

Nuclear Level Density

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The nuclear level density $\rho(E)$

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Summary

The nuclear level density $\rho(E)$ is a characteristic property of every nucleus and it is defined as the number of levels per unit energy at a certain excitation energy. **Average level density $\rho(E) = dN/dE$**

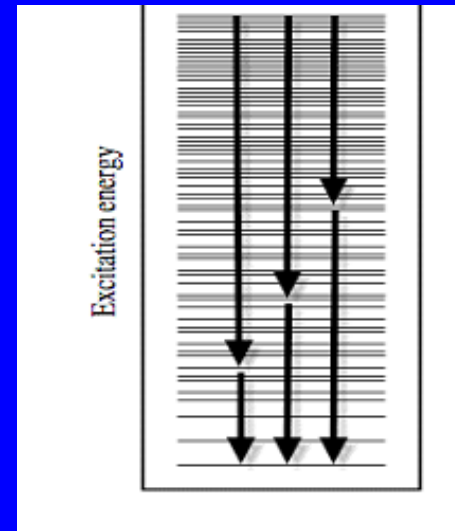
In other words it is the number of different ways in which individual nucleons can be placed in the various single particle orbitals such that the excitation energy lies in the range E to $E+dE$. It increases rapidly with excitation energy.

Importance of nuclear level density:

Estimation of reaction cross section
(Hauser-Feshbach theory)

$$d\sigma(E) \sim T(E) \rho(E^*) dE$$

Reactor design



To understand the microscopic structure of nucleus

Astrophysics (thermonuclear rates for nuclear synthesis)

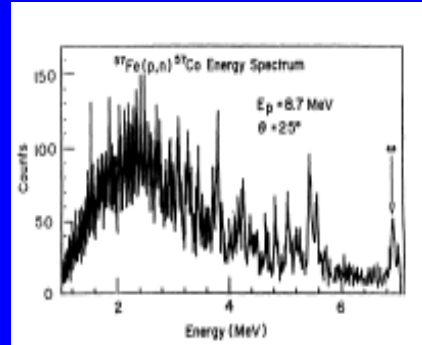
Experimental techniques to study nuclear level densities.

Counting of levels at low excitation energy.

$E^* < 4 - 5 \text{ MeV}$

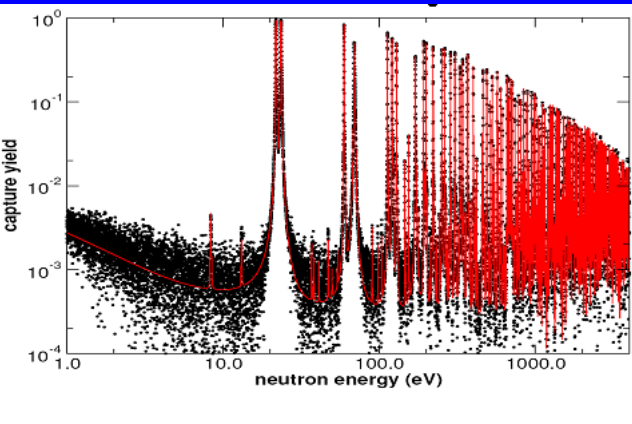
(p,n) (d,n) reactions.

Ohio group $\Delta E \sim 5\text{KeV}$, FP 29.6 m

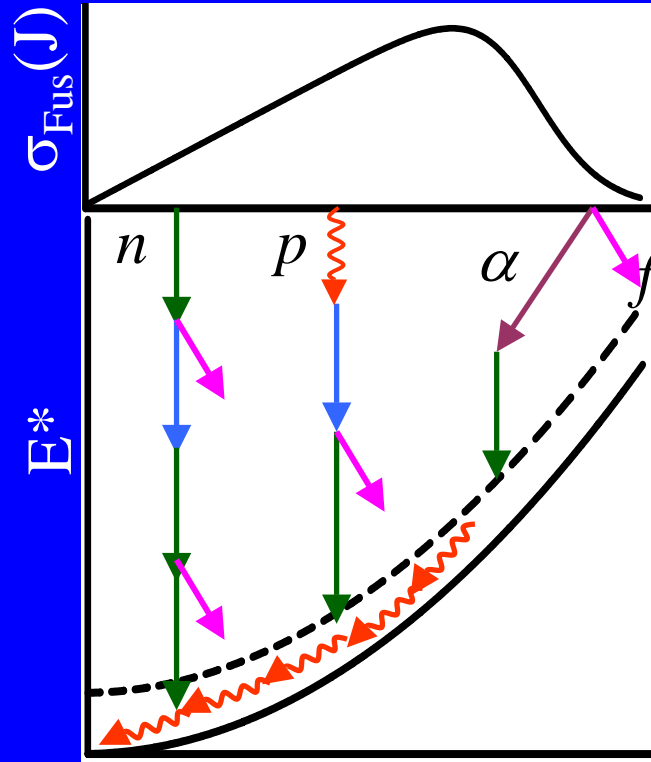


PRC 49, 750 (1994)

Counting of neutron resonances at $E^* \sim B_n$



Measurement at n_TOF facility ²³²Th (n, γ)



Evaporation from compound nucleus at high excitation energy, need to isolate from direct reaction channels. Average nuclear level density

Only method to study NLD at high E^* , J

Nuclear level density models: We do not have data for every nucleus in wide range of excitation energy and angular momentum so we have largely depends on models to predict the nuclear level density.

