Shear viscosity of a pion gas due to $\rho\pi\pi$ and $\sigma\pi\pi$ interactions

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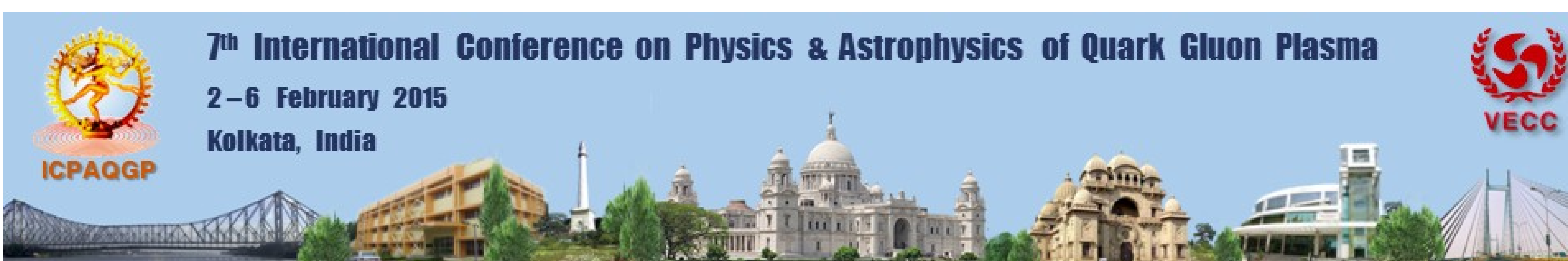
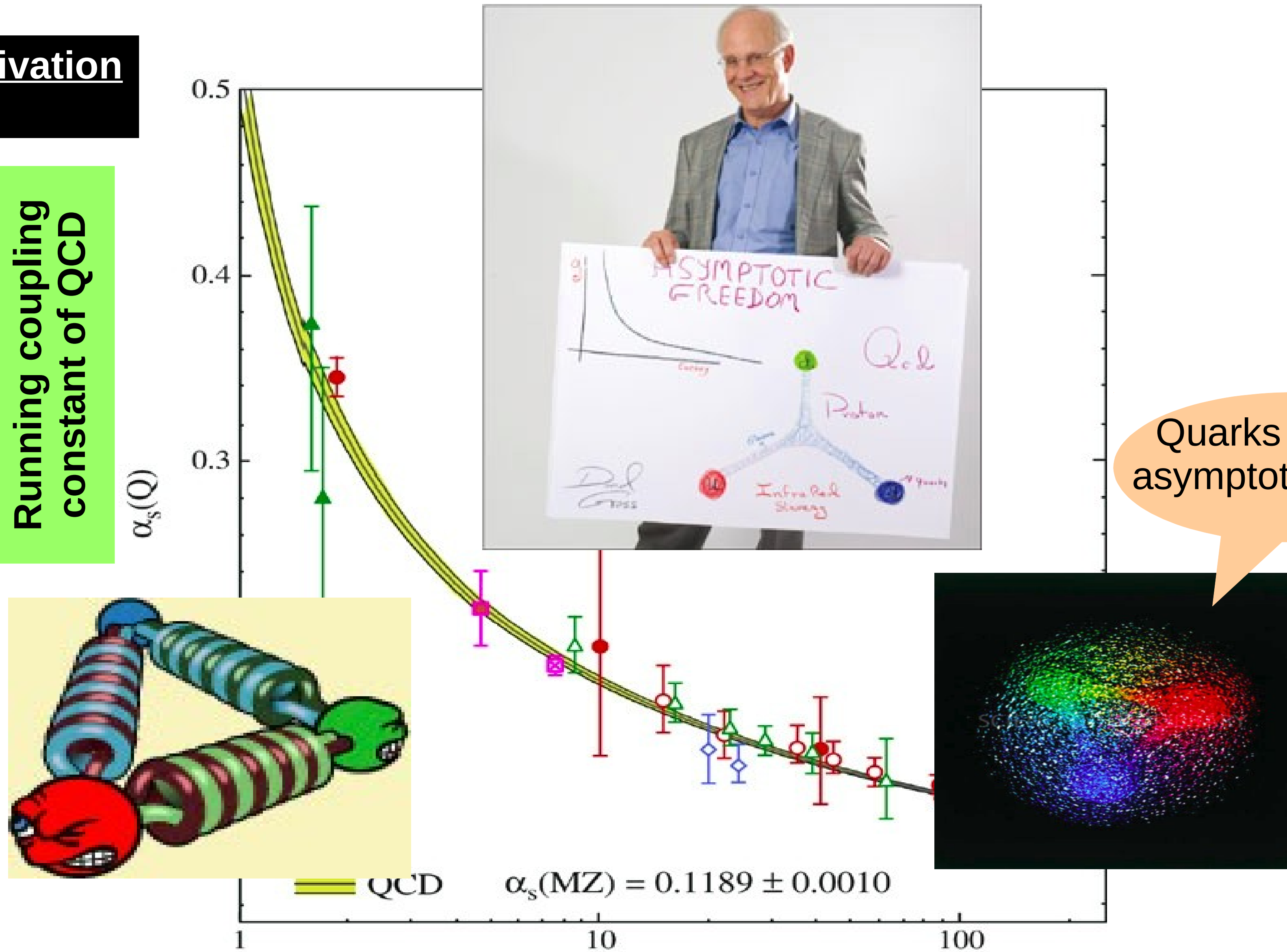
- **<u>Motivation of the Calculations</u>**
- **<u>Formalism part of the calculation</u>**
- **<u>Discussion on numerical results</u>**



