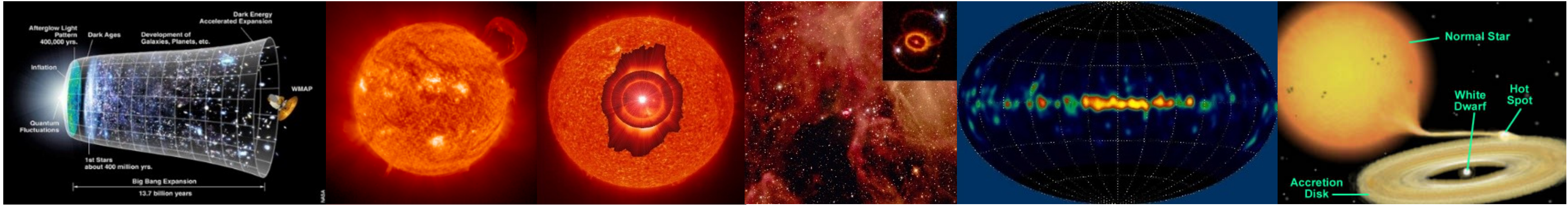


Stellar Reaction Rates:
Non-resonant (direct) & resonant rates

Faïrouz Hammache (IPN-Orsay)
hammache@ipno.in2p3.fr

Plan of lecture II

- **Stellar reaction rates**

  - for direct neutron capture

  - for direct charged induced reactions

  - for direct charged particle reactions

  - for resonant reactions

- **Additional effects on reaction rate in stellar environment**

Thermonuclear Reaction Rates: Reminder

Total reaction rate $$R_{aX} = \frac{1}{1+\delta_{aX}} N_a N_X \langle \sigma v \rangle_{aX}$$ reactions cm^{-3} s^{-1}

N_i = number density

Energy production rate: $\varepsilon_{aX} = R_{aX}\, Q_{aX}$

Mean lifetime of nuclei X against destruction by nuclei a

$$\tau_a(X) = \frac{1}{N_a < \sigma v >}$$

energy production as star evolves ⟸ $\langle \sigma v \rangle$ = KEYquantity ⟹ change in abundance of nuclei X

to be determined from experiments and/or theoretical considerations

as star evolves, T changes $\Rightarrow$ evaluate $\langle \sigma v \rangle$ for each temperature

What is needed to determine thermonuclear reaction rates?

- The cross section as a function of energy (velocity)

The stellar reaction rate can then be calculated by integrating over the Maxwell Boltzmann distribution.

$$<\sigma v>_{aX} = \left(\frac{8}{\pi\mu_{aX}}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty \sigma(E) \exp\left(-\frac{E}{kT}\right) E\, dE$$

The cross section depends **sensitively** on the reaction mechanism and the properties of the nuclei involved. It can vary by many (tens) orders of magnitude. It can either be measured experimentally or calculated. Both are difficult.

Typical energies for astrophysical reactions are of the order of kT

Sun T ~ 15 MK

Si burning in a massive star: T ~ 1 GK

There is no nuclear theory that can predict the relevant properties of nuclei accurately enough. In practice, a combination of experiments and theory is needed

Thermonuclear Reaction Rates: Non-resonant (Direct) reactions

$a + A \rightarrow B + \gamma$

Direct transition from initial state |a+A> to final state <f| (some state in B)

$$\sigma \propto \pi \lambda_a^2 \cdot \left| \langle f | H | a + A \rangle \right|^2 \cdot P_l(E)$$

geometrical factor (deBroglie wave length of projectile - “size” of projectile)

Interaction matrix element

Penetrability: probability for projectile to reach the target nucleus for interaction.
Depends on projectile Angular momentum l and Energy E

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

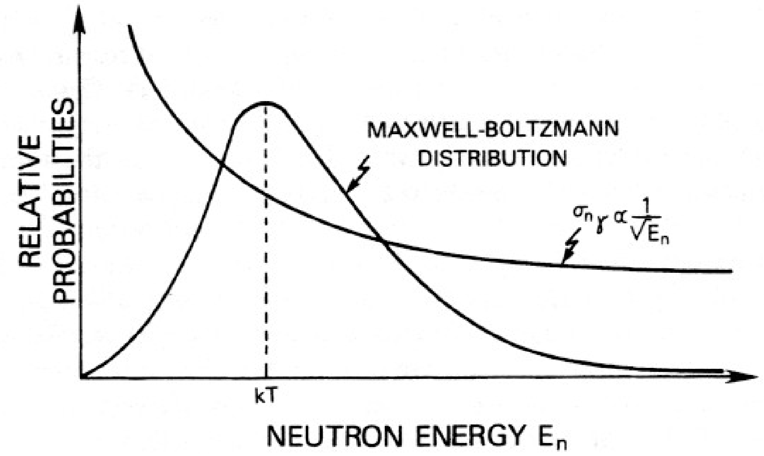
$$\sigma \propto \frac{1}{E} \cdot \left| \langle f | H | a + A \rangle \right|^2 \cdot P_l(E)$$

Stellar Reaction Rates for s-wave direct neutron capture

- As the cross section for s-wave (l = 0) neutron capture can be written

$$\sigma \propto \frac{1}{v} \longrightarrow \sigma v = \text{const} = <\sigma v>$$

the most probable capture energy is ~kT



Thermal neutron cross section:

Many neutron capture cross sections have been measured at reactors using a "thermal" (room temperature) neutron energy distribution at **T= 293.6 K (20 ^{0}C), kT=25.3 meV**

The measured cross section is an average over the neutron **flux** spectrum Φ(E) used:

$$<\sigma>=\frac{\int \sigma(E)\Phi(E)dE}{\int \Phi(E)dE} \quad \text{(all Lab energies)}$$

For a thermal spectrum $\Phi(E) = E\,e^{-\frac{E}{kT}}$ so $<\sigma>_{th} = \frac{\int \sigma(E) E\, e^{-\frac{E}{kT}} dE}{\int E\, e^{-\frac{E}{kT}} dE}$

Why is a flux of thermalized particles distributed as $\Phi(E) = E\,\mathrm{e}^{-\frac{E}{kT}}$ **?**

The number density n of particles in the beam is Maxwell Boltzmann (MB) distributed

$$\frac{dn}{dE} \propto \sqrt{E}\,\mathrm{e}^{-E/kT}$$

BUT the flux is the number of neutrons hitting the target per second and area. This is a current density **j = n * v**

$$\Rightarrow \qquad \frac{dj}{dE} = \mathrm{v}\frac{dn}{dE} \propto E\,\mathrm{e}^{-E/kT}$$

The cross section is averaged over the neutron flux

Same situation than in the center of a star. The number density of particles is M.B. distributed, but the number of particles passing through an area per second is $\propto E\,\mathrm{e}^{-E/kT}$ distributed & so is the stellar reaction rate

Stellar Reaction Rates for s-wave direct neutron capture

With these definitions one can show that the measured averaged cross section and the stellar reaction rate are related simply by

$$<\sigma v>=\frac{2}{\sqrt{\pi}} v_T <\sigma>_{th} = v_T \sigma_{th}$$

with $v_T = \sqrt{\frac{2kT}{\mu}}$ (most frequent velocity, corresponding to E_{CM}=kT)

for reactor neutrons (thermal neutrons) $v_T = 2.2\times 10^5$ cm/s

and $$\sigma_{th} = \frac{2}{\sqrt{\pi}} <\sigma>_{th} = \frac{<\sigma v>}{v_T}$$

that's usually tabulated as "thermal cross section"

For s-wave neutron capture one can relate the thermal cross section to the cross section value at the energy kT

$$\sigma_{th} = \sigma(kT)$$

Stellar Reaction Rates for direct neutron capture with higher l

For neutron capture, the only barrier is the angular momentum barrier

The penetrability scales with

$$P_l(E) \propto E^{1/2+l}$$

and therefore the cross section:

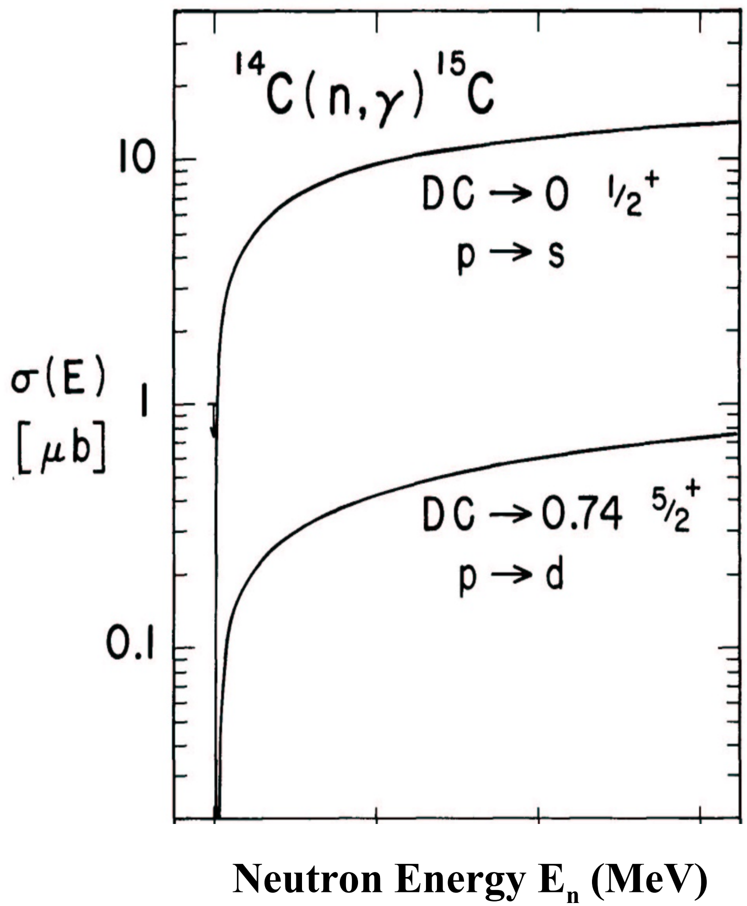
$$\sigma \propto E^{l-1/2}$$

for **l>0** → σ↘ with E↘ (centrifugal barrier)

•s-wave capture dominates at low energies, in particular at thermal energies.

•Higher l-capture usually plays only a role at higher energies.

p-wave capture in $^{14}C(n,\gamma)^{15}C$



(from Wiescher et al. ApJ 363 (1990) 340)

Note: sometimes s-wave is strongly suppressed because of angular momentum selection rules (as it would then require higher gamma-ray multipolarities)

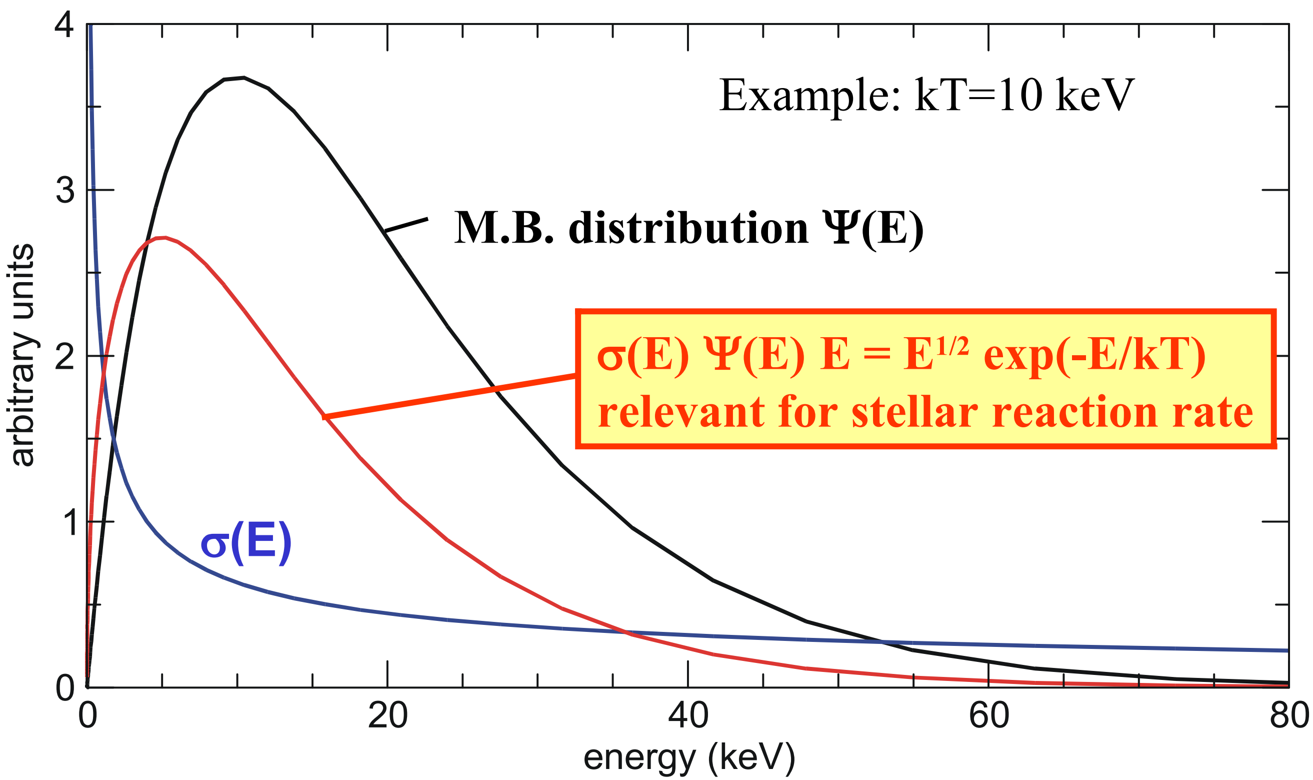
Stellar Reaction Rates for direct neutron capture

The energy range the cross section needs to be known to determine the stellar reaction rate for n-capture ?

This depends on cross section shape and temperature:

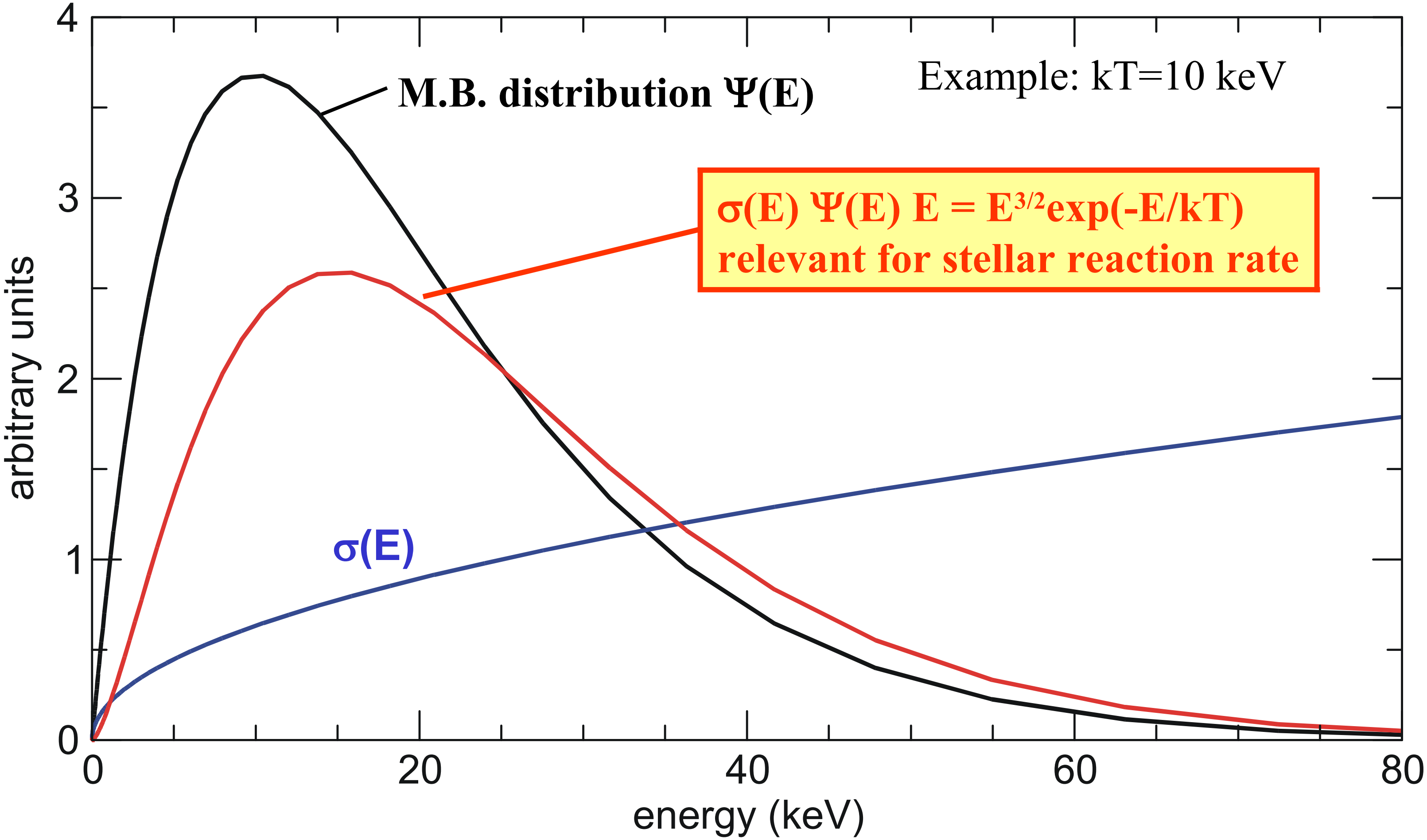
$$<\sigma v>= \int \sigma(v)\Phi(v)v dv = \int \sigma(E)\Psi(E)E dE$$