Fermi-gas model : consider non interacting Fermi-particles moving in a common potential

$$\rho(U) = \frac{\exp[2\sqrt{a(U - E_1)}]}{12\sqrt{2}\sigma a^{1/4} (U - E_1)^{5/4}}$$

Constant temperature model : considering classical ideal gas, used at low excitation energy. This model indicates an independence of temperature with respect to excitation energy.

$$\rho(U) = \frac{1}{T} \exp[(U - E_0)/T]$$

Gilbert Cameron model : Combination of constant temperature and Fermi gas model

$$\rho(U) = \frac{1}{T} \exp[(U - E_0)/T] \quad U \leq B_n$$

$$\rho(U) = \frac{\exp[2\sqrt{a(U - E_1)}]}{12\sqrt{2}\sigma a^{1/4} (U - E_1)^{5/4}} \quad U \geq B_n$$

Microscopical models : calculations based on different representation of nuclear potentials plus collective effects.
Hartree Fock BCS model

Widely used phenomenological nuclear level density expression

$$\rho(E^*, J) = \frac{(2J+1)}{12} \frac{\hbar^2}{2I_{\text{eff}}} \sqrt{a} \frac{\exp(2\sqrt{aU})}{U^2}$$

E_{rot} is the rotation energy

I = Moment of inertia

$$U = E^* - E_{\text{rot}} - \Delta P$$

$$a = \frac{A}{k}$$

ΔP = Pairing term

Collective excitation and its contribution to nuclear level density

For ground state deformed nucleus, there is a collective enhancement of NLD, which was formulated by Ignatyuk.

$$\rho(E^*, J) = \rho_{\text{int}}(E^*, J) K_{\text{coll}}(E^*)$$

$$K_{\text{coll}}(E^*) = K_{\text{vib}}(E^*) K_{\text{rot}}(E^*)$$



Bjornholm, Bohr and Mottleson have suggested a critical temperature, T_c beyond which the collective enhancement in NLD is expected to fade out

$$T_c = \hbar \omega_0 \beta_2 = 40 A^{-1/3} \beta_2 \text{ MeV}$$

Dependence of level density parameter

$$\rho(E_x, J) = \frac{(2J+1) \hbar^2}{12 \cdot 2I} \sqrt{a} \frac{\exp(2\sqrt{aU})}{U^2}$$

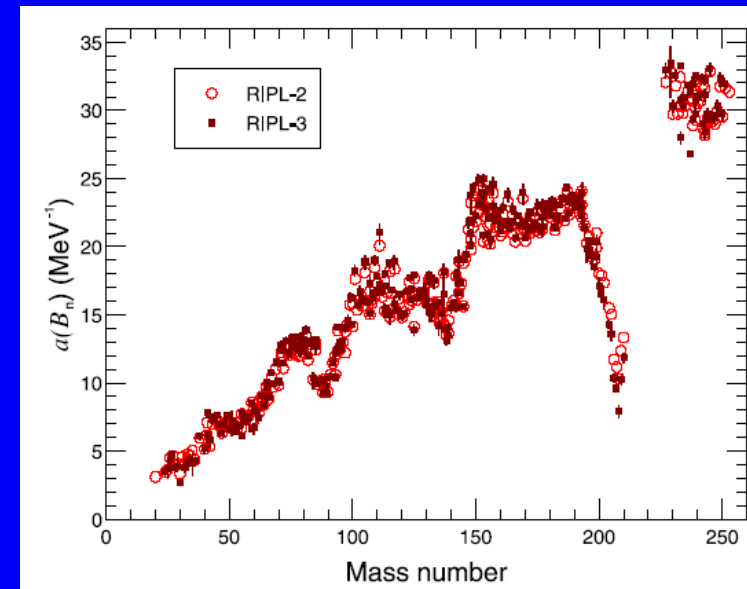
a is called level density parameter

1. Excitation Energy
2. Mass
3. Angular momentum

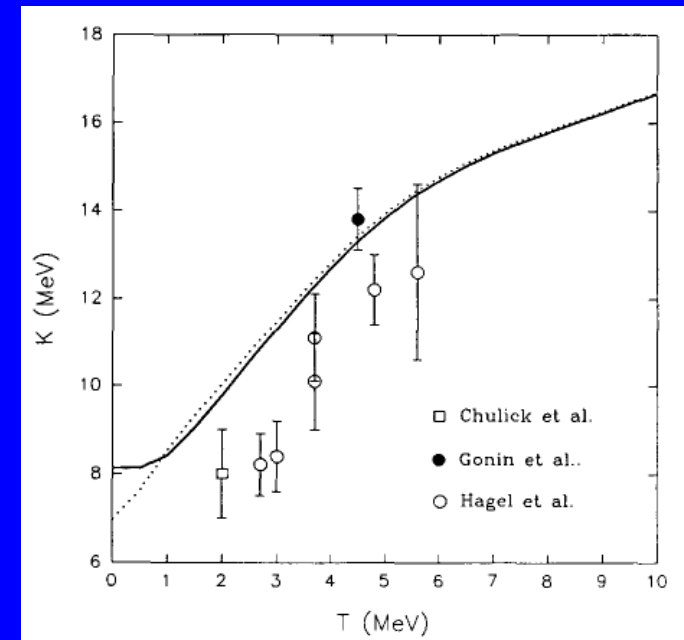
$$a = \tilde{a} \left[1 - \frac{\Delta S}{U} \{ 1 - \exp(-\gamma U) \} \right]$$