Motivation

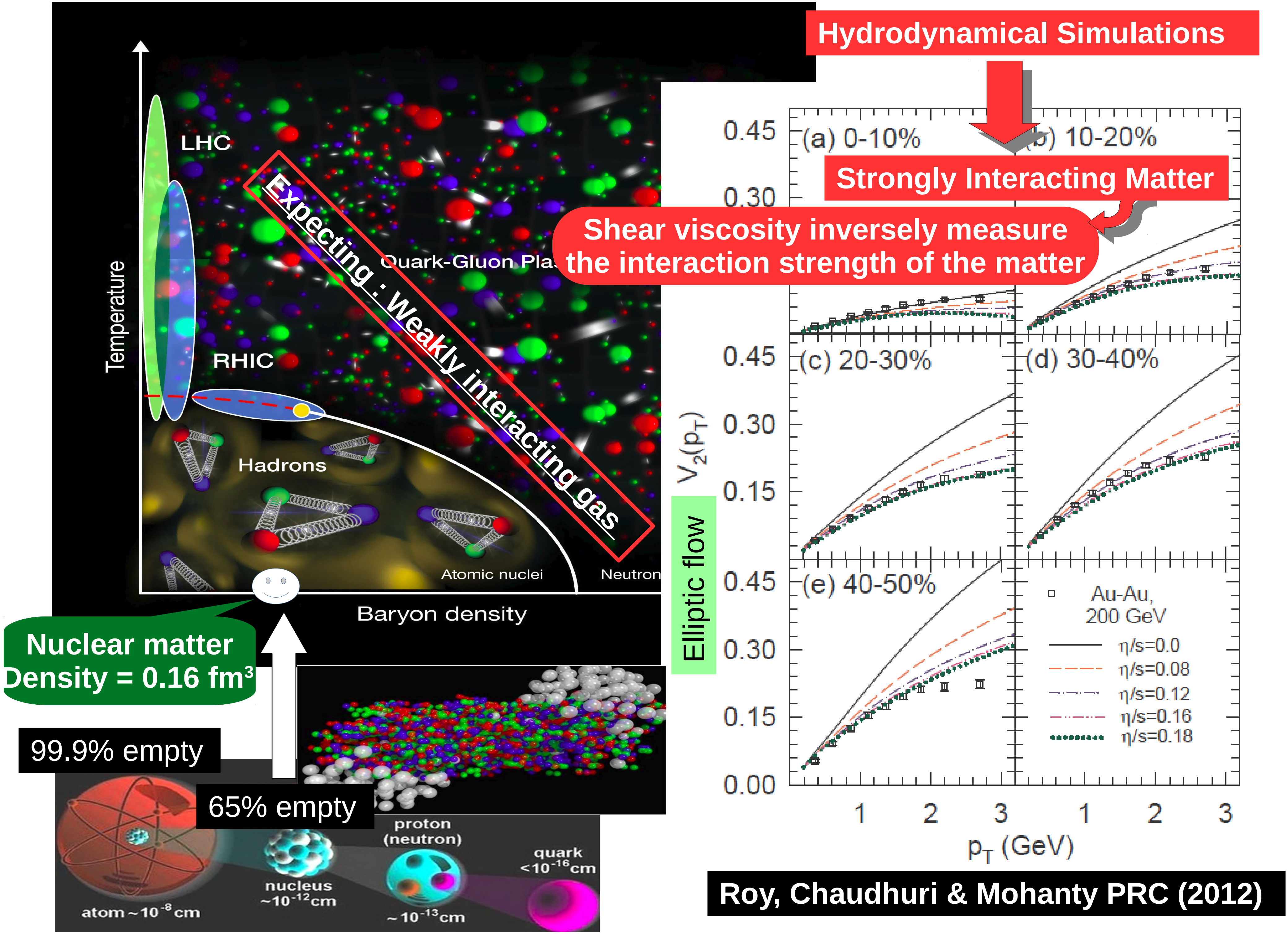
Running coupling constant of QCD



Quarks become asymptotically free

Momentum transfer

Distant between two quarks


Hydrodynamical Simulations
Strongly Interacting Matter
Shear viscosity inversely measure
the interaction strength of the matter
Expecting : Weakly interacting gas
Temperature
LHC
RHIC
Quark-Gluon Plas
Hadrons
Atomic nuclei
Neutron
Baryon density
Nuclear matter
Density = 0.16 fm³
99.9% empty
65% empty
atom ~10⁻⁸cm
nucleus ~10⁻¹²cm
proton (neutron)
~10⁻¹³cm
quark <10⁻¹⁶cm
(a) 0-10%
(b) 10-20%
(c) 20-30%
(d) 30-40%
(e) 40-50%
V₂(p_T)
Elliptic flow
0.45
0.30
0.15
0.00
1
2
3
p_T (GeV)
Au-Au, 200 GeV
η/s=0.0
η/s=0.08
η/s=0.12
η/s=0.16
η/s=0.18
Roy, Chaudhuri & Mohanty PRC (2012)

Kubo relation :

Shear viscosity

(Dissipative quantity)

$$\eta = \frac{1}{20} \lim_{q_0, \mathbf{q} \to 0} \frac{A_\eta(q_0, \mathbf{q})}{q_0} ,$$

$$A_\eta(q_0, \mathbf{q}) = \int d^4x \, e^{iq\cdot x} \langle [\pi_{ij}(x), \pi^{ij}(0)] \rangle_\beta$$

Thermal correlator (fluctuation) of

Viscous Stress Tensor

$$\pi_{\mu\nu} = t^{\rho\sigma}_{\mu\nu} T_{\rho\sigma} ,$$

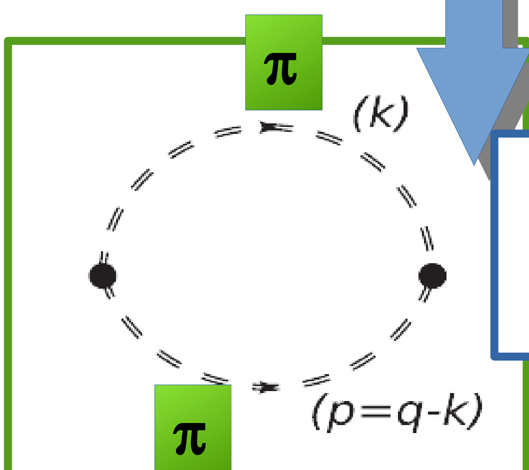
$$t^{\rho\sigma}_{\mu\nu} = \Delta^\rho_\mu \Delta^\sigma_\nu - \frac{1}{3} \Delta_{\mu\nu} \Delta^{\rho\sigma} .$$