<u>s-wave n-capture:</u> of the order of KT (somewhat lower than MB distribution)



Stellar Reaction Rates for direct neutron capture

<u>p-wave n-capture:</u> of the order of KT (close to MB distribution)



Stellar Reaction Rates for direct neutron capture

The concept of the astrophysical S-factor (for n-capture)

recall: $$\sigma \propto \underbrace{\frac{1}{E} \cdot P_l(E)}_{\text{"trivial" strong energy dependence}} \cdot \underbrace{\left| \langle f | H | a + A \rangle \right|^2}_{\text{"real" nuclear physics}}$$

"trivial" strong energy dependence

"real" nuclear physics weak energy dependence (for direct reactions !)

<u>S-factor concept: write cross section as</u>

strong "trivial" energy dependence × weakly energy dependent S-factor

The S-factor can be

- easier graphed
- easier fitted and tabulated
- easier extrapolated
- and contains all the essential nuclear physics

Stellar Reaction Rates for direct neutron capture

For neutron capture with **strong s-wave dominance** with corrections.

$$\sigma = \frac{1}{v} S(E)$$

expand S(E) around E=0 as powers of $\sqrt{E}$

$$\sigma \approx \frac{1}{v}\left(S(0) + \dot{S}(0) E^{1/2} + \frac{1}{2}\ddot{S}(0) E \right)$$

$$\sigma \approx \frac{S(0)}{v}\left(1 + \frac{\dot{S}(0)}{S(0)} E^{1/2} + \frac{1}{2}\frac{\ddot{S}(0)}{S(0)} E \right)$$

with • denoting $\frac{\partial}{\partial \sqrt{E}}$

in practice, these are tabulated fitted parameters

typical S(E) units with this definition: barn $\mathrm{MeV}^{1/2}$

Stellar Reaction Rates for direct neutron capture

Astrophysical reaction rate

$$< \sigma v > \approx S(0) + \dot{S}(0)\frac{2}{\sqrt{\pi}}(kT)^{1/2} + \ddot{S}(0)\frac{3}{4}kT$$

For pure s-wave capture

$$< \sigma v >= S(0)$$

for pure s-wave capture the S-factor is entirely determined by the thermal cross section measured with room temperature reactor neutrons:

using $< \sigma \, v >= \sigma_{th} v_T = S(0)$ one finds

$v_T=2.2\times10^5$cm/s

$$S(0) = 2.20 \cdot 10^{-19} \sigma_{th}[\text{barn}] \quad \text{cm}^3 / s$$

Stellar Reaction Rates for direct neutron capture

<u>For neutron capture that is dominated by p-wave</u>: one can define a p-wave S-factor:

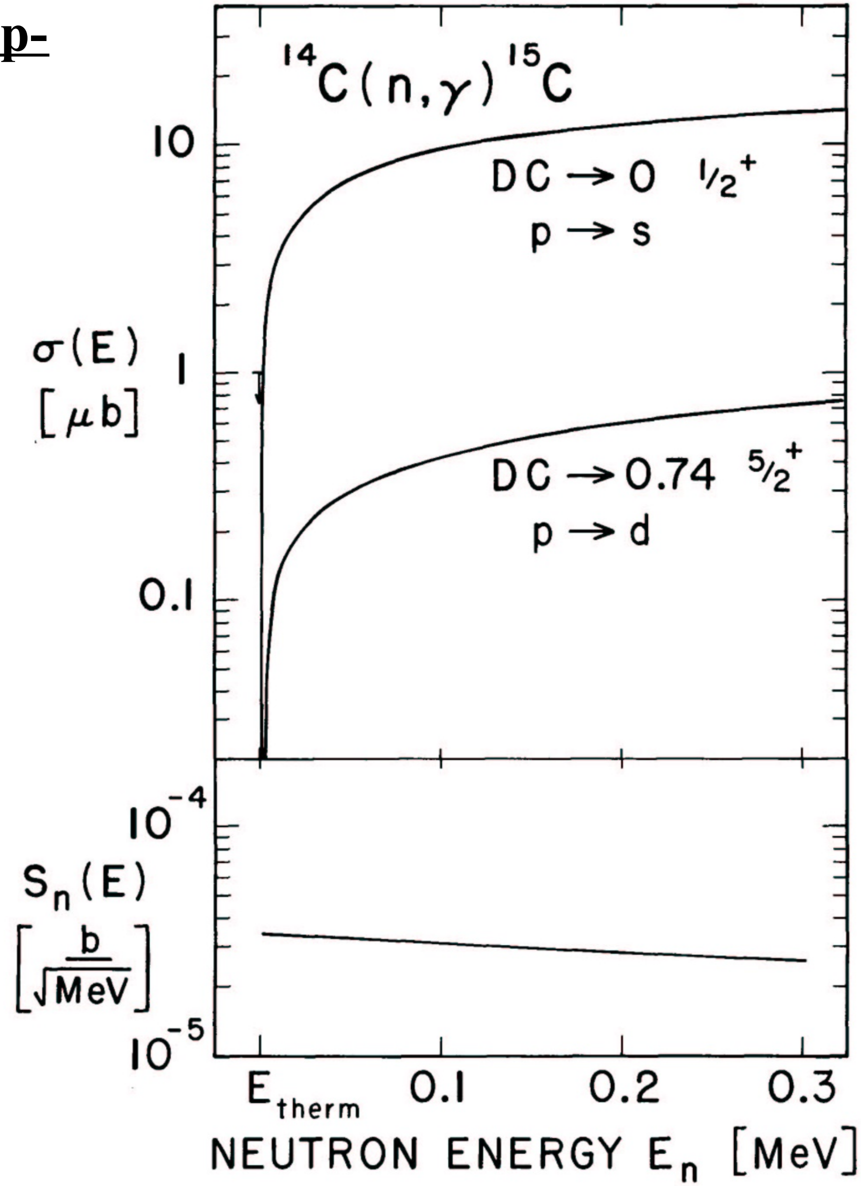
$$\sigma = \sqrt{E}S(E) \quad \text{or} \quad S(E) = \frac{\sigma}{\sqrt{E}}$$

which leads to a relatively constant S-factor because of

$$\sigma \propto \sqrt{E}$$

(typical unit for S(E) is then barn/MeV$^{1/2}$)

S-factor →



Stellar Reaction Rates for direct charged particle reactions

Recall (This lecture & lecture I)

$$\sigma \propto \frac{1}{E} \cdot P_l(E) \cdot \left| \langle f | H | a + A \rangle \right|^2$$

Now the incoming particle has to overcome Coulomb barrier → $P_l(E) = e^{-2\pi\eta}$

with $\eta = \frac{Z_1 Z_2 \mathrm{e}^2}{\hbar} \sqrt{\frac{\mu}{2E}}$

$$2\pi\eta = 31.29 Z_1 Z_2 \sqrt{\frac{\mu_{\mathrm{amu}}}{E_{\mathrm{keV}}}}$$

The S-factor for charged particle reactions is defined via:

$$\sigma(E) = \frac{1}{E} \times e^{-2\pi\eta} \times S(E) \quad \rightarrow \quad \sigma(E) = \frac{1}{E} \times e^{-\frac{b}{E^{1/2}}} \times S(E)$$

typical unit for S(E): keV barn

Stellar Reaction Rates: Gamow peak (relevant energy range)

$$\sigma(E) = S(E)\frac{1}{E}\exp(-2\pi\eta)$$

$$<\sigma v>_{aX} = \left(\frac{8}{\pi\mu_{aX}}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int S(E) \exp\left[-\frac{E}{kT} - \frac{b}{E^{1/2}}\right] dE \quad \textbf{(I)}$$

