# Power spectrum of flow fluctuations

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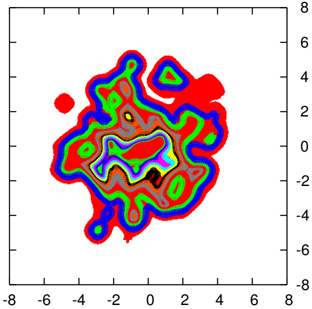
# Outline

1. Introduction
2. Inflationary density fluctuations and flow fluctutaions
3. Power spectra
4. Effect of magnetic field on flow
5. Summary and Conclusions

- Inhomogeneities of all scales are present in the initial state energy density distribution in heavy-ion collisions.
- We argued that these inhomogeneities will generate flow fluctuations, both even and odd, for harmonics upto, say $n = 30$.
- It is important to calculate the root-mean-square values of these flow fluctuations than the averages.
- A plot of the rms values of the Fourier transforms of the flow fluctuations $v_n^{rms}$ vs. $n$ will contain important information about the initial state fluctuations and their evolution.

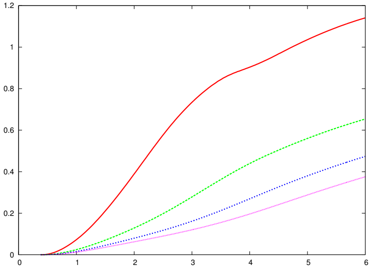
# Initial energy density for $Pb - Pb$ collision at 200 GeV/$A$



- These density fluctuations are similar to the density fluctuations in the early universe which leave their imprints in the cosmic microwave background radiation as temperature fluctuations.
- Inflationary density fluctuations are present on all scales and after inflation, the scales larger than the horizon scale 'enter' the horizon, the shortest scale first and evolve.
- In heavy-ion collisions, thermalisation happens in less than a fm/$c$. The system has fluctuations of orders larger than this thermalisation scale, all the way upto the system size.

- These energy density fluctuations grow into momentum anisotropies, the shortest modes first. The higher harmonics grow first. Elliptic flow being the largest mode will grow to it maximum last.
- The relevant scale one should compare with the modes is the acoustic horizon at a given time.
- The larger modes like elliptic flow may not completely grow to the corresponding momentum anisotropy before freeze-out.

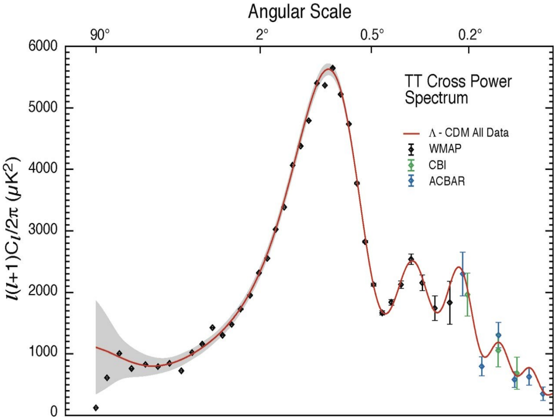
- We have argued that the growth of a spatial fluctuation to the corresponding momentum fluctuation will depend on the size of the mode compared to the acoustic horizon scale.
- Thus the elliptic mode of spatial eccentricity in smaller systems should grow faster compared to that in larger systems if the thermalisation time scales are similar.



Growth of $\epsilon_p/\epsilon_x$ for systems of different size. Smooth Glauber initial conditions are used.

- Another important aspect of inflationary density fluctuations is the acoustic oscillations. The over/under densities which enter the horizons grow and oscillate acoustically.
- We have argued that the sub-horizon modes density fluctuations in heavy-ion collisions also should display an oscillatory behavior.
- Following CMBR fluctuation analysis, a natural observable to look at would be the Fourier co-efficients of the azimuthal anisotropies in the final particle momenta in a given rapidity bin. (CMBR anisotropic fluctuations are studied by expanding them in spherical harmonics.)
- We study the Fourier co-efficients of the azimuthal anisotropy of momenta given by $\frac{p(\phi)-p_{av}}{p_{av}}$.

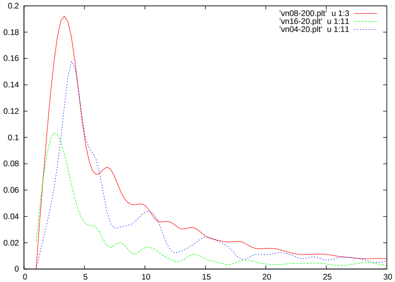
# CMB angular power spectrum



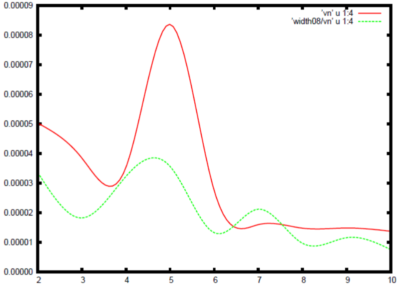
# Relativistic hydrodynamic simulations

*Digal, Saumia, Srivastava*

Plots of $v_n^{rms}$ vs. $n$ for Gaussian fluctuations of width s



Woods-Saxon density profile, 2 fm radius with 10 Gaussian fluctuations $s$=0.4,0.8,1.6 (red, blue, green)