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

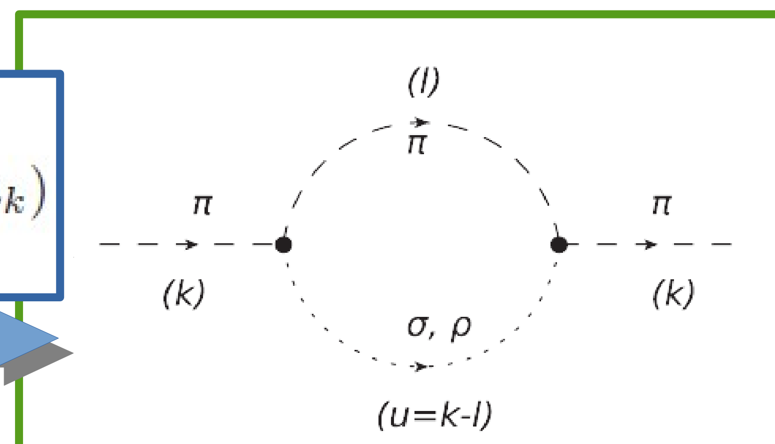
Energy-Momentum Tensor

$$T_{\rho\sigma} = -g_{\rho\sigma}\mathcal{L} + \frac{\partial \mathcal{L}}{\partial(\partial^\rho \phi)} \partial_\sigma \phi + \frac{\partial \mathcal{L}}{\partial(\partial^\sigma \phi)} \partial_\rho \phi$$
$$= -g_{\rho\sigma}\mathcal{L} + \partial_\sigma \phi \partial_\rho \phi$$

$$t^{\rho\sigma}_{\alpha\beta} t^{\alpha\beta}_{\mu\nu} i \int d^4x \, e^{iqx} \Big[\Big\langle T \partial_\sigma \phi(x) \partial_\rho \phi(x) \partial^\mu \phi(0) \partial^\nu \phi(0) \Big\rangle_\beta + \Big\langle T \partial_\rho \phi(x) \partial_\sigma \phi(x) \partial^\mu \phi(0) \partial^\nu \phi(0) \Big\rangle_\beta \Big]$$



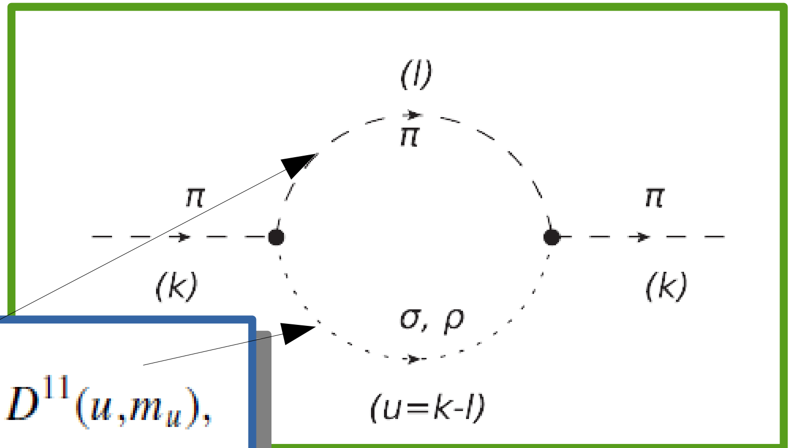
$$\eta_\pi = \frac{\beta}{10\pi^2} \int_0^\infty \frac{d\mathrm{k}\, \mathrm{k}^6}{\omega_k^2 \Gamma_\pi} n_k (1 + n_k)$$



Pion self-energy for πσ and πρ loops

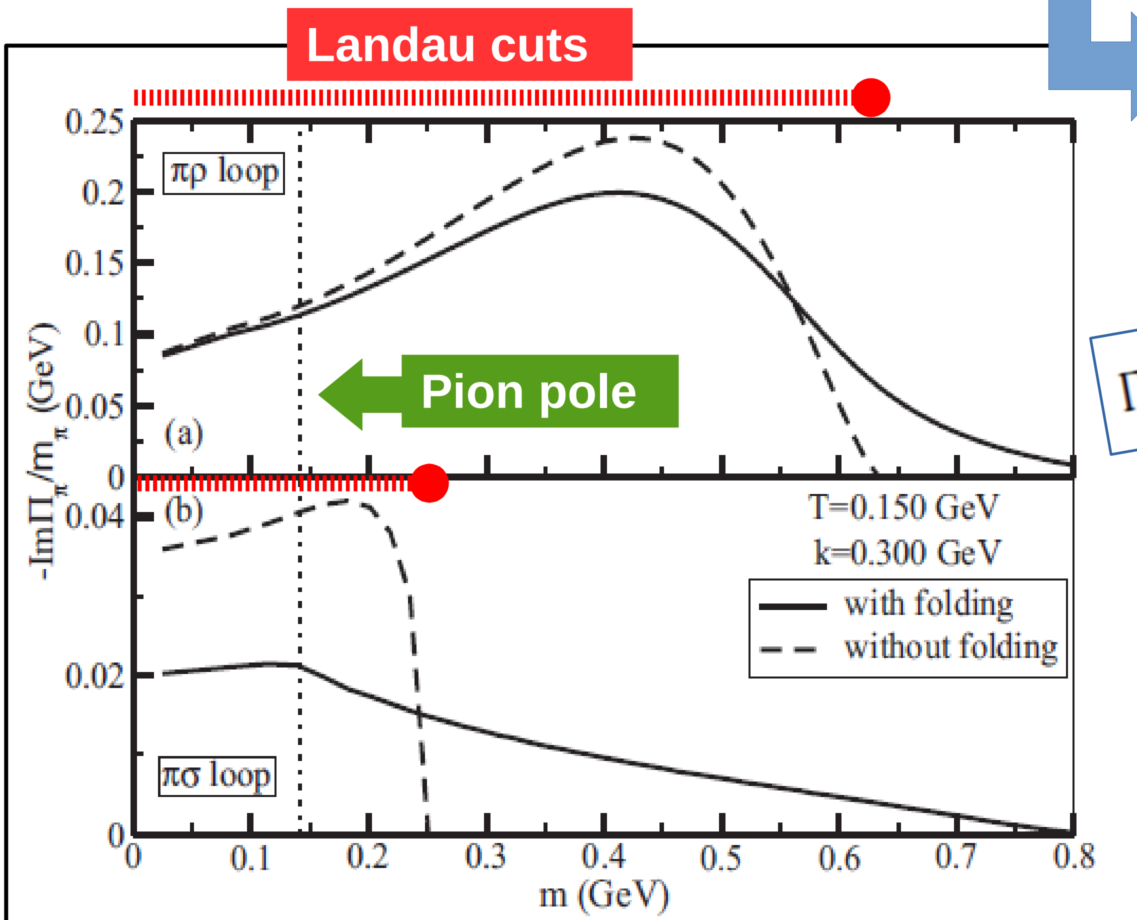
$$\Pi^{11}_{\pi}(k) = \Pi^{11}_{\pi}(k,\sigma) + \Pi^{11}_{\pi}(k,\rho),$$

$$\Pi^{11}_{\pi}(k,u) = -i \int \frac{d^4 l}{(2\pi)^4} L(k,l)\, D^{11}(l,m_l)\, D^{11}(u,m_u),$$