$S(E)$: varies smoothly with energy

$f(E) = -\frac{E}{kT} - \frac{b}{E^{1/2}}$: governs energy dependence

$$b = \sqrt{2\mu}\,\frac{\pi Z_1 Z_2 e^2}{\hbar}$$

MAXIMUM reaction rate:

$$\frac{df(E)}{dE} = 0 \rightarrow E_0 = \left(\frac{bkT}{2}\right)^{2/3}$$

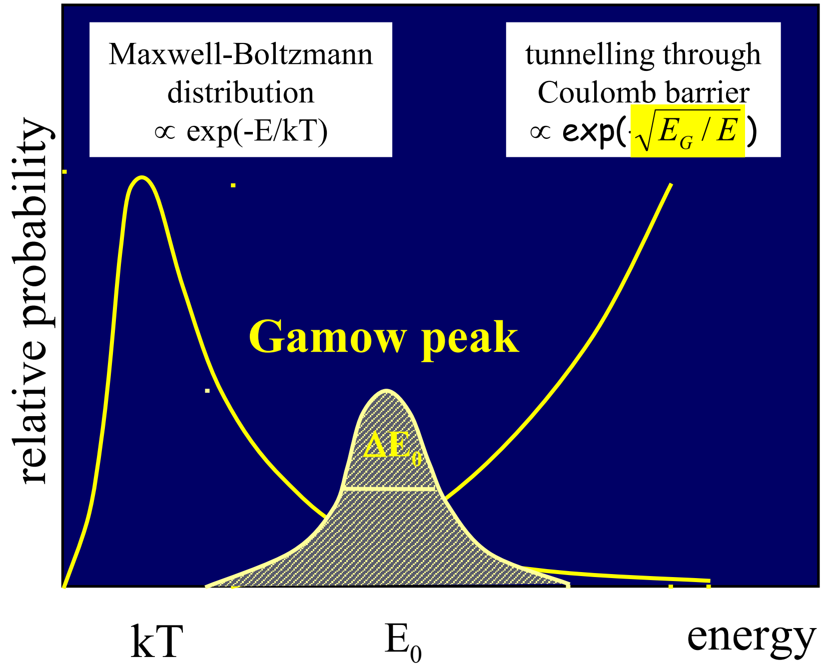
$$E_0 = \pi kT\eta(E_0) = 1.22\left(Z_1^2 Z_2^2 \mu_{amu} T_6^2\right)^{1/3} \text{ keV}$$

$$\Delta E_0 = 4\sqrt{E_0 kT/3} = 0.749\left(Z_1^2 Z_2^2 \mu_{amu} T_6^5\right)^{1/6} \text{ keV}$$

$$T_6 = T\,(\text{MK})$$

ΔE0<E0 → only small energy range contributes to reaction rate

⇒ OK to set $S(E) \sim S(E_0) = \text{const.}$



Stellar Reaction Rates: Gamow peak

Gamow peak: most effective energy region for thermonuclear reactions

$E_0 \pm \Delta E_0/2$ energy window of astrophysical interest

$E_0 = f(Z_a, Z_X, T)$

↓

depends on reaction and/or temperature

Examples: $T \sim 15\times10^6$ K (T_6=15) → kT=1.34 keV

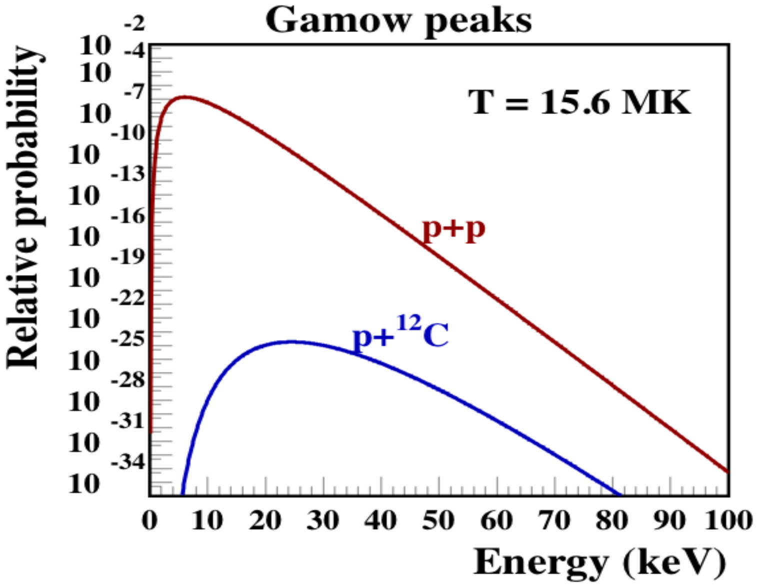
reaction	Coulomb barrier (MeV)	E_0 (keV)	$\exp(-3E_0/kT)\ \Delta E_0$
p + p	0.55	5.9	7.0×10^{-6}
$\alpha + {}^{12}C$	3.43	56	5.9×10^{-56}
${}^{16}O + {}^{16}O$	14.07	237	2.5×10^{-237}

➡ area of Gamow peak (height ×width) ~ $<\sigma v>$

STRONG sensitivity to Coulomb barrier

separate stages: H-burning, He-burning, C/O-burning

Stellar Reaction Rates: non-resonant (direct) capture



- Maximum of the Gamow peak ($E=E_0$):

$$I_{\max} = \exp(-\tau)$$

where $$\tau = \frac{3E_0}{kT} = 42.46\left(Z_a^2 Z_X^2 \mu_{amu} / T_6\right)^{1/3}$$

$\Rightarrow I_{\max}$ is strongly dependent of the product $Z_a Z_X \Rightarrow$ successive nuclear burning phases

For **non-resonant capture** (**direct**), one approximates the rate calculation by assuming the S-factor is constant over the Gamow Window

Reaction rate:

$$\langle \sigma v \rangle_{aX} = \left(\frac{8}{\mu\pi}\right)^{1/2} \frac{1}{(kT)^{3/2}} S(E_0)\sqrt{\pi/2}\, I_{\max} \Delta E_0$$

with $S(E_0)$ in keV b :

$$\langle \sigma v \rangle_{aX} = 7.20\times 10^{-19} \frac{\tau^2 \exp(-\tau)}{\mu_{amu} Z_a Z_X} S(E_0)\ \text{cm}^3\ \text{s}^{-1} \qquad \textbf{(II)}$$

Stellar Reaction Rates: non-resonant (direct) capture

- For many non-resonant reactions S-factor is not a constant & varies with E
  → Expand the experimental or theoretical S(E) around E=0 as powers of E to second order:

$$S(E) \approx S(0) + S'(0)E + \frac{1}{2}S''(0)E^2$$

If one integrates this over the Gamow window in **Eq. I**, one finds that one can use **Eq. II** by replacing **$S(E_0)$** with the **effective S-factor S_{eff}**

$$S_{eff} = S(0)\left[\left(1+\frac{5}{12\tau}\right) + \frac{S'(0)}{S(0)}\left(E_0 + \frac{35}{36}kT\right) + \frac{1}{2}\frac{S''(0)}{S(0)}\left(E_0^2 + \frac{89}{36}E_0 kT\right)\right]$$

F(τ): the correction factor due to the asymmetry of the Gamow peak

Corrections due to the S-factor variation with energy

Fowler, Caughlan and Zimmerman 1967

Stellar Reaction Rates: Resonant reaction

Recall

$$\langle \sigma v \rangle_{aX} = \left(\frac{8}{\mu\pi}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty \sigma(E) E e^{-E/kT} dE$$

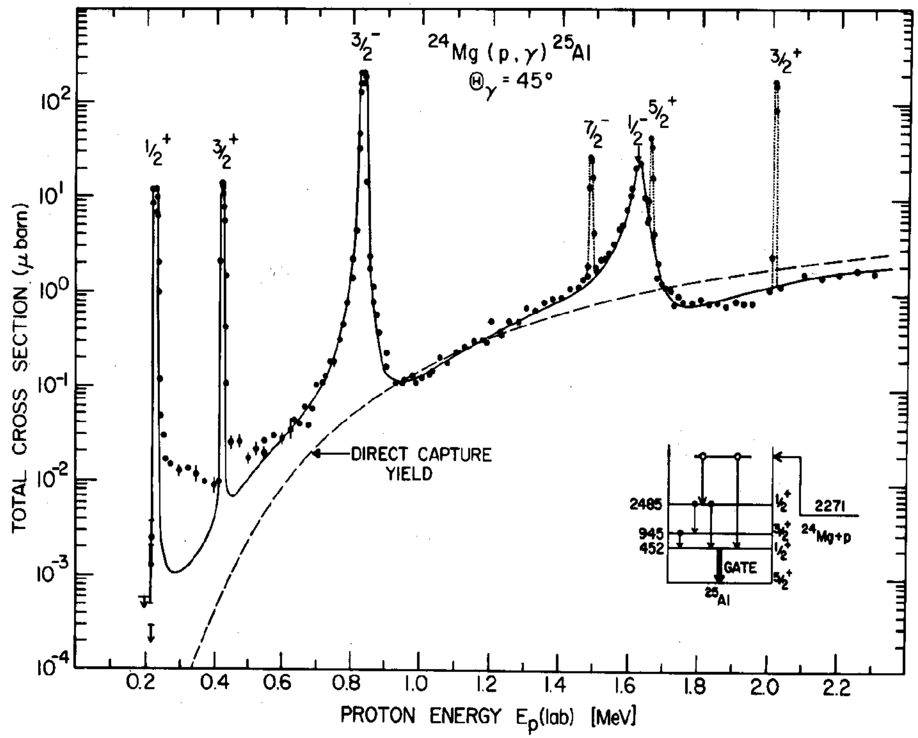
If in the energy range of interest (Gamow window for charged particle reaction & nearly KT for neutron captures) there is an excited state (or part of it, as states have a width) in the Compound nucleus then the reaction rate will have a resonant contribution.