$$\tilde{a} = \frac{A}{k} \quad \gamma^{-1} = \frac{0.4A^{4/3}}{\tilde{a}}$$

ΔS is shell correction, γ shell damping factor



Nuclear Data Sheets 110 (2009) 3107



Variation of level density with angular momentum

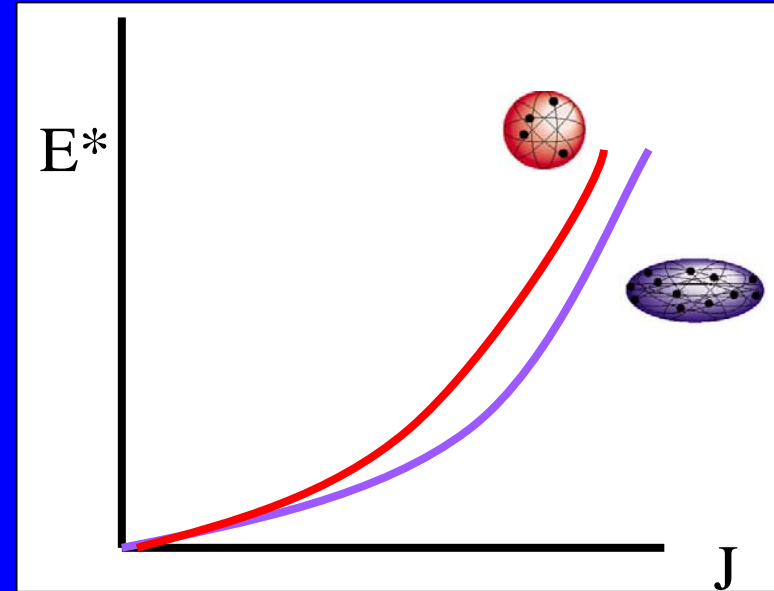
Approach 1 : used at low E^* and J .
used in neutron resonance
measurements.

$$\rho \propto \exp \sqrt{2aE^*} \exp\left[\frac{(-J + 1/2)}{2\sigma^2}\right]$$

$$\sigma^2 = \frac{I_{rig} T}{\hbar^2}$$

σ Is called spin cut off factor.

Approach 2 : Angular momentum dependent
deformation. Used in high E^* and J , but mostly
tested in inclusive spectra



$$\rho \propto \frac{\exp \sqrt{2a(E^* - E_{rot})}}{(E^* - E_{rot})^2}$$

$$E_{rot} = \frac{\hbar^2}{2I_0(1 + \delta_1 J^2 + \delta_2 J^4)} J(J + 1)$$

$E^* \gg E_{rot}$ two prescriptions become equivalent.

Recent Measurements:

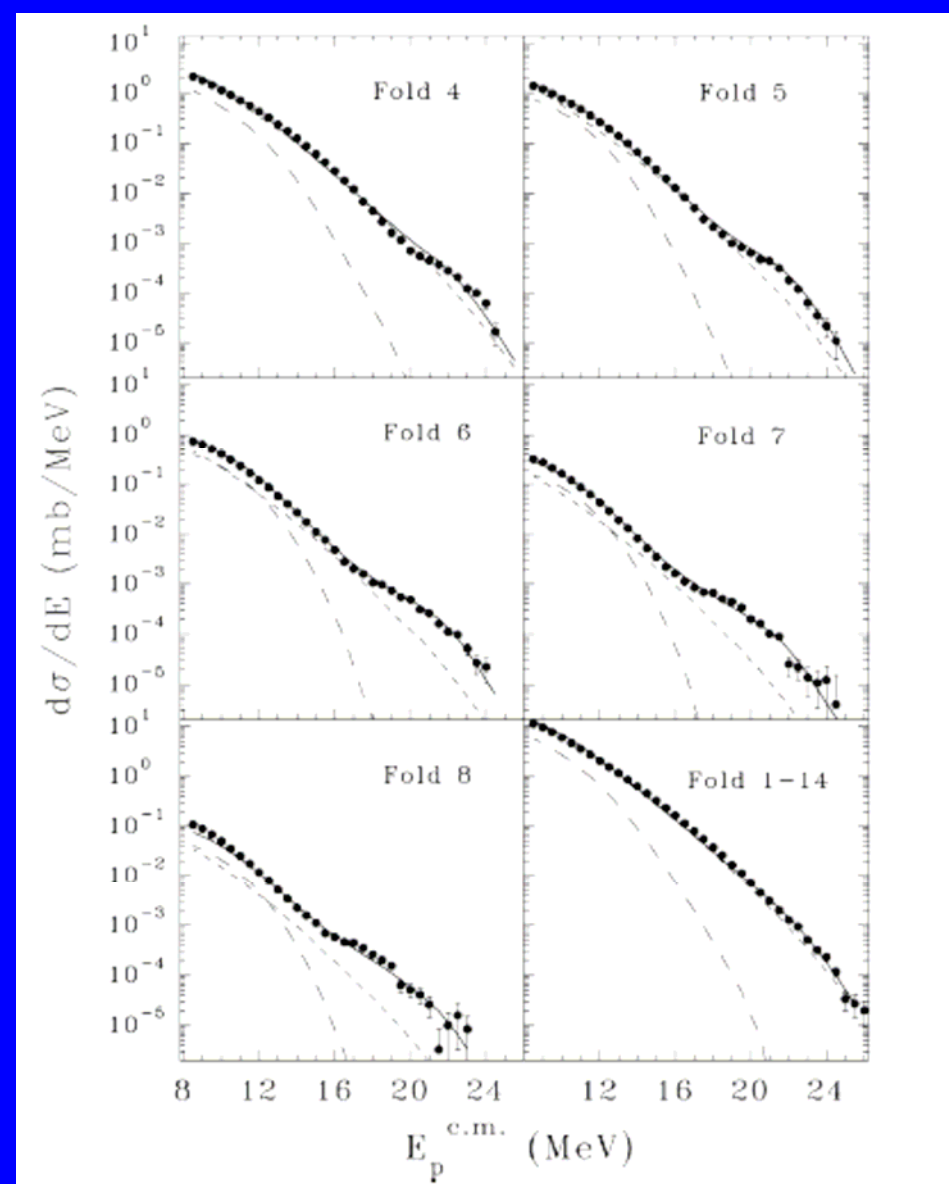
${}^{92}\text{Zr}({}^{64}\text{Ni}, 1n){}^{155}\text{Er}$