Without folding

$$\Gamma^{\rm nw}_{\pi}(k,T,u) = \frac{1}{16\pi |k| m_\pi} \int_{\omega_+}^{\omega_-} d\omega\, L(\omega) \times [n(\omega) - n(\omega_k + \omega)],$$



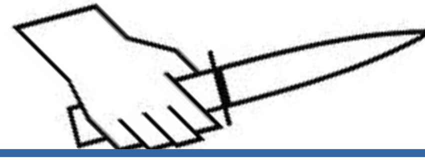
With folding

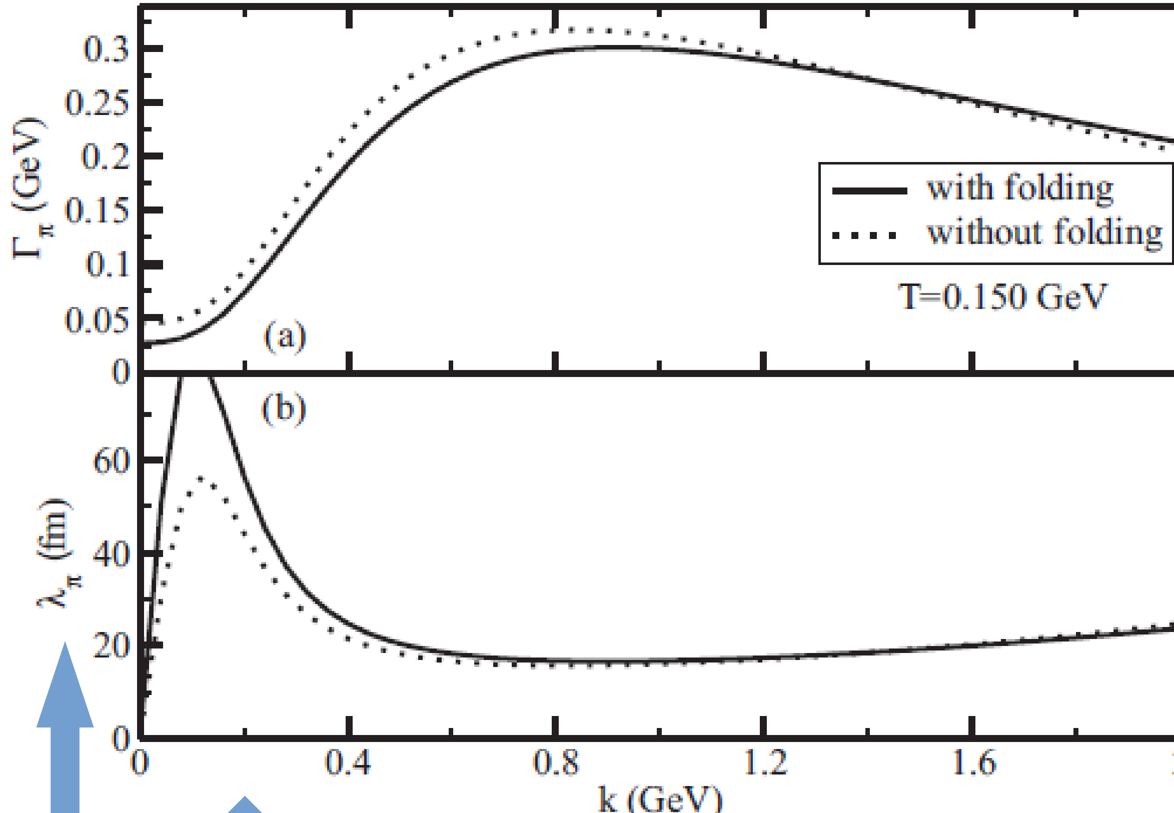
$$\Gamma_{\pi}(k,T,m_u) = \frac{1}{N_u} \int_{(m_u^-)^2}^{(m_u^+)^2} dM^2\, \rho_u(M)\, \Gamma^{\rm nw}_{\pi}(k,T;M),$$

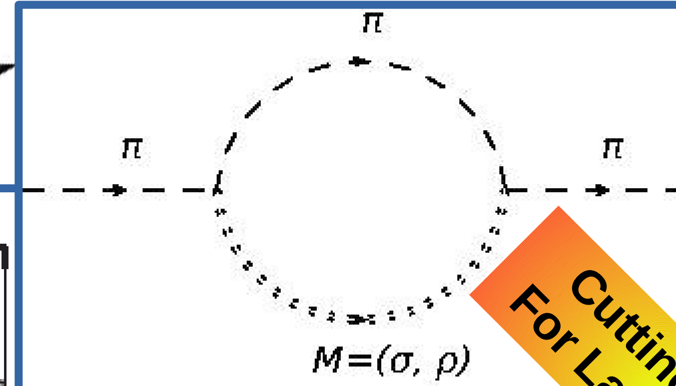
$$\rho_u(M) = \frac{1}{\pi} \mathrm{Im}\left[\frac{-1}{M^2 - m_u^2 + iM\Gamma_u(M)} \right]$$

$$N_u = \int_{(m_u^-)^2}^{(m_u^+)^2} dM^2\, \rho_u(M)$$

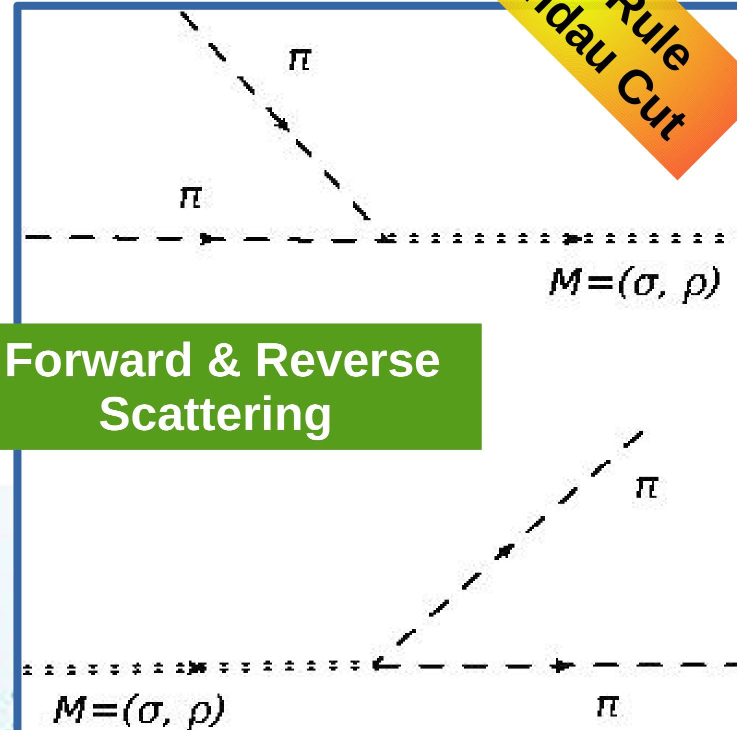
Pion Thermal Width as a function of its momentum