Contribution to the reaction rate of a resonance at the energy E_R near E_0:

$$\langle \sigma v \rangle_{aX} = \left(\frac{8}{\mu\pi}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty \sigma_{BW}(E) E e^{-E/kT} dE$$

with

$$\sigma_{BW}(E) = \pi \lambda\!\!\!^{-2} \omega \frac{\Gamma_a \Gamma_b}{(E - E_R)^2 + (\Gamma/2)^2}$$



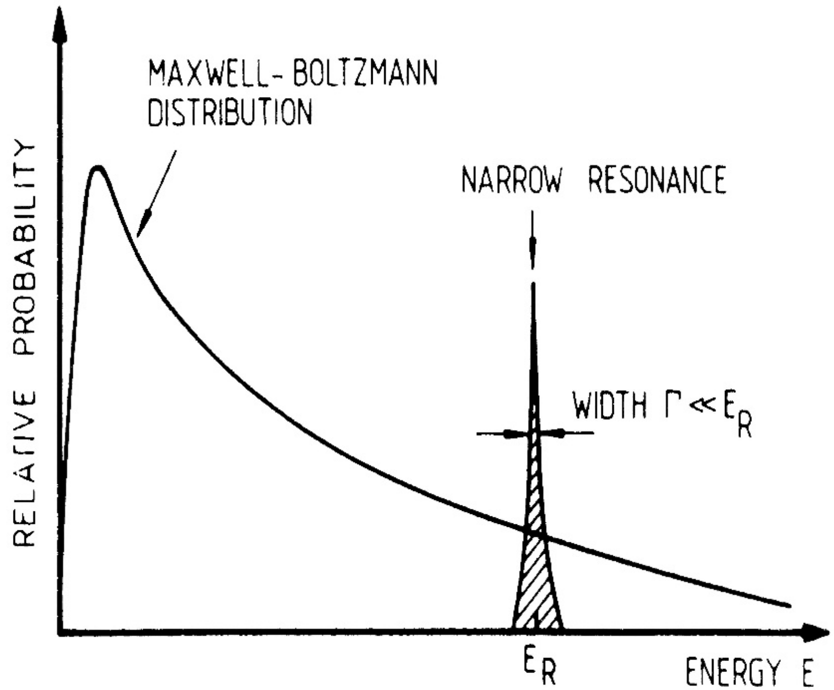
➢The reaction rate becomes extremely sensitive to the properties of the resonant state

The case of a narrow resonance $\Gamma << E_R$

➢ The resonance energy must be "near" the relevant energy range ΔE to contribute to the stellar reaction rate.

➢ Maxwell-Boltzmann distribution ~ cst

$$\langle \sigma v \rangle_{aX} = \left(\frac{8}{\mu\pi} \right)^{1/2} \frac{E_R e^{-E_R/kT}}{(kT)^{3/2}} \int_0^\infty \sigma_{BW}(E) dE$$



- If the Γ_i are constants over $\Gamma << E_R$:

$$\int_0^\infty \sigma_{BW}(E) dE = 2\pi^2 \lambda\!\!\!^{-}{}_R^2 \omega \frac{\Gamma_a \Gamma_b}{\Gamma}$$

$$\Rightarrow \langle \sigma v \rangle_{aX} = \left(\frac{2\pi}{\mu kT} \right)^{3/2} \hbar^2 e^{-E_R/kT} \omega\gamma$$

$$\omega\gamma = \omega \frac{\Gamma_a \Gamma_b}{\Gamma}$$

is the strength of the resonance

The case of a narrow resonance $\Gamma \ll E_R$

For the contribution of a single narrow resonance to the stellar reaction rate:

$$N_A < \sigma v >= 1.54 \cdot 10^{11} (A T_9)^{-3/2} \omega\gamma\, [\mathrm{MeV}]\, e^{\frac{-11.605\, E_R[\mathrm{MeV}]}{T_9}} \quad \frac{\mathrm{cm}^3}{\mathrm{s\ mole}}$$

The rate is entirely determined by the "resonance strength":

$$\omega\gamma = \frac{2J_R + 1}{(2J_a + 1)(2J_X + 1)} \frac{\Gamma_a \Gamma_b}{\Gamma}$$

→ depends mainly on the total and partial widths of the resonance

Often $\Gamma = \Gamma_a + \Gamma_b$ Then for $\Gamma_a \ll \Gamma_b \longrightarrow \Gamma \approx \Gamma_b \longrightarrow \frac{\Gamma_a \Gamma_b}{\Gamma} \approx \Gamma_a$

$\Gamma_b \ll \Gamma_a \longrightarrow \Gamma \approx \Gamma_a \longrightarrow \frac{\Gamma_a \Gamma_b}{\Gamma} \approx \Gamma_b$

And reaction rate is determined by the smaller one of the widths !

The case of broad resonances $\Gamma \sim E_R$

- Partial and total widths depend sensitively on the decay energy. Therefore:
  - widths depend sensitively on the excitation energy of the state
  - widths for a given state are energy-dependent

(they are NOT constants in the Breït-Wigner Formula)

Particle widths:

$$\Gamma_a = 2P_l(E)\gamma_a^2$$

Penetrability: Main energy dependence (calculated) → $P_l(E)$

"reduced width"→ contains the nuclear physics → γ_a^2

Photon widths:

$$\Gamma_\gamma = B(l)\, E_\gamma^{2l+1}$$

$B(l)$ → Reduced matrix element

$$\langle \sigma v \rangle_{aX} = \left(\frac{8}{\mu\pi}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty \sigma_{BW}(E) E e^{-E/kT} dE$$

$\sigma_{BW}(E)$ → Breït-Wigner formula (energy-dependant partial widths)

The case of broad resonances $\Gamma \sim E_R$

$$\langle \sigma v \rangle_{aX} = \sqrt{2\pi} \frac{\omega \hbar^2}{(\mu k T)^{3/2}} \int_0^\infty e^{-E/kT} \frac{\Gamma_a(E)\Gamma_b(E+Q-E_f)}{(E-E_R)^2 + \Gamma(E)^2/4} dE$$

Rate can be obtained from numerical integration

Rate of reaction through the wing of a broad resonance

A simple case

➢ Resonances outside the energy window for the reaction can contribute through their wings

Assume Γ_b= const & Γ=const

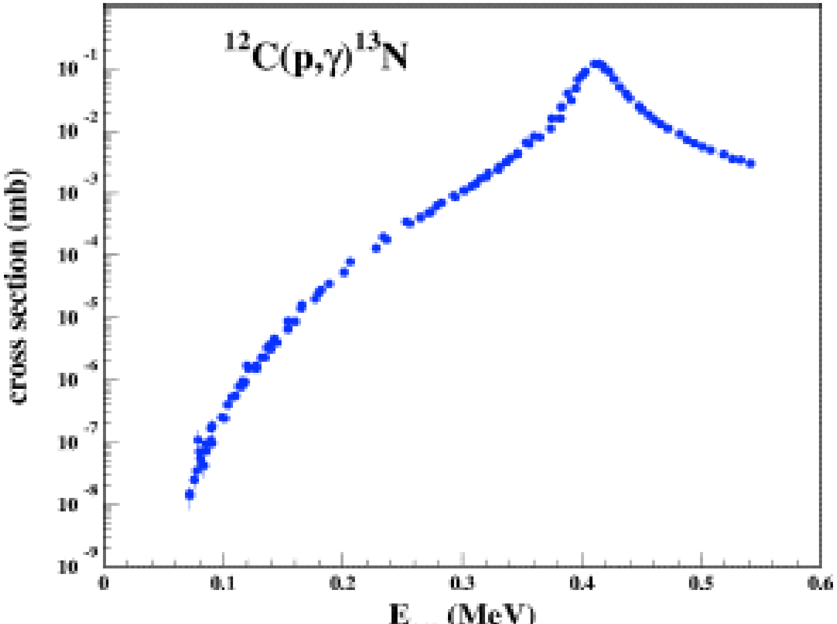
$$\sigma(E) = \pi \lambda\!\!\!^{-2} \omega \Gamma_a(E) \frac{\Gamma_b}{(E-E_R)^2 + (\Gamma/2)^2}$$