$$a = A/(8.8 \pm 1.3) \text{MeV}^{-1}$$

$$\langle J \rangle = 52 \hbar$$

$$E^* = 30 - 36 \text{MeV}$$

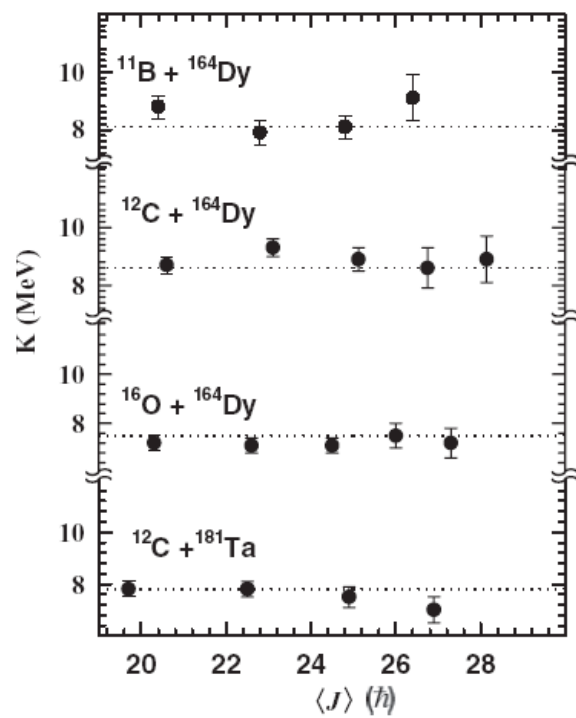
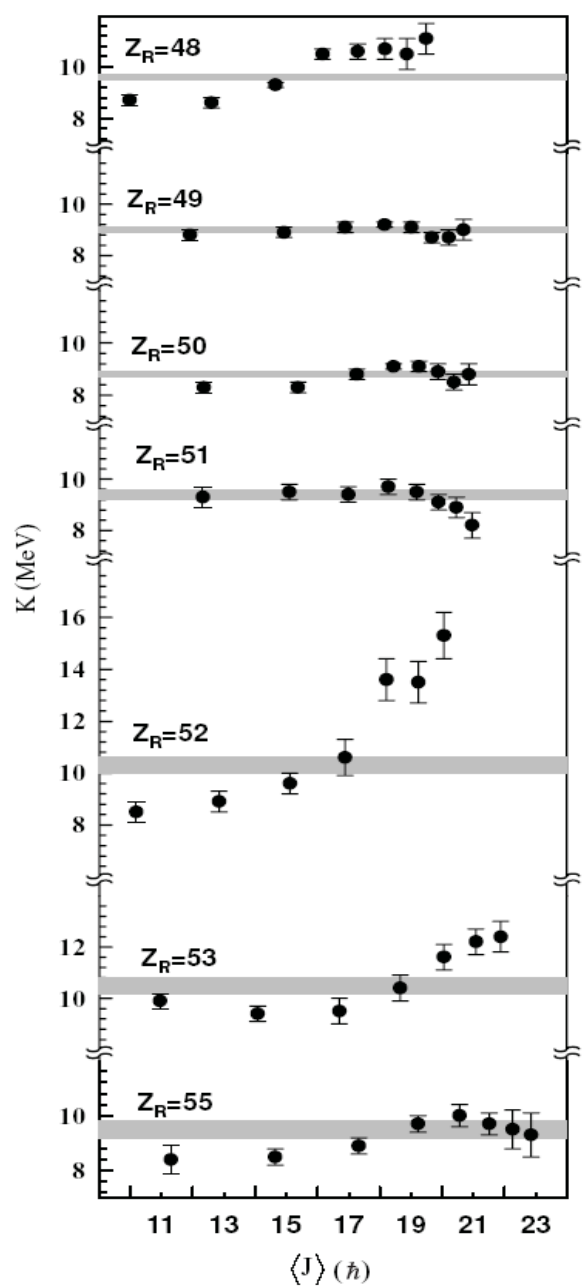
Darmstadt-Heidelberg
Crystal ball. S. Henss et.
al. **PRL60, 11 (1988)**



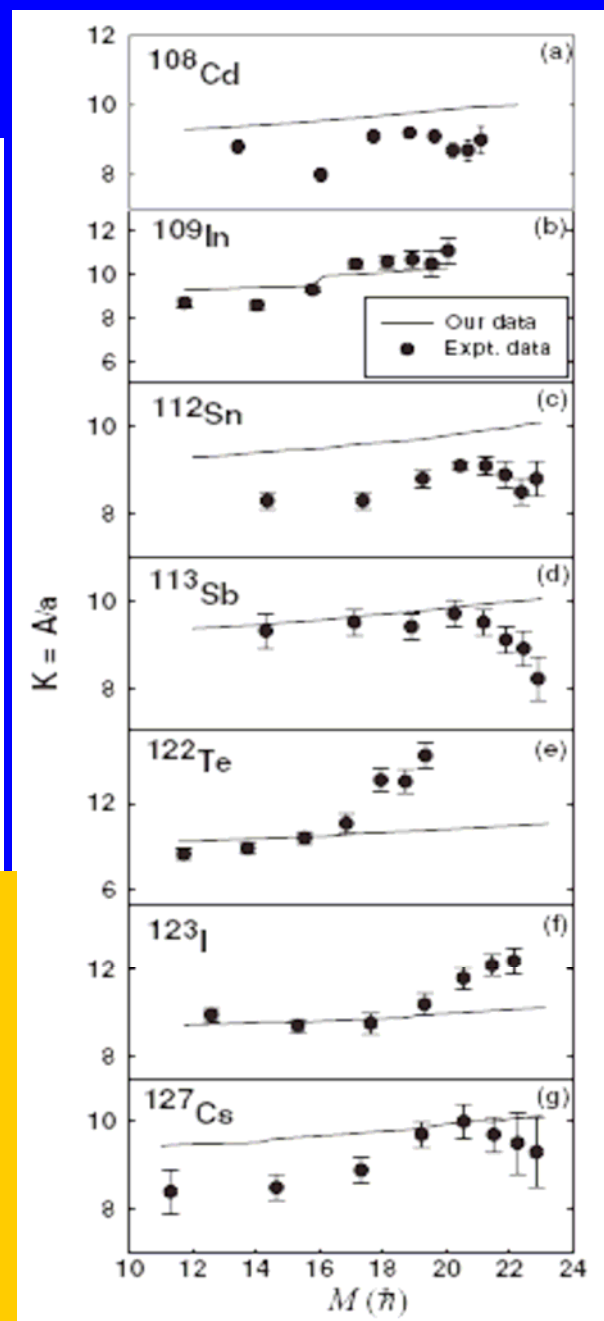
Mitra et. al. Nucl Phys A 707 (2002) 343
 ${}^{12}\text{C} + {}^{93}\text{Nb}$. Proton spectra, unusual structure
seen when gated with gamma fold