Cutting Rule For Landau Cut



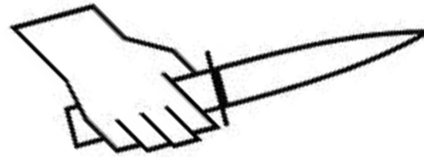
Forward & Reverse Scattering

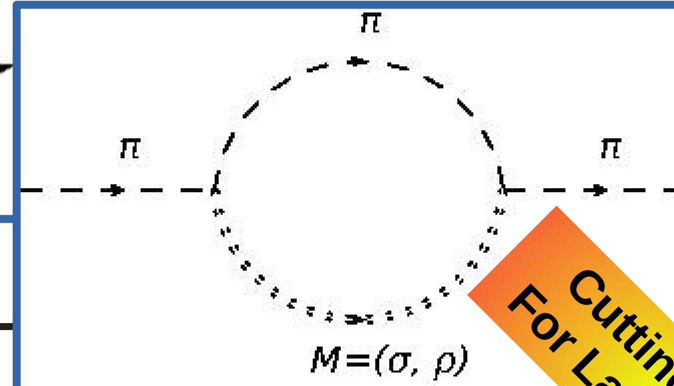
Mean Free Path

$$\lambda_\pi(k,T) = \frac{|k|}{\omega_k \, \Gamma_\pi(k,T)}$$

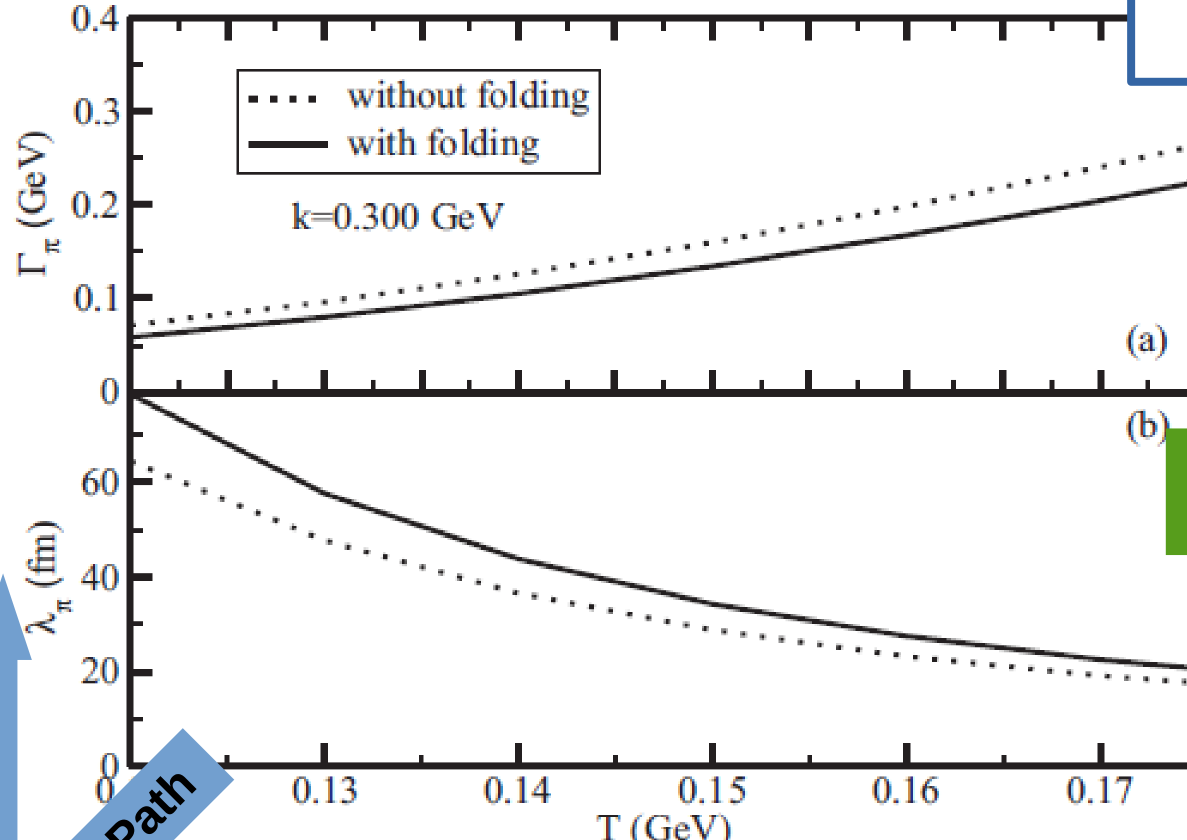
Propagating of pion (off-equilibrated) through pionic medium

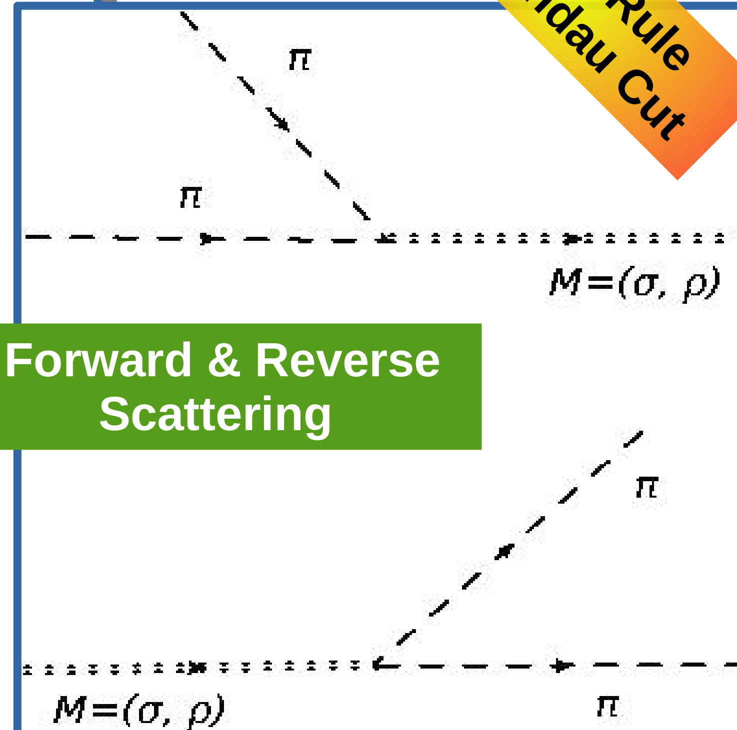
Pion Thermal Width as a function of temperature





