Effects of magnetic field on chiral symmetry breaking and CP violation

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Introduction and Motivation

1. Symmetries of QCD allows a parity violating topological term in the Lagrangian

$$\mathcal{L}^{\theta}_{QCD} = \mathcal{L}^{\theta=0}_{QCD} + \mathcal{L}^{\theta} = \mathcal{L}^{\theta=0}_{QCD} + \frac{\theta}{64\pi^2} g^2 F^a_{\mu\nu} \tilde{F}^{a\mu\nu}.$$

2. Axial anomaly

$$\partial^{\mu} J^f_{\mu 5} = 2 m_f i \bar{\psi}_f \gamma_5 \psi_f + \frac{N_f g^2 \theta}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{a\mu\nu}.$$

3. Electric dipole moment of neutron sets an upper bound

$$|\theta| < 0.7 \times 10^{-11}.$$

4. Strong CP problem

Introduction and Motivation

1. Parity violation is forbidden in vacuum for $\theta = 0 \implies$ Vafa-Witten Theorem.
2. Parity may be violated for $T, \mu \neq 0$.
3. Parity odd degenerate vacuum states are allowed for $\theta = \pi$ $\implies$ Dashen phenomena.
4. Non perturbative calculation is necessary.
5. Heavy ion collision may produce parity odd metastable state.
   - Chiral magnetic effect $\implies$ May be responsible for charge separation observed in STAR.
   - Possibility of excess dilepton production.
6. Magnetic fields can be of the order of 10^{18} Gauss - 10^{20} Gauss.

KMT-NJL model

1. The 3-flavor Nambu-Jona Lasinio(NJL) Lagrangian with Kobayashi-Maskawa-t'Hooft(KMT) term is given by

$$\begin{aligned} \mathcal{L}_{NJL}^{KMT} &= \bar{\psi} i \partial\!\!\!/ \psi - \bar{\psi} m \psi + \sum_{a=0}^{8} G \left[(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi} i \lambda_a \gamma_5 \psi)^2 \right] \\ &+ K \left[e^{-i\theta} det \bar{\psi}_i (1-\gamma_5)\psi_j + h.c \right] \\ &= \mathcal{L}_{NJL} + \mathcal{L}_{KMT}. \end{aligned}$$

2. The axial anomaly equation becomes

$$\partial^\mu J_{\mu 5} = 2mi\bar{\psi}\gamma_5\psi + 2iN_f K \left(e^{-i\theta} det\Phi - h.c \right),$$

where

$$\Phi_{ij} = \bar{\psi}_i (1-\gamma_5)\psi_j = 2\bar{\psi}_{iR}\psi_{jL}.$$

3. CP violation is embedded in $\mathcal{L}_{KMT}$.

Dirac Equation in magnetic field

1. The Dirac equation in presence of magnetic field is

$$(i\partial\!\!\!/ - qA\!\!\!/ - m)\psi = 0.$$

2. We choose the magnetic field to be of strength B and along the z direction.
3. We choose $A_\mu = (0, 0, Bx, 0)$.
4. For $E > 0$, the energy levels are given by

$$E_n^2 = m^2 + p_z^2 + (2n+1)|q|B - qB\alpha.$$

5. For $E < 0$, the energy levels are given by

$$E_n^2 = m^2 + p_z^2 + (2n+1)|q|B + qB\alpha.$$

$\alpha = 1$ for spin up and $\alpha = -1$ for spin down.

Ground state

1. We take the ground state with quark-antiquark pairs as

$$|\Omega\rangle = U_{II} U_{I} |vac\rangle$$

where

$$U_I = \exp\left[\sum_{n=0}^{\infty}\int d\boldsymbol{p}_{\not{x}}\, q_r^{i\,\dagger}(n,\boldsymbol{p}_{\not{x}}) a_{r,s}^{i}(n,p_z) f^i(n,\boldsymbol{p}_{\not{x}}) \tilde{q}_s^i(n,-\boldsymbol{p}_{\not{x}}) - h.c.\right]$$

with

$$a_{r,s}^{i} = \frac{1}{|\boldsymbol{p}^i|}\left[-\sqrt{2n|q^i|B}\,\delta_{r,s} - ip_z\delta_{r,-s}\right]$$

and

$$U_{II} = \exp\left[\sum_{n=0}^{\infty}\int d\boldsymbol{p}_{\not{x}}\, q_r^{i\,\dagger}(n,\boldsymbol{p}_{\not{x}}) r g^i(n,\boldsymbol{p}_{\not{x}}) \tilde{q}_s^i(n,-\boldsymbol{p}_{\not{x}}) - h.c.\right]$$

Thermodynamic potential

1. Expectation value of the scalar and pseudo scalar condensates are

$$I_s^i = -\frac{2N_c|qB|}{(2\pi)^2}\sum_{n=0}^{\infty}\alpha_n\int dp_z \cos\phi^i \cos 2g^i(1-n_-^i-n_+^i),$$

$$-iI_p^i = -i\frac{2N_c|qB|}{(2\pi)^2}\sum_{n=0}^{\infty}\alpha_n\int dp_z \sin 2g^i(1-n_-^i-n_+^i),$$

where $\alpha_n = 2-\delta_{n0}$. ϕ_i and g_i are the scalar and pseudoscalar condensate function.