Extracted from alpha spectra

Gupta et. al. PRC 80, 054611 (2009).
Extracted from alpha spectra



Although there have been couple of experiments to study level density and its variation with angular momentum. But no consistent picture evolved out these experiments



Plan of Study:

System:

Beam Energy = 30 - 42 MeV

$${}^4\text{He} + {}^{58}\text{Ni} \rightarrow {}^{62}\text{Zn} \quad \beta_2({}^{62}\text{Zn})= 0.209, \beta_2({}^{61}\text{Zn})= 0.208$$

$${}^4\text{He} + {}^{93}\text{Nb} \rightarrow {}^{97}\text{Tc} \quad \beta_2({}^{97}\text{Tc})= 0.134, \beta_2({}^{96}\text{Tc})= 0.053$$

$${}^4\text{He} + {}^{115}\text{In} \rightarrow {}^{119}\text{Sb} \quad \beta_2({}^{119}\text{Sb})= -0.122, \beta_2({}^{118}\text{Sb})= -0.138$$

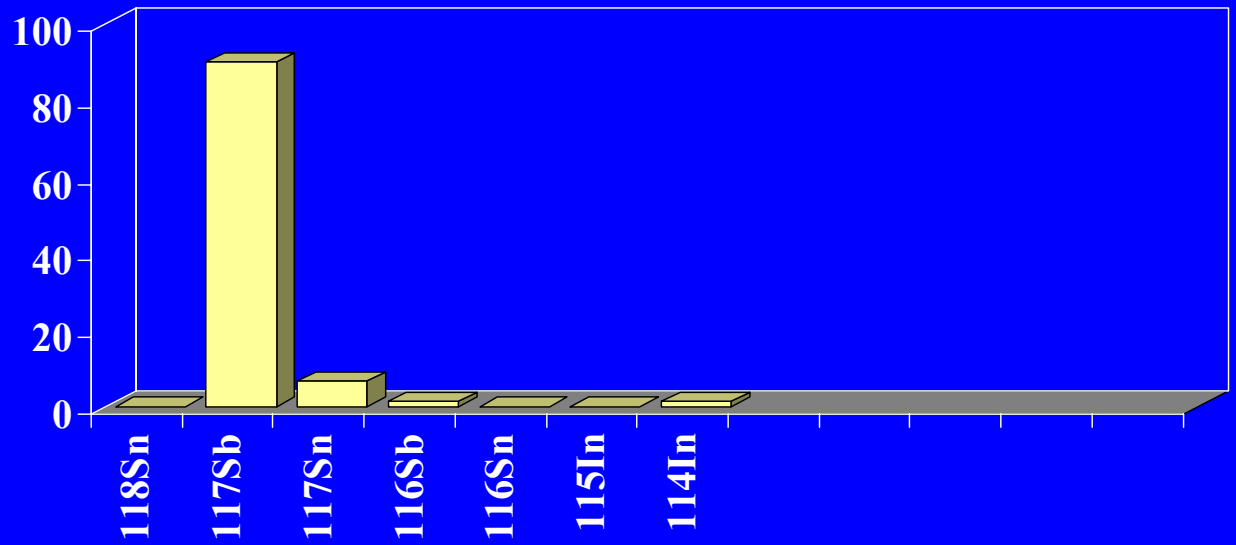
$${}^4\text{He} + {}^{165}\text{Ho} \rightarrow {}^{169}\text{Tm} \quad \beta_2({}^{169}\text{Tm})= 0.295, \beta_2({}^{168}\text{Tm})= 0.294$$

$${}^4\text{He} + {}^{181}\text{Ta} \rightarrow {}^{185}\text{Re} \quad \beta_2({}^{185}\text{Re})= 0.221, \beta_2({}^{184}\text{Re})= 0.230$$

$${}^4\text{He} + {}^{197}\text{Au} \rightarrow {}^{201}\text{Tl} \quad \beta_2({}^{201}\text{Tl})= -0.044, \beta_2({}^{200}\text{Tl})= -0.044$$

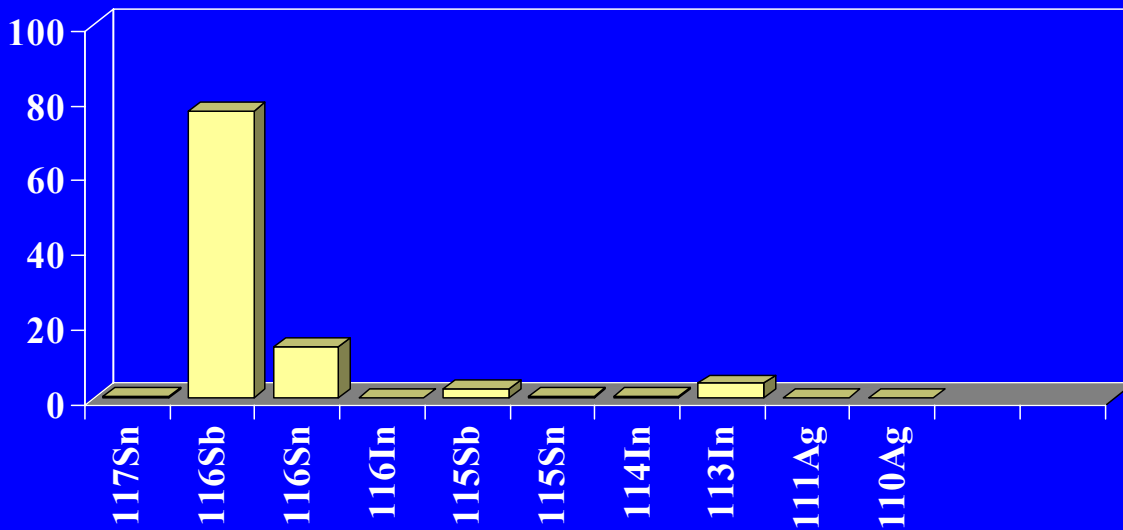
${}^4\text{He} + {}^{115}\text{In}$ @
 $E_{\text{lab}} = 30 \text{ MeV}$