Cutting Rule For Landau Cut





Forward & Reverse Scattering

Mean Free Path

$$\lambda_\pi(k,T) = \frac{|k|}{\omega_k\, \Gamma_\pi(k,T)}$$

Propagating of pion (off-equilibrated) through pionic medium

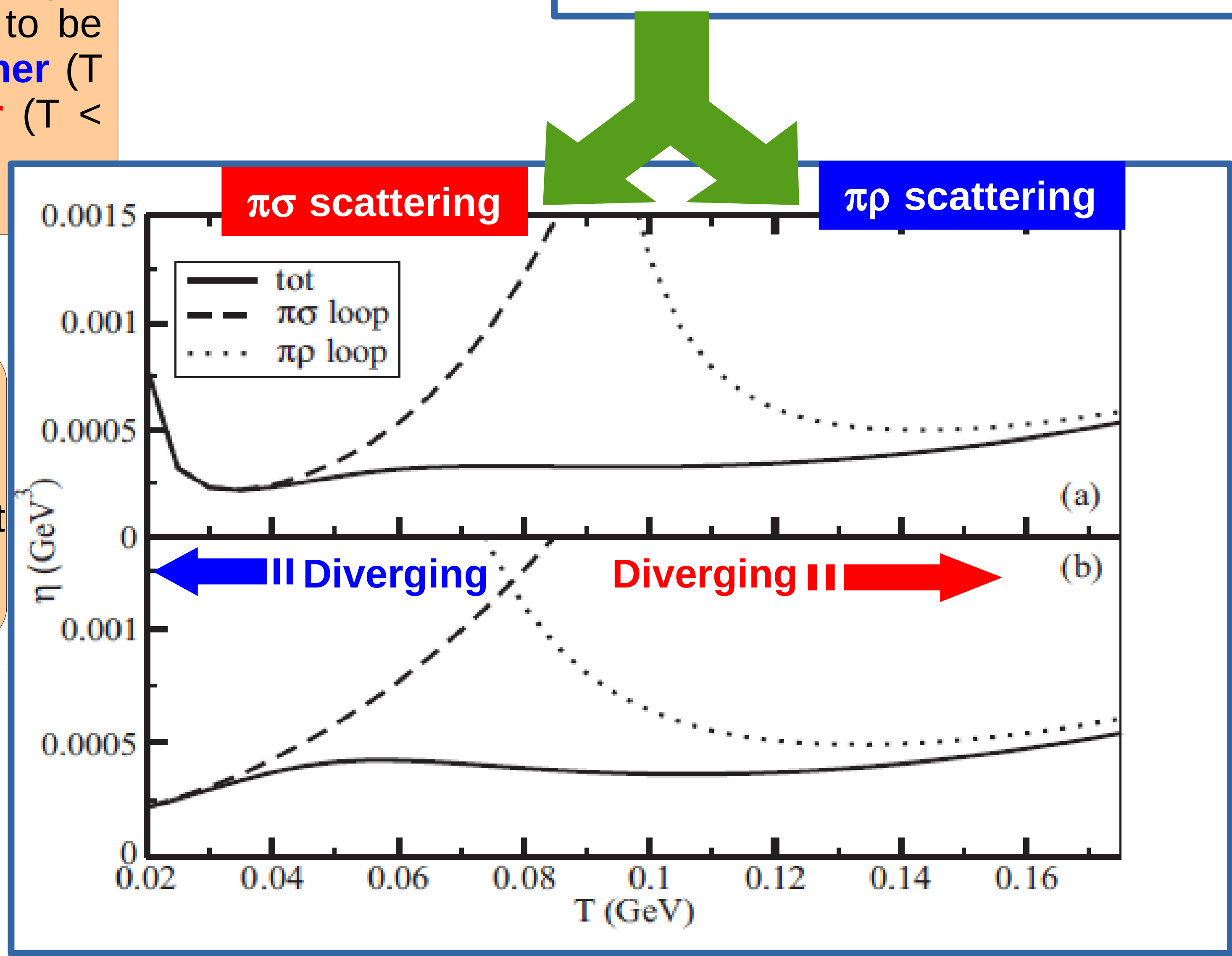
Complementary role of rho and sigma resonances in shear viscosity

$$\eta = \frac{\beta}{10\pi^2}\int \frac{d^3k\, k^6}{\Gamma_\pi(k,T)\,\omega_k^2}\, n(\omega_k)[1+n(\omega_k)],$$