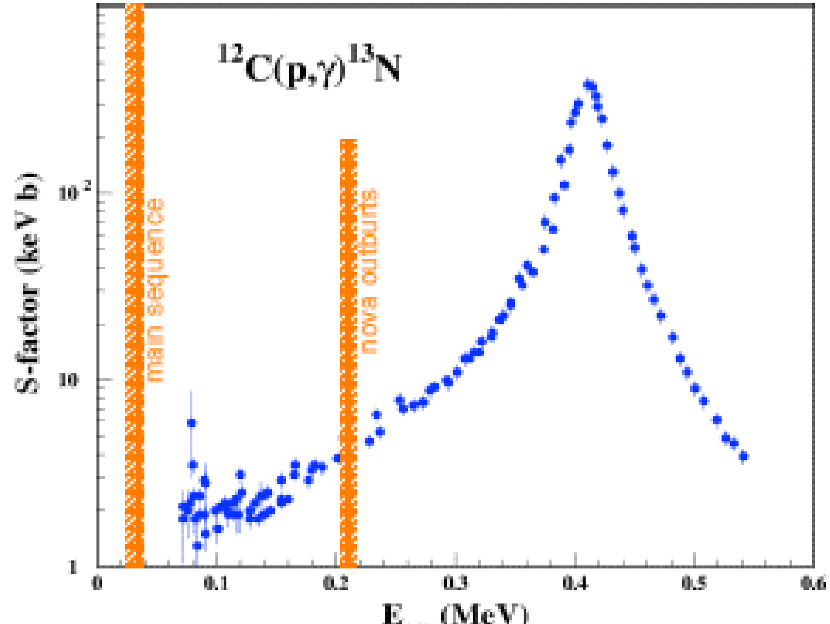
Same energy dependence than direct reaction

For E << E_R very weak energy dependence

Example of resonant reaction

$^{12}C(p,\gamma)^{13}N$: Proceeds mainly through tail of 0.46 MeV resonance





data from NACRE (Angulo et al.,NP A656)

Resonance parameters:

$J^{\pi}=1/2^{+}$ $E_R=0.42$ MeV

$\Gamma_{\gamma} = 0.50 \pm 0.04$ eV

$\Gamma = 31.7 \pm 0.8$ keV $\rightarrow = \Gamma_p$

Burning conditions:

$T_6=20$ $E_0= 29$ keV

CNO-cycle (main sequence, M =2 $M_{\odot}$)

$T_9= 0.4$ $E_0= 213$ keV

Hot CNO-cycle (Novae)

Rate of reaction through the wing of a broad resonance (Summary)

- Far from the resonance the contribution from wings has a similar energy dependence than the direct reaction mechanism.

- In particular, for s-wave neutron capture there is often a 1/v contribution at thermal energies through the tails of higher lying s-wave resonances.

- Therefore, resonant tail contributions and direct contributions to the reaction rate can be parametrized in the same way (for example S-factor). Tails and DC are often mixed up in the literature.

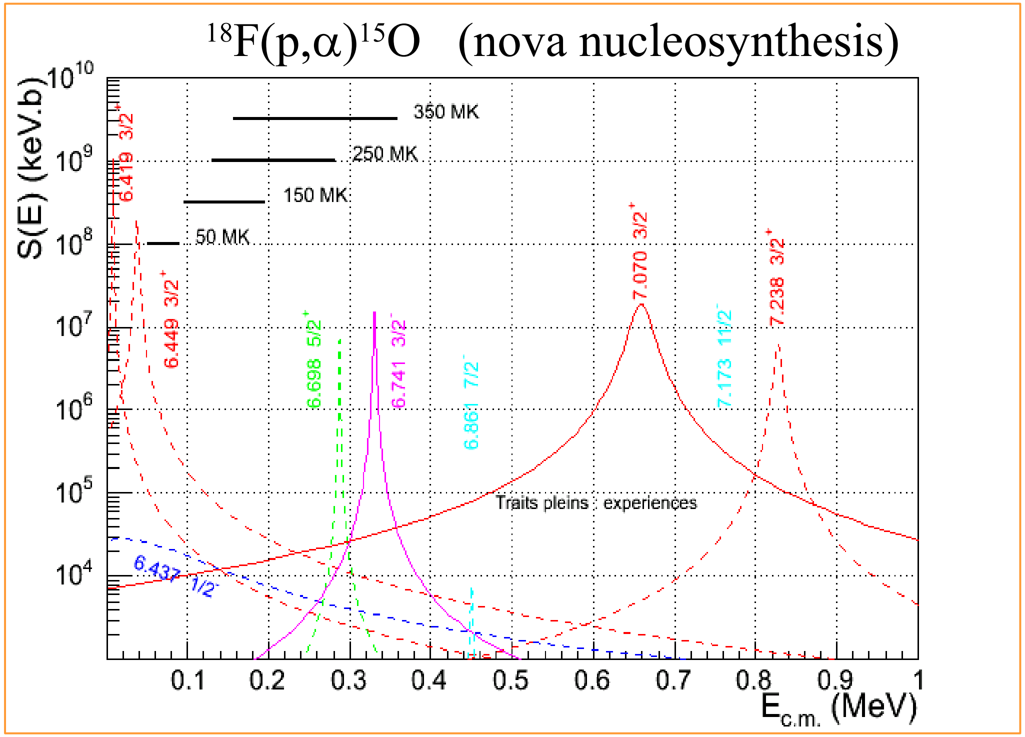
- Though they look the same, direct and resonant tail contributions are different things:
  - in direct reactions, no compound nucleus forms
  - resonance contributions can be determined from resonance properties measured at the resonance, far away from the relevant energy range (but need to consider interference !)

Stellar Reaction Rates: Summary

The stellar reaction rate of a nuclear reaction is determined by the sum of

- sum of direct transitions to the various bound states
- sum of all narrow resonances in the relevant energy window
- tail contribution from higher lying resonances or sub-threshold resonances

$$<\sigma v> = \sum_i <\sigma v>_{\mathrm{DC->state\,i}} + \sum_i <\sigma v>_{\mathrm{Res;\,i}} + <\sigma v>_{\mathrm{tails}}$$



Caution:

Interference effects are possible (constructive or destructive addition) among:

- Overlapping resonances with same quantum numbers
- Same wave direct capture and resonances

Example of non-resonant and resonant reaction

TABLE V. Nonresonant direct capture transitions and the astrophysical S factors; resonance energies, γ widths, proton widths, and resonance strengths for $^{32}Cl(p,\gamma)^{33}Ar$.

$^{32}Cl(p,\gamma)^{33}Ar$ $Q = 3.34$ MeV

E_x	J^π	ℓ_i	$n\ell_f$	C^2S_f	$S(E_0)$ (MeV b)
0.00	$\frac{1}{2}^+_1$	p	$2s_{1/2}$	0.080	7.00×10^{-3}
		p	$1d_{3/2}$	0.672	6.14×10^{-3}
1.34	$\frac{3}{2}^+_1$	p	$1d_{3/2}$	0.185	2.62×10^{-3}
1.79	$\frac{5}{2}^+_1$	p	$1d_{3/2}$	0.145	2.74×10^{-3}
2.47	$\frac{3}{2}^+_2$	p	$2s_{1/2}$	0.031	6.16×10^{-3}
		p	$1d_{3/2}$	0.167	1.67×10^{-3}
3.15	$\frac{3}{2}^+_3$	p	$2s_{1/2}$	0.068	1.46×10^{-2}
		p	$1d_{3/2}$	0.516	3.01×10^{-3}
E_x	**E_p**	**J^π**	**Γ_γ (eV)**	**Γ_p (eV)**	**$\omega\gamma$ (eV)**
3.43	0.09	$\frac{5}{2}^+_2$	1.77×10^{-2}	8.7×10^{-18}	8.7×10^{-18}
3.56	0.22	$\frac{7}{2}^+_2$	1.94×10^{-3}	1.13×10^{-9}	1.51×10^{-9}
3.97	0.63	$\frac{5}{2}^+_3$	1.54×10^{-2}	2.22×10^{-2}	9.09×10^{-3}
4.19	0.85	$\frac{1}{2}^+_2$	1.54×10^{-1}	46.74	5.12×10^{-2}
4.73	1.39	$\frac{3}{2}^+_4$	8.48×10^{-2}	100.3	5.65×10^{-2}

Direct

Res.