Probability of popu
(%)



Predominantly only one residue, populated through 2n and 3n channel at 30, 42 MeV respectively.

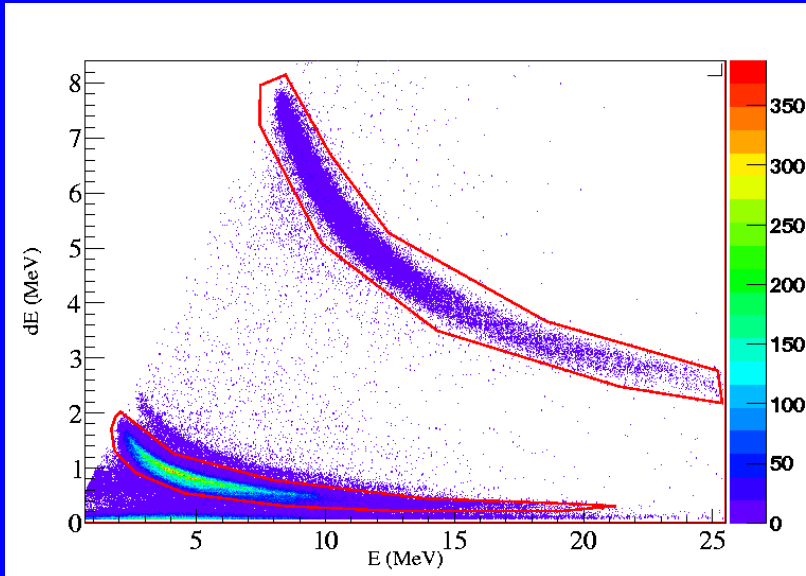
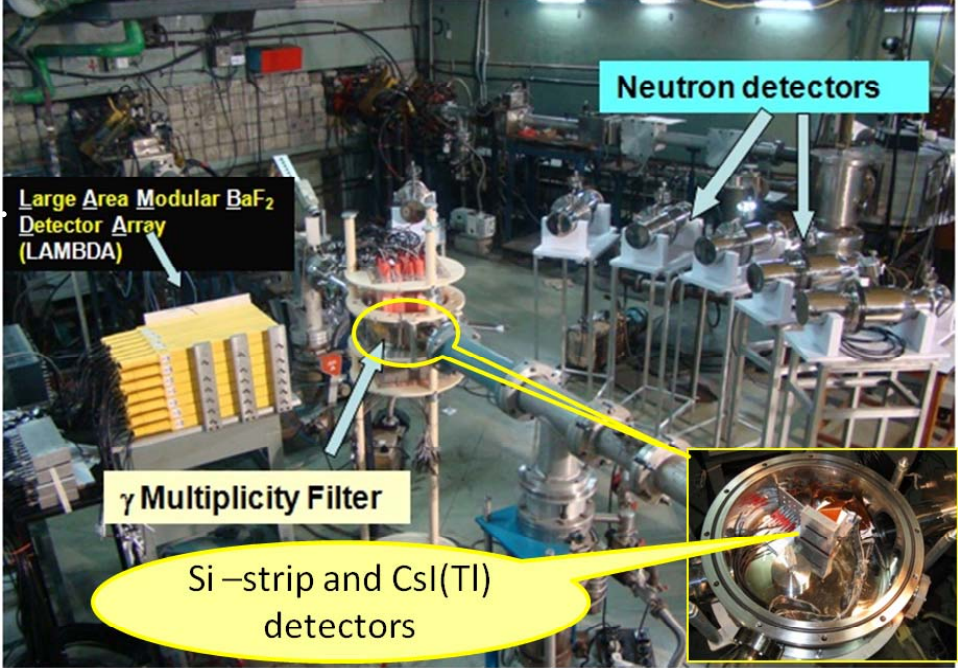
Probability of popu
(%)