2. The thermodynamic potential is given by

$$\Omega = T + V - \mu\langle\psi^\dagger\psi\rangle - \frac{S}{\beta}$$

Gap equation

1. Minimization of the thermodynamic potential gives

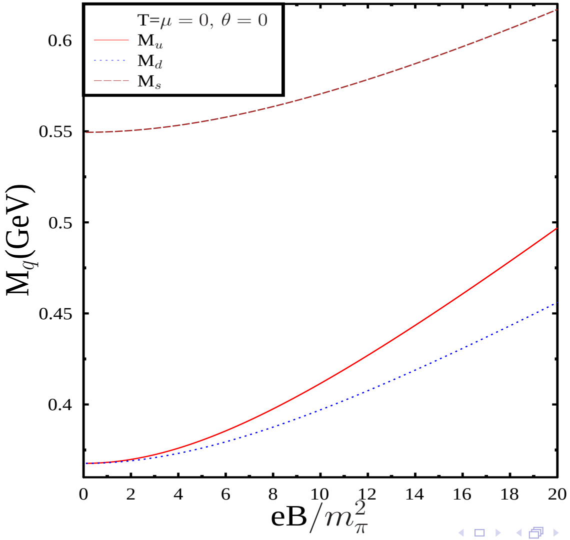
$$
\begin{aligned}
M_s^i &= m^i + 4GI_s^i + K|\epsilon_{ijk}|\{\cos\theta(I_s^j I_s^k - I_p^j I_p^k) - \sin\theta(I_s^j I_p^k + I_p^j I_s^k)\} \\
M_p^i &= 4GI_p^i - K|\epsilon_{ijk}|\{\cos\theta(I_s^j I_p^k + I_p^j I_s^k) - \sin\theta(I_p^j I_p^k - I_s^j I_s^k)\}.
\end{aligned}
$$

2. The total mass is $M^i = \sqrt{{M_s^i}^2 + {M_p^i}^2}$.
3. All these can be obtained by taking $eB = 0$ and using

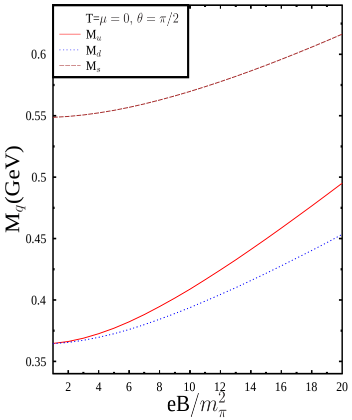
$$
\int \frac{d\boldsymbol{p}}{(2\pi)^3} \to \frac{|qB|}{(2\pi)^2} \sum_{n=0}^{\infty} \alpha_n \int dp_z .
$$

4. The gap equations are to be solved self consistently.

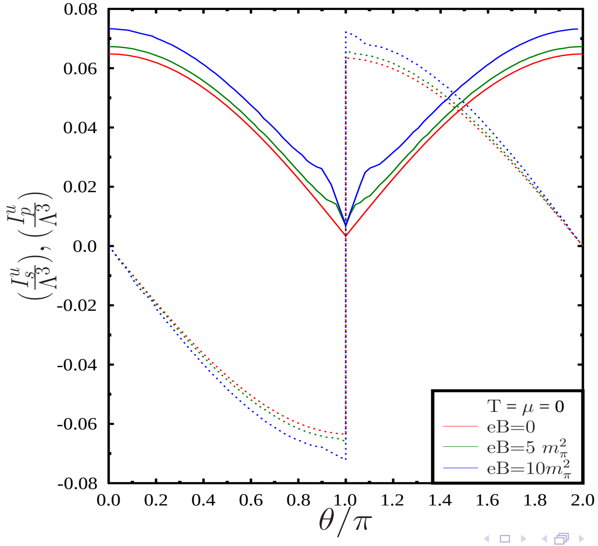
Magnetic catalysis at $T = \mu = 0$, $\theta = 0$.



