Prediction of c and b their discovery
- Neutral meson mixing
- Different types of CPV
  - Kaon CPV - yes
  - Charm CPV - no