${}^4\text{He} + {}^{115}\text{In}$ @
 $E_{\text{lab}} = 42 \text{ MeV}$

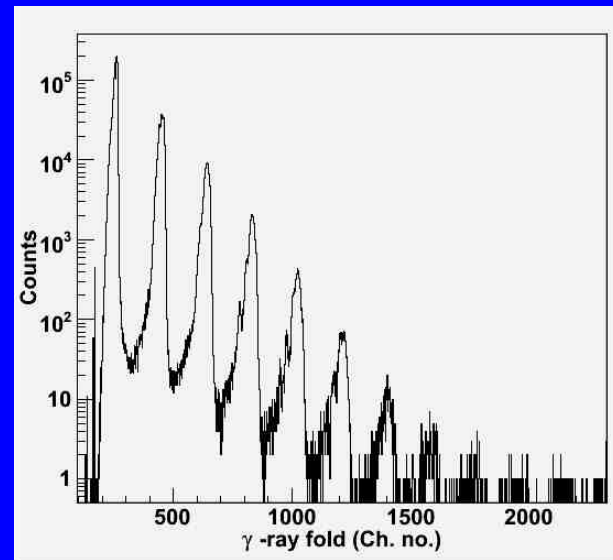
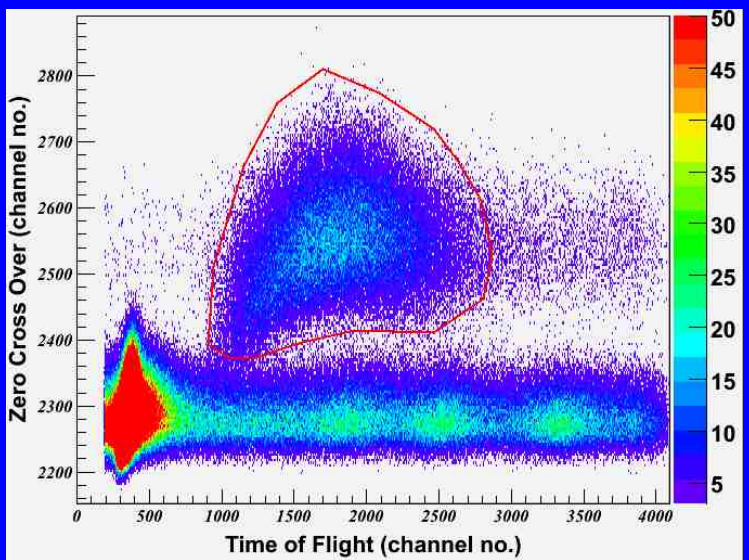
Experimental Setup:

Charged particle: 50 μ Si -SSSD (ΔE) +
500 μ Si -DSSD ($\Delta E/E$) + 4cm CsI(Tl) (E)



Gamma: 50 BaF₂ detector
(3.5 x 3.5 x 5 cm³)

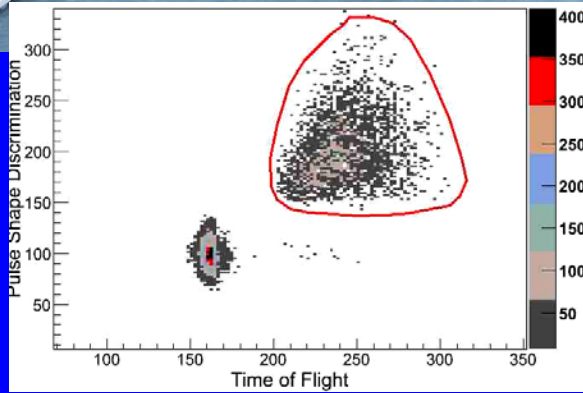
Neutron: 7 liquid
scintillator (BC501A)
detector 5in x 5in



Neutron Detectors @ VECC

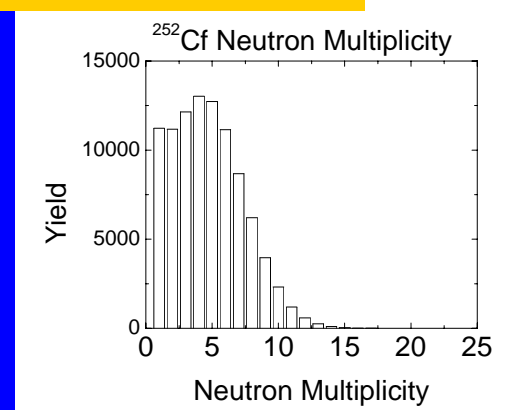
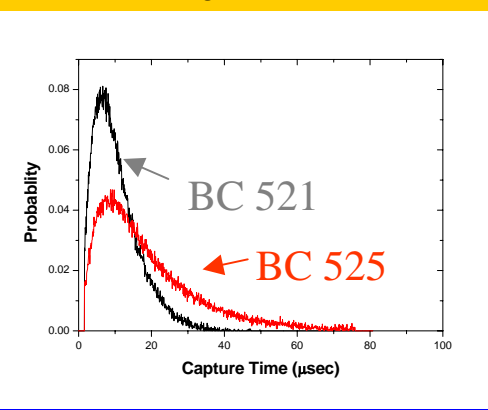
Prototype for
MONSTER