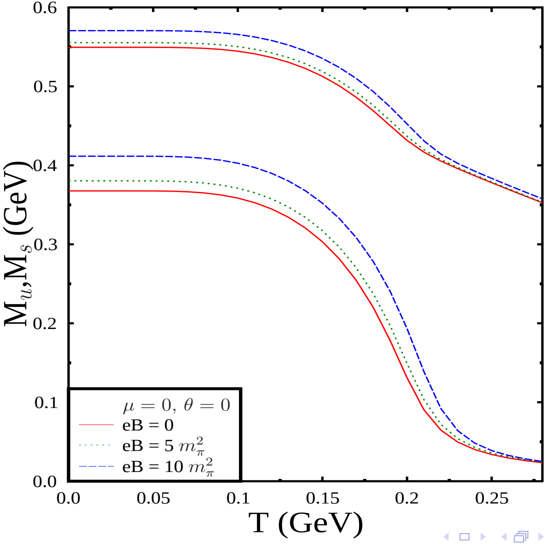
Magnetic catalysis at $T = \mu = 0$.



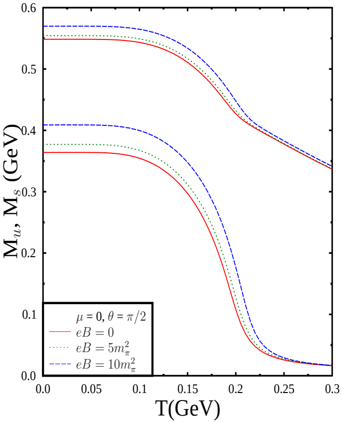
Degenerate states at $T = \mu = 0$.

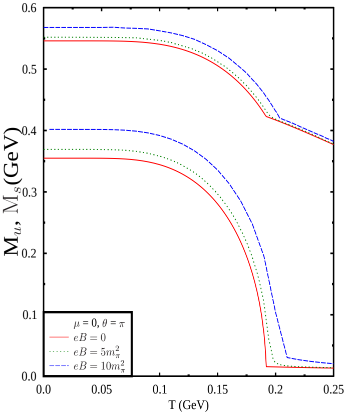


Magnetic catalysis $T \neq 0, \mu = 0$.