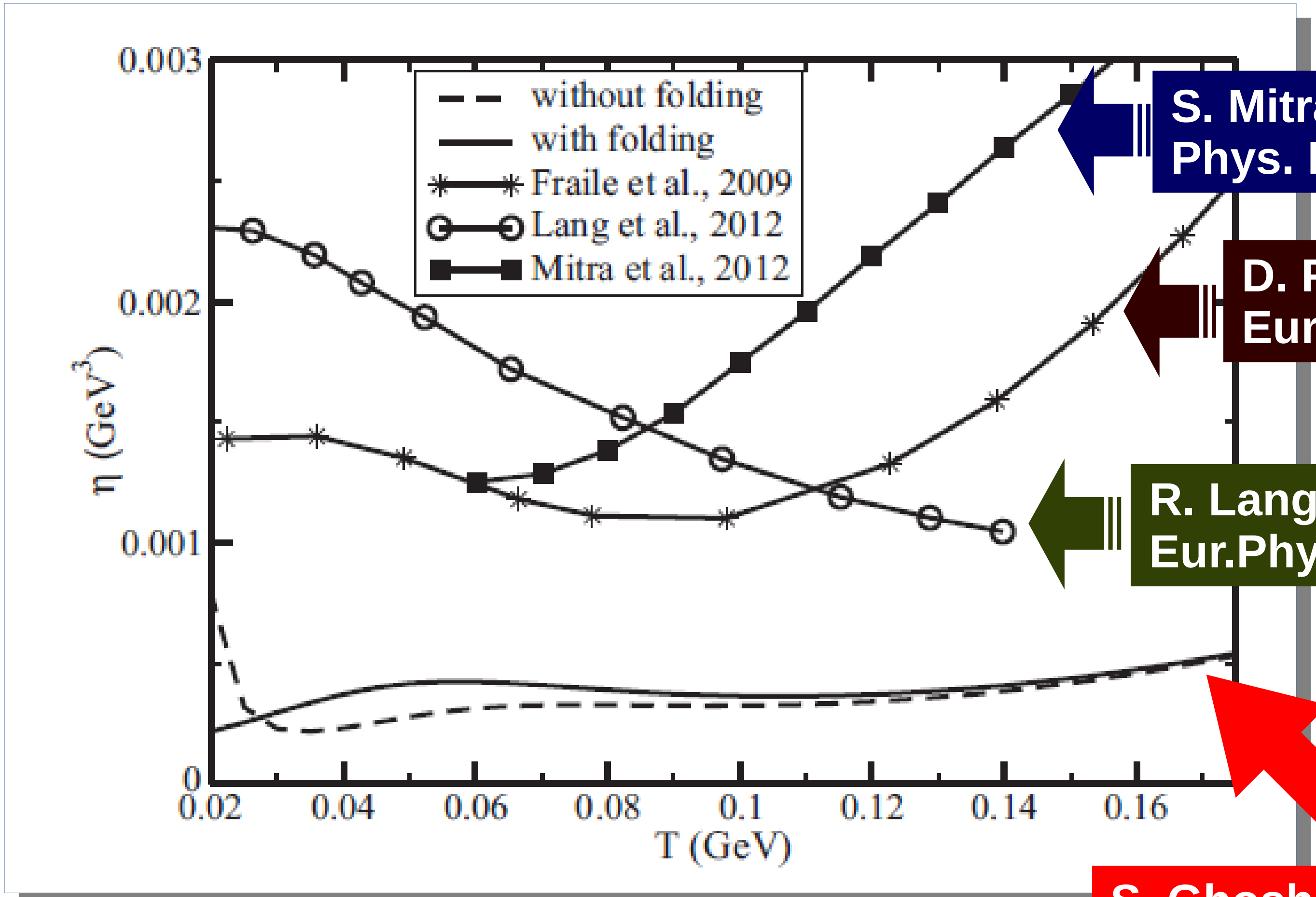
$$\Gamma_\pi(k,T) = \Gamma_\pi(k,T,\rho) + \Gamma_\pi(k,T,\sigma)$$

Interestingly, we see that the **πρ** and **πσ** contributions play a complementary role in η to be **nondivergent** in the **higher** (T > 0.100 GeV) and **lower** (T < 0.100 GeV) temperature regions, respectively.

both resonances in π-π scattering is strictly necessary to obtain a smooth, nondivergent η at the hadronic temperature domain



Comparison with the other earlier results



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(Present Work)

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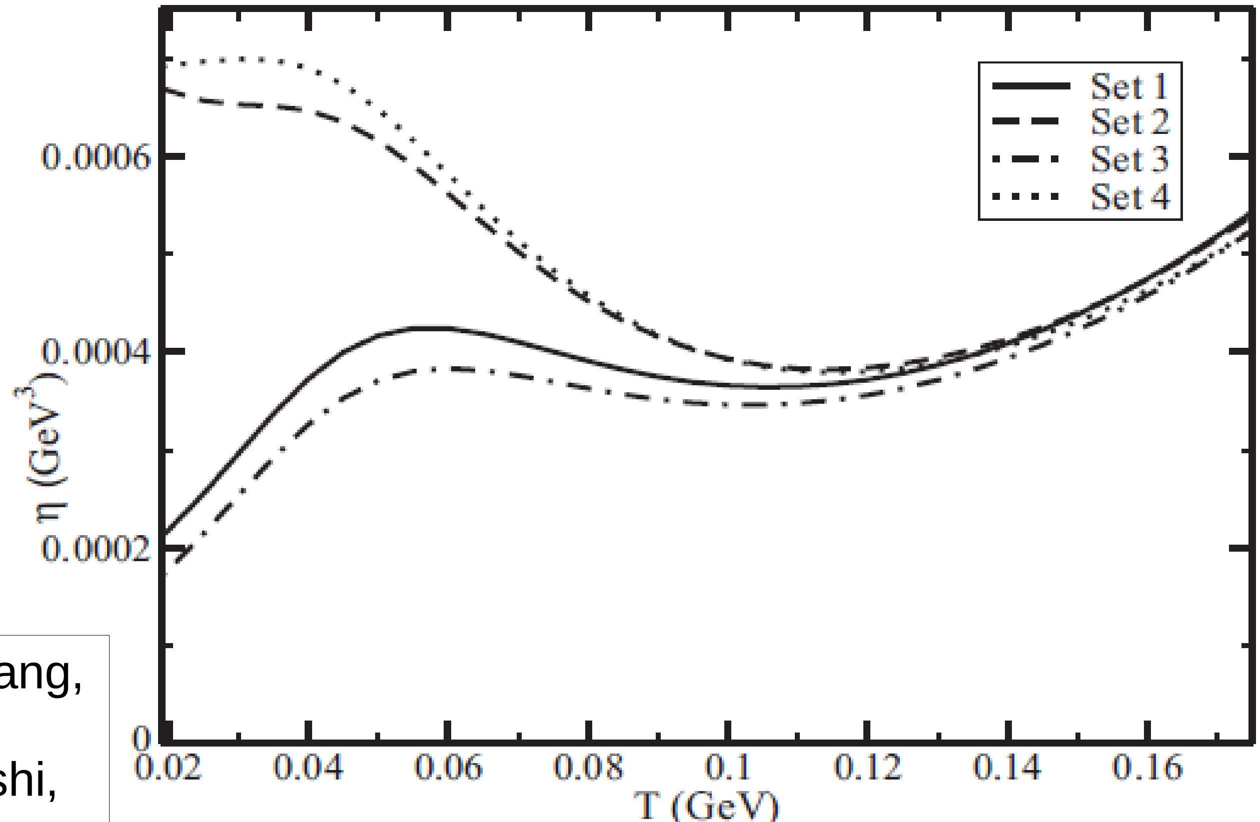
Numerical band of our Estimations

phenomenological uncertainty of the parameters of the σ resonance



	m_σ	Γ^0_σ	g_σ
Set 1 (BES) [57]	0.390	0.282	5.82
Set 2 (E791) [58]	0.489	0.338	5.73
Set 3 (PDG min) [59]	0.400	0.400	6.85
Set 4 (PDG max) [59]	0.550	0.700	7.03

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[58] E. M. Aitala et al. (Fermilab E791 Collaboration), **Phys. Rev. Lett. 86, 770 (2001)**.