- TOF type neutron detector



Liquid Scintillator :
BC501A

K. Banerjee et. al. NIM A 608 (2009) 440

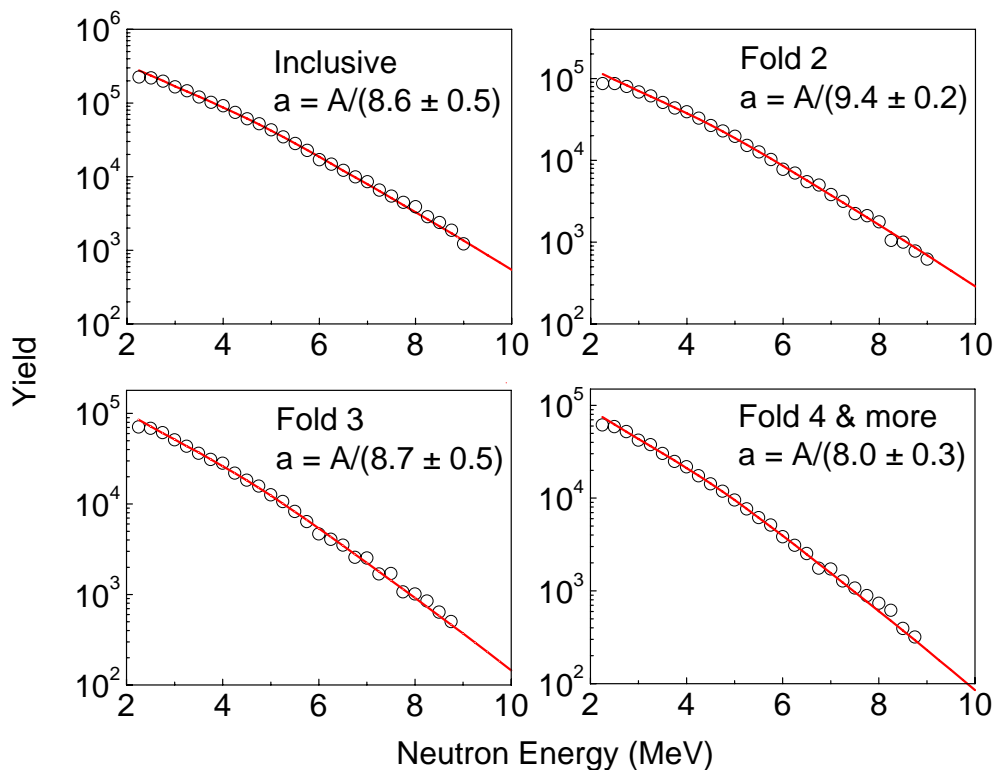


Neutron Multiplicity detector

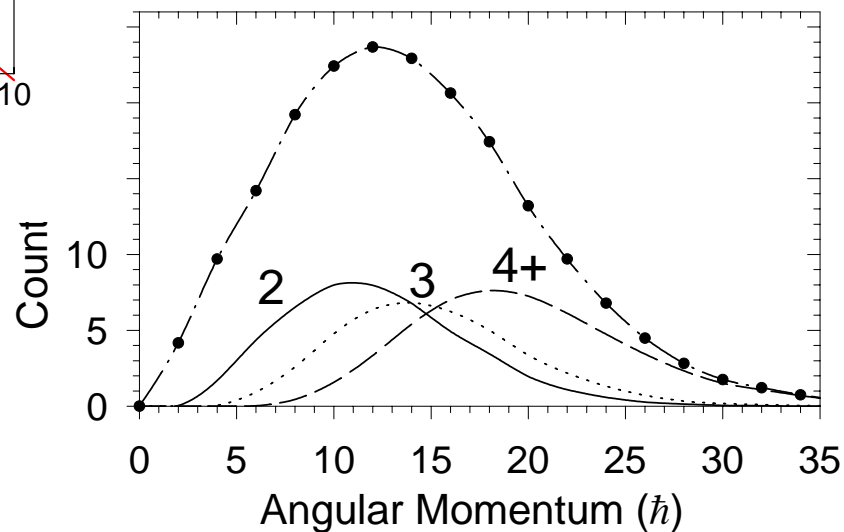
Technical Details
Liquid Scintillator = 0.5% Gd loaded liquid
Scintillator BC521
5" photo-multiplier tubes five per section

K. Banerjee et. al. NIM A 580 (2007) 1383

Experimental neutron energy spectrum with CASCADE prediction at $\theta_{\text{lab}} = 150^\circ$

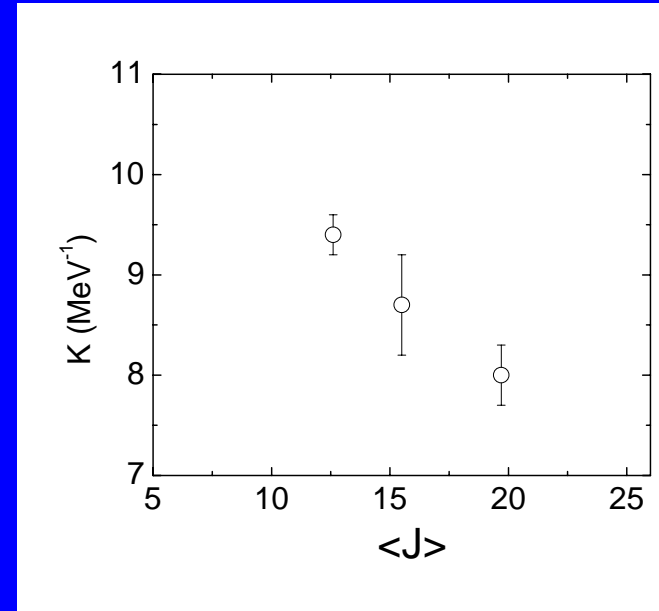


The sampling of the total angular momentum distribution by gating different folds observed in the fold distribution of the multiplicity filter.



$^4\text{He} + ^{115}\text{In}$

Beam Energy (MeV)	Fold	$\langle J \rangle$ h	K (MeV)
30	All	15.0 ± 5.9	8.6 ± 0.5
30	2	12.6 ± 4.9	9.4 ± 0.2
30	3	15.5 ± 5.2	8.7 ± 0.5
30	4	19.7 ± 6.2	8.0 ± 0.3
42	All	16.9 ± 6.4	9.8 ± 0.2
42	2	14.1 ± 5.2	11.1 ± 0.3
42	3	16.8 ± 5.4	9.5 ± 0.5
42	4	21.1 ± 6.8	8.9 ± 0.3



$$U = E^* - E_{rot} - S_n - \langle E_n \rangle$$

$$U = aT^2$$

In the last stage of decay cascade $T < T_c$, so the possibility of collective enhancement exists. However the β_2 value ($= -0.122$) is quite small and the empirical relation available for collective enhancement is independent of J. So the above trend can not be explained quantitatively.

Summary:

Particle evaporation spectra in coincidence with γ - ray multiplicity have been measured in ${}^4\text{He} + {}^{58}\text{Ni}$, ${}^{93}\text{Nb}$, ${}^{115}\text{In}$. Particle spectra have also been estimated through statistical model calculation, considering level density from independent particle Fermi Gas model.

Probable correction in the nuclear level density has been incorporated such as shell effect, rotation induced deformation. It is observed from the data k value decreases with the increase in angular momentum.

We also consider the effect of collective enhance, however the present observation couldn't be explained quantitatively. Microscopic calculation using all possible effect like shell, pairing and collectivity may be interesting to study the above trend.

Thank You