S_p=3.34 MeV

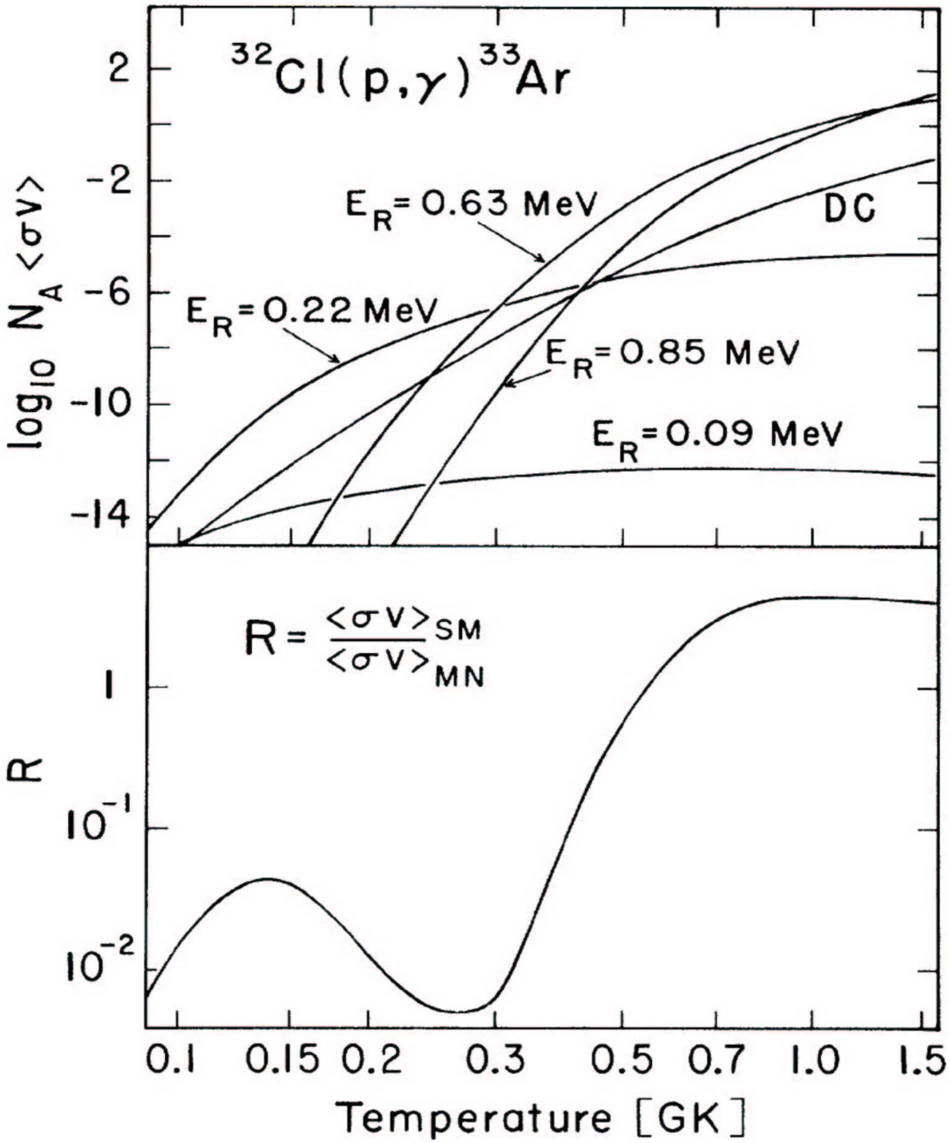
Weak changes in gamma width

Strong energy dependence of proton width

Resonance strengths

Herndl et al. PRC52(95)1078)

Example of non-resonant and resonant reaction



Gamow Window:

0.1 GK: 130-220 keV
0.5 GK: 330-670 keV
1GK: 500-1100 keV

The Gamow window moves to higher energies with increasing temperature

→ **different resonances** play a role at different temperatures.

Stellar reaction rates: Other Remarks

- If a resonance is in or near the Gamow window it tends to dominate the reaction rate by orders of magnitude

- As the level density increases with excitation energy in nuclei, higher temperature rates tend to be dominated by resonances, lower temperature rates by direct reactions.

- As can be seen from the reaction rate equation for narrow resonance, the reaction rate is extremely sensitive to the resonance energy. For p-capture this is due to the $\exp(E_R/kT)$ term **AND** $\Gamma_p(E)$ (Penetrability) !

As $E_r=E_x-Q$ one needs accurate excitation energies **and** masses !

Stellar reaction rates: Complications in stellar environment

Beyond temperature and density, there are additional effects related to the extreme stellar environments that affect reaction rates.

In particular, experimental laboratory reaction rates need a (theoretical) correction to obtain the stellar reaction rates.

The most important two effects are:

1. Thermally excited target

At the high stellar temperatures photons can excite the target. Reactions on excited target nuclei can have different angular momentum and parity selection rules and have a somewhat different Q-value.

2. Electron screening

Atoms are fully ionized in a stellar environment, but the electron gas still shields the nucleus and affects the effective Coulomb barrier.

Reactions measured in the laboratory are also screened by the atomic electrons, but the screening effect is different (see lecture III).

Stellar reaction rates: Thermally excited target nuclei

Ratio of nuclei in a thermally populated excited state to nuclei in the ground state is given by the Saha Equation:

$$\frac{n_{\mathrm{ex}}}{n_{\mathrm{gs}}} = \frac{g_{\mathrm{ex}}}{g_{\mathrm{gs}}} e^{-\frac{E_x}{kT}} \qquad g = (2J+1)$$

Ratios of order 1 for $E_x \sim kT$

In nuclear astrophysics, kT=1-100 keV, which is small compared to typical level spacing in nuclei at low energies (~ MeV).

→ usually only a very small correction, but can play a role in some cases:

- a low lying (~100 keV) excited state exists in the target nucleus
- temperatures are high
- the populated state has a very different rate (for example due to very different angular momentum or parity or if the reaction is close to threshold and the slight increase in Q-value 'tips the scale' to open up a new reaction channel)

The correction for this effect has to be calculated. NACRE compilation gives a correction.

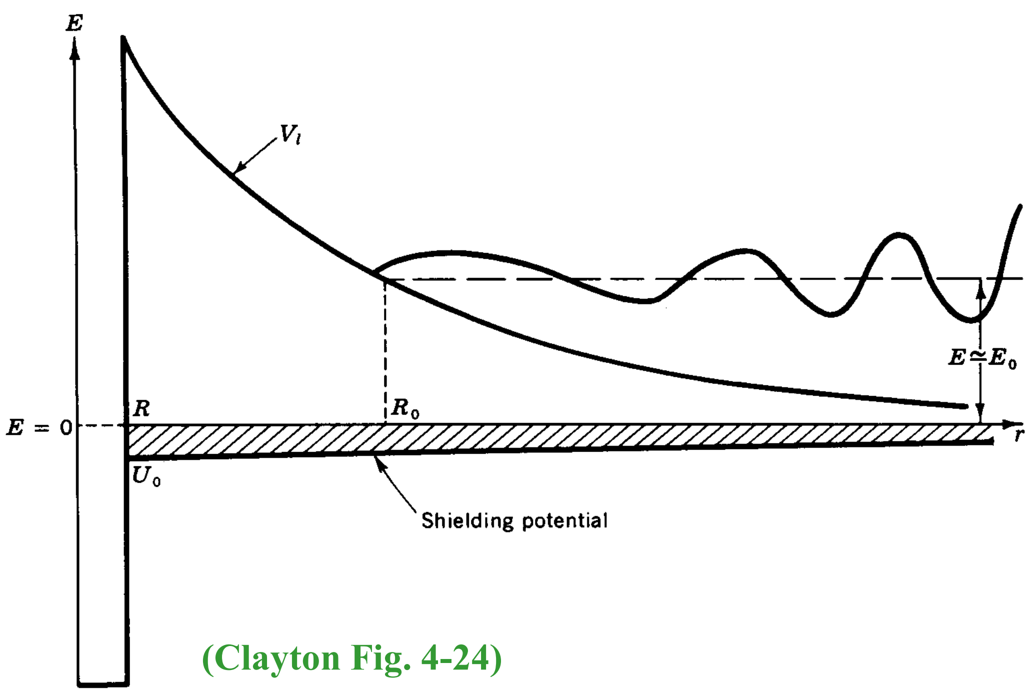
Stellar reaction rates: Electron Screening (1)

The nuclei in an astrophysical plasma undergoing nuclear reactions are fully ionized.