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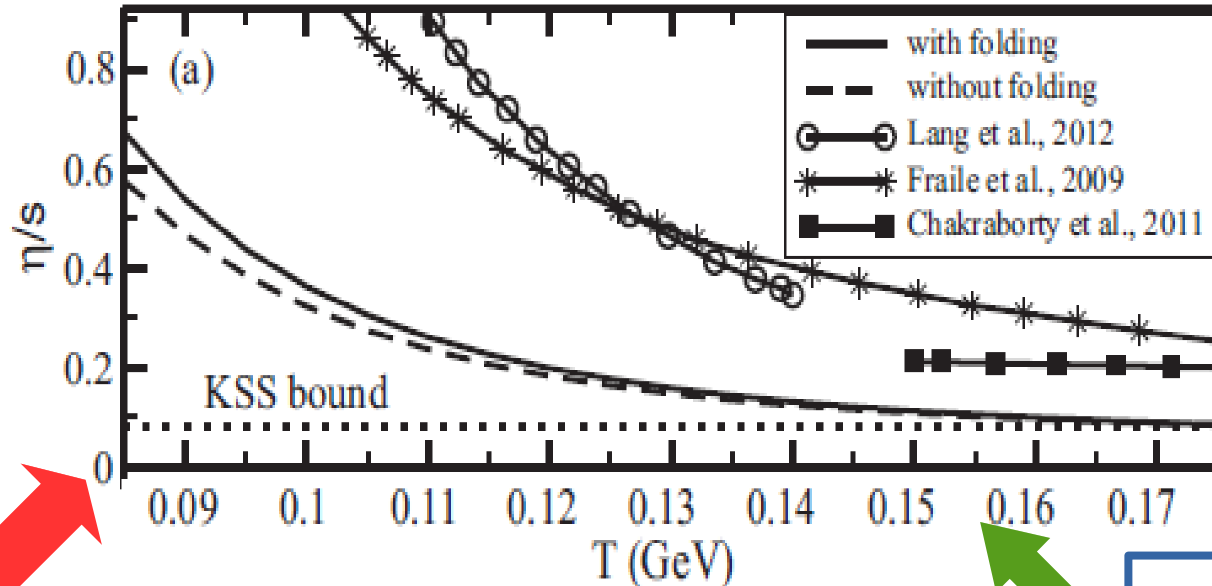
Viscosity to entropy density ratio



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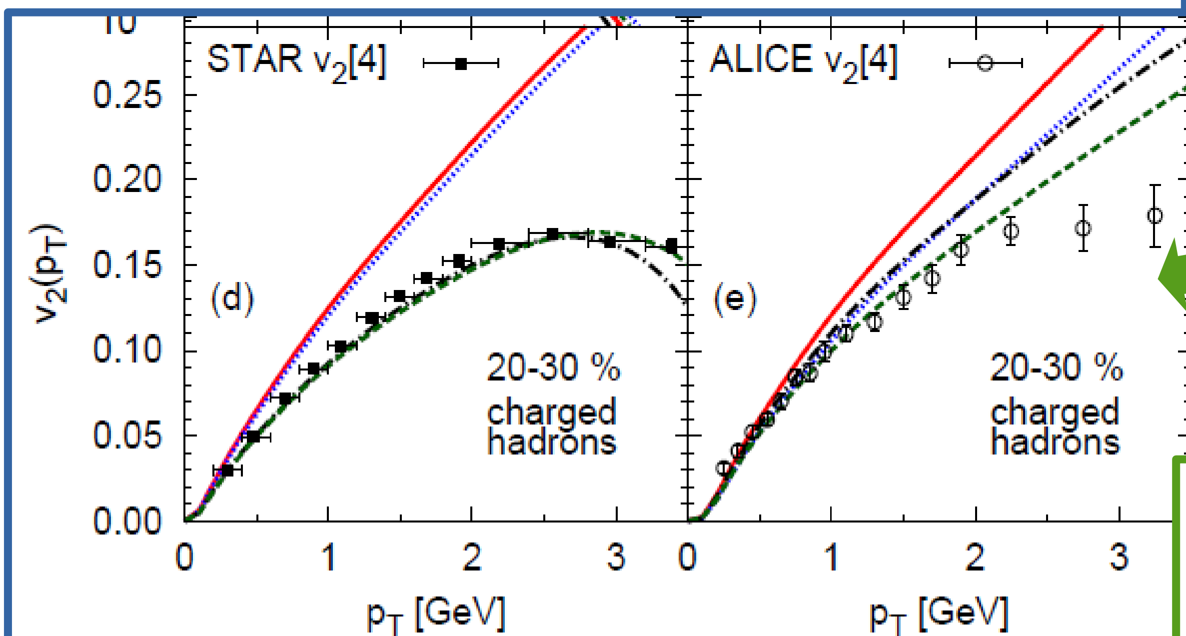

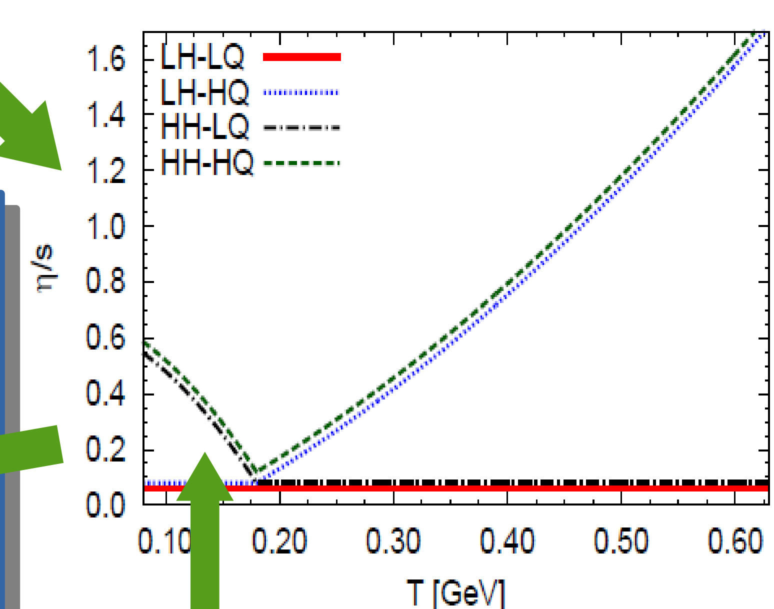

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