Glauber model density profile and fluctuations $Pb - Pb$, 200 GeV
$s$=0.4,0.8 fm (red,green)

# Effect of magnetic field on flow

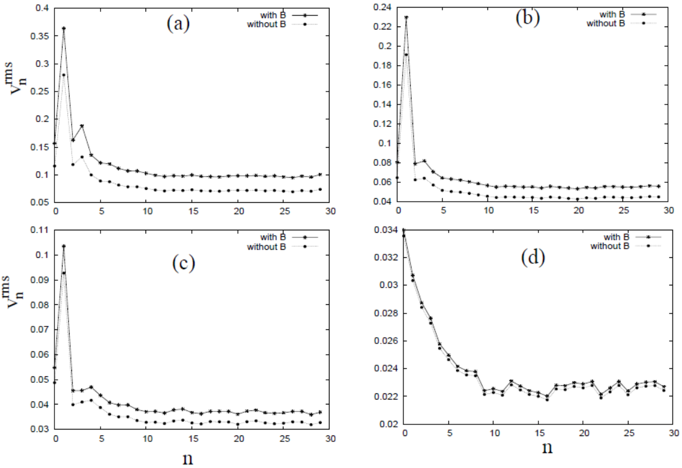
- Presence of magnetic field in the early universe distorts the power spectrum of CMBR. This is due to the complex dependence of magnetosonic waves on magnetic fields and density gradients.
- We studied the effect of magnetic field on flow in heavy-ion collisions.
- In non-central collisions, a large magnetic field is generated during the collisions. The time scale of this field can be several fm due to induced currents in the plasma.
- We argued that this can enhance flow anisotropies. It is interesting to see whether larger values of viscosity can be accommodated in this scenario.

- In presence of magnetic field, there are three types of waves in the plasma in place of ordinary sound waves.
- We calculated the flow anisotropies by using the generalised sound waves, the fast magnetosonic waves which have significant contributions from the magnetic pressure.
- The expression of group velocity of the fast magnetosonic waves is given by

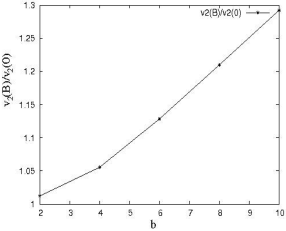
$$v_{gr} = v_{ph}\left[\mathbf{n} + \mathbf{t}\frac{\sigma \pm 2\delta(a \mp (1+\cos^2\theta)\sin\theta\cos\theta}{2(1+\delta\cos^2\theta \pm a)a}\right]$$

- $\delta = \frac{c_s^2 v_A^2}{\left(\frac{4\rho}{3\omega}c_s^2 + v_A^2\right)}$, $\sigma = \frac{4c_s^2 v_A^2}{[(\frac{4\rho}{3\omega})c_s^2 + v_A^2)]^2}$, $\omega = \left(\frac{4\sigma}{3}\right) + B^2$

Root-mean square values of flow co-efficients for $b = 10, 8, 6, 2$ fm.



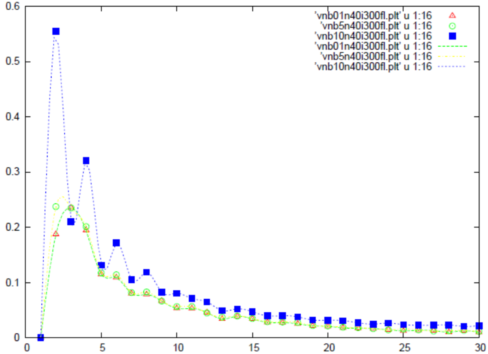
Enhancement of elliptic flow with magnetic field.
Mohapatra, Saumia, Srivastava, MPLA26, 2477 (2011).



Subsequent analysis by Tuchin: Results in agreement with this
K. Tuchin, J.Phys. G 39, 025010 (2012)

# Summary and Conclusions

- The first peak contains information about the freezeout stage, and the dominant scale of fluctuations.
- We plot $v_n$ up to $n = 30$, which corresponds to wavelength of fluctuation of order 1 fm. There will be some scale (probably less than 1 fm) below which hydrodynamics breaks down. A changeover in the plot of $v_n$ at some large $n$ will indicate applicable regime of hydrodynamics.
- One important factor which can affect the shape and inter-spacings of these peaks, is the nature and presence of the quark-hadron transition.
- Magnetic field leads to complex flow patterns, vorticity generation. There are some interesting results already from MHD. The odd harmonics seem to be suppressed in presence of magnetic field.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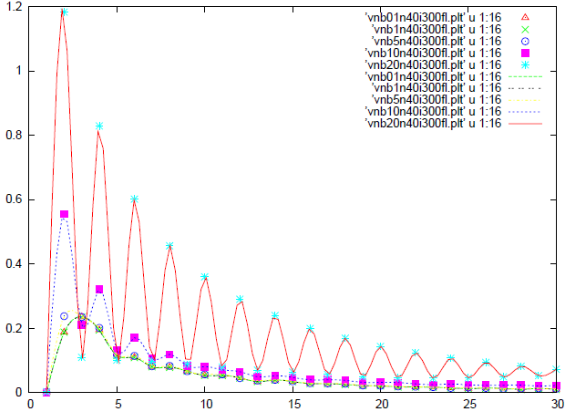
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