However, they are immersed in a dense electron gas, which leads to some shielding of the Coulomb repulsion between projectile and target for charged particle reactions.

Charged particle reaction rates are therefore enhanced in a stellar plasma, compared to reaction rates for bare nuclei.

The Enhancement depends on the stellar conditions



(Clayton Fig. 4-24)

$$V(r) = \frac{Z_1 Z_2 e^2}{r} + U(r)$$

Bare nucleus Coulomb

Extra Screening potential

(attractive so <0)

Stellar reaction rates: Electron Screening (2)

Screening factor f definition:

$$<\sigma v>_{\text{screened}} = f <\sigma v>_{\text{bare}}$$

Case 1: Weak Screening

Definition of weak screening regime:

Average Coulomb energy between ions << thermal Energy

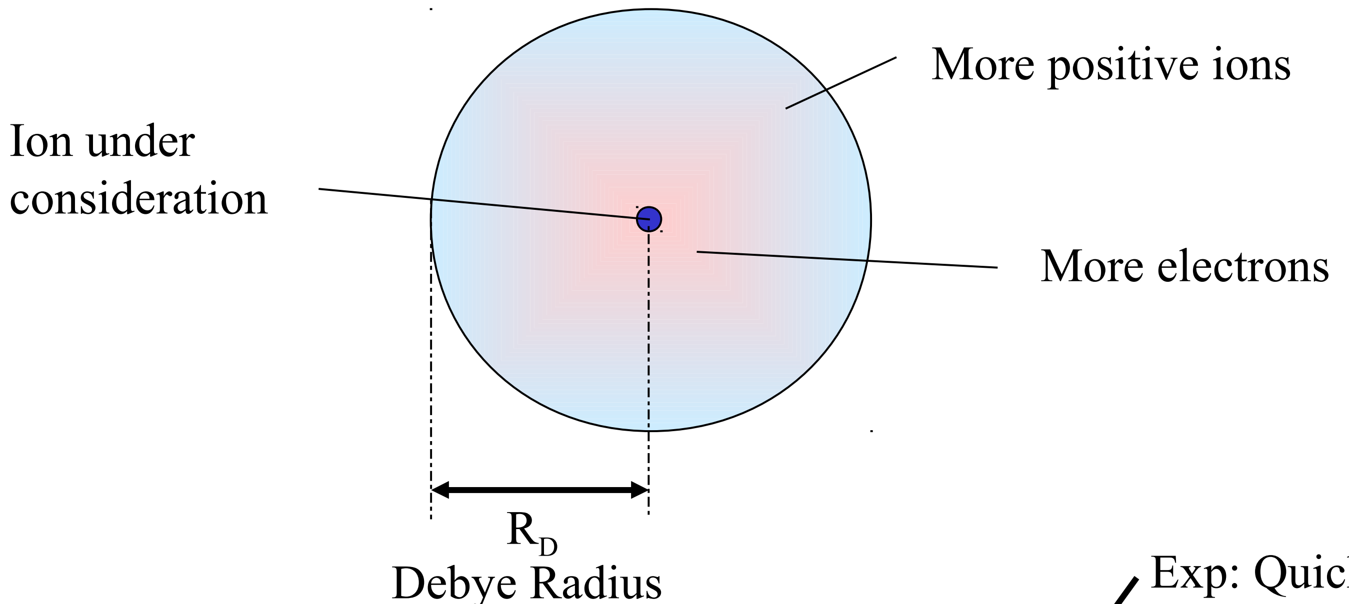
$$\frac{e^2 Z^2}{n^{-1/3}} << kT$$ (for a single dominating species)

Means:
- high temperature
- low density

(typical for example for stellar hydrogen burning)

Stellar reaction rates: Electron Screening (3)

For weak screening, each ion is surrounded by a sphere of ions and electrons that are somewhat polarized by the charge of the ion (Debeye Huckel treatment)



Exp: Quicker drop off due to screening

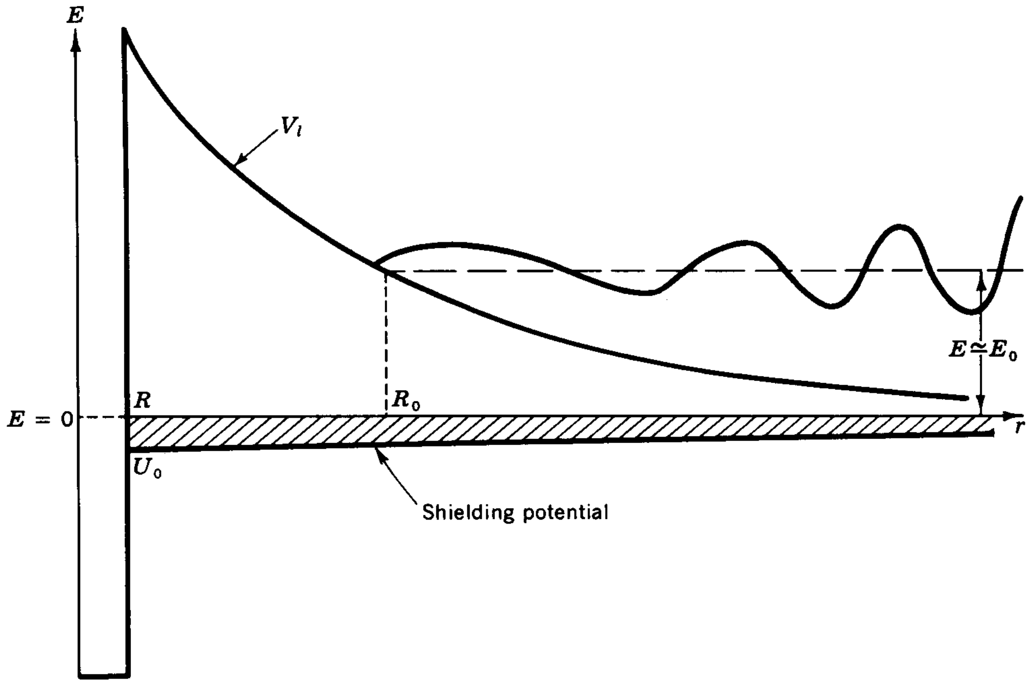
Then potential around ion $$V_1(r) = \frac{eZ}{r} e^{-r/R_D}$$

With $$R_D = \sqrt{\frac{kT}{4\pi e^2 \rho N_A \xi^2}}$$ $$\xi = \sqrt{\sum_i (Z_i^2 + Z_i \Theta_e) Y_i}$$

So for r>>R_D complete screening

Stellar reaction rates: Electron Screening (4)

But effect on barrier penetration and reaction rate only for potential between R and classical turning point R_0



In weak screening regime, $R_D >> (R_0\text{-}R)$

And therefore one can assume $U(r) \sim const \sim U(0)$.

In other words, we can expand V(r) around r=0:

$$V_1(r) = \frac{eZ_1}{r}\left(\underbrace{1 - \frac{r}{R_D}}_{\text{To first order}} + \frac{r^2}{2R_D^2} - \ldots\right)$$

So to first order, barrier for incoming projectile:

$$V(r) = eZ_2V_1(r) = \frac{e^2Z_1Z_2}{r} - \frac{e^2Z_1Z_2}{R_D}$$

Comparison with

$$V(r) = \frac{Z_1Z_2e^2}{r} + U(r)$$

Yields for the screening potential:

$$U(r) = U(0) = U_0 = -\frac{e^2Z_1Z_2}{R_D}$$

These 2 equations describe a corrected Coulomb barrier for the astrophysical environment.

Stellar reaction rates: Electron Screening (6)

One can show, that the impact of the correction on the barrier penetrability and therefore on the astrophysical reaction rate can be approximated through a screening factor f:

$$f = \mathrm{e}^{-\mathrm{U}_0/kT}$$

In weak screening $U_0 << kT$ and therefore $f \approx 1 - \frac{U_0}{kT}$ $\quad U_0 = -\frac{e^2 Z_1 Z_2}{R_D}$

Summary weak screening:

$$<\sigma v>_{\text{screened}} = f <\sigma v>_{\text{bare}}$$

$$f = 1 + 0.188\, Z_1 Z_2 \rho^{1/2} \xi T_6^{-3/2}$$

$$\xi = \sqrt{\sum_i (Z_i^2 + Z_i \Theta_e) Y_i}$$

Stellar reaction rates: Electron Screening (7)

Other cases:

Strong screening:

Average coulomb energy larger than kT – for high densities and low temperatures
Again simple formalism available, for example in Clayton

Intermediate screening:

Average Coulomb energy comparable to kT – more complicated but formalisms available in literature