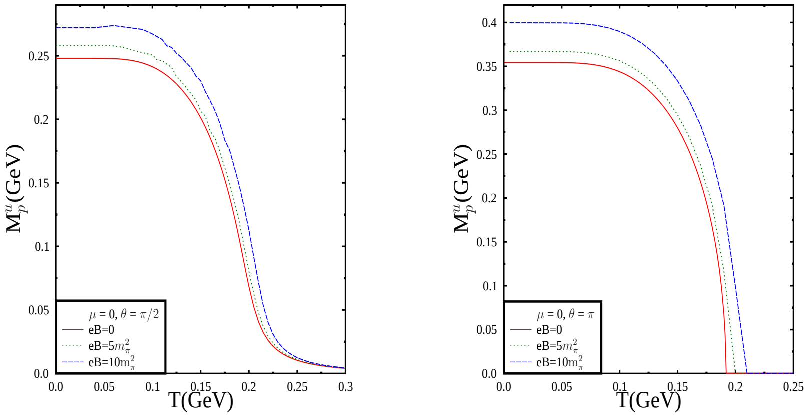


Magnetic catalysis $T \neq 0, \mu = 0.$

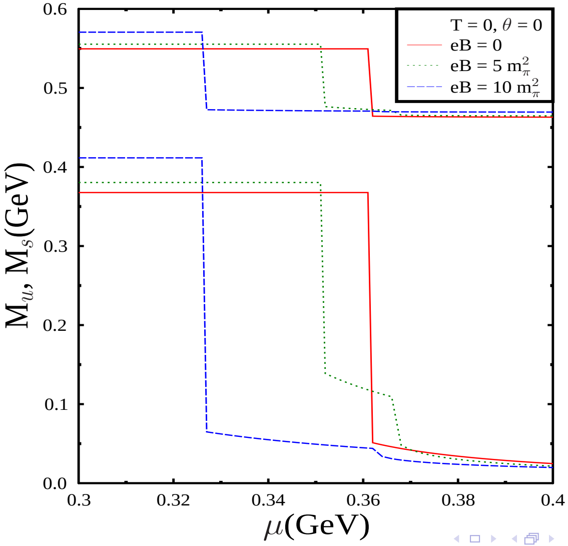




Magnetic catalysis $T \neq 0, \mu = 0.$

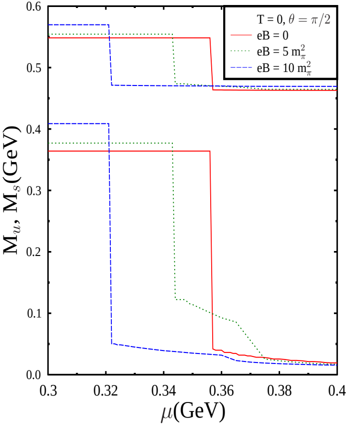


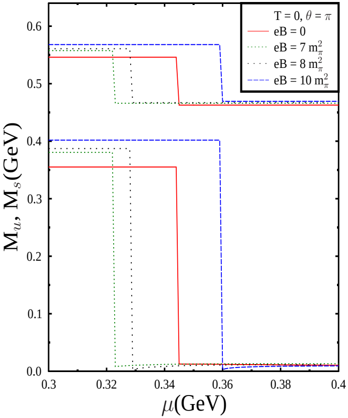
$T = 0, \mu \neq 0.$



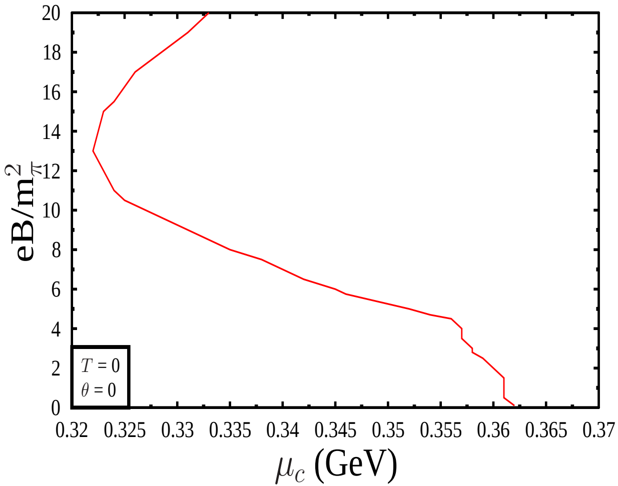
T = 0, θ = 0
eB = 0
eB = 5 m_π^2
eB = 10 m_π^2
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.3
0.32
0.34
0.36
0.38
0.4
M_u, M_s(GeV)
μ(GeV)

$T = 0, \mu \neq 0.$


T = 0, θ = π/2
eB = 0
eB = 5 m$_\pi^2$
eB = 10 m$_\pi^2$
M$_u$, M$_s$(GeV)
μ(GeV)
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.3
0.32
0.34
0.36
0.38
0.4

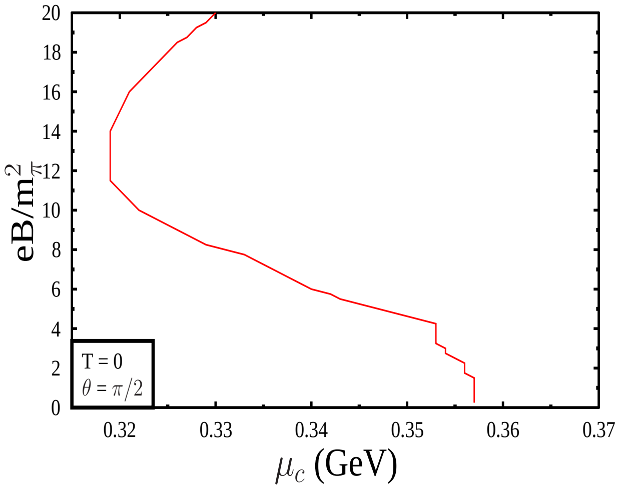

T = 0, θ = π
eB = 0
eB = 7 m$_\pi^2$
eB = 8 m$_\pi^2$
eB = 10 m$_\pi^2$
M$_u$, M$_s$(GeV)
μ(GeV)
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.3
0.32
0.34
0.36
0.38
0.4

μ_c Vs eB

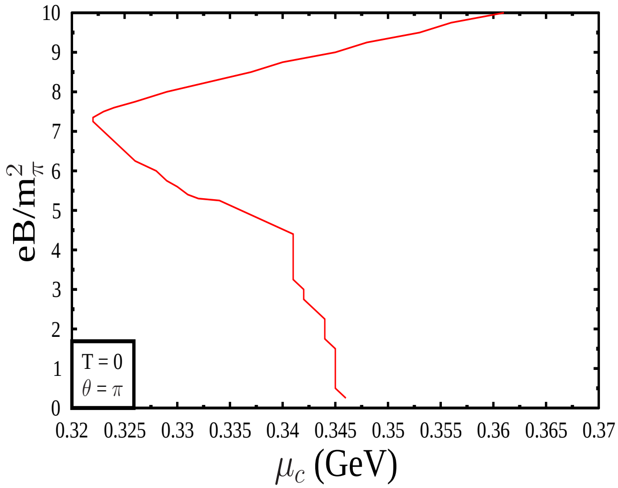


20
18
16
14
12
10
8
6
4
2
0
eB/m$_\pi^2$
0.32 0.325 0.33 0.335 0.34 0.345 0.35 0.355 0.36 0.365 0.37
μ_c (GeV)
$T = 0$
$\theta = 0$

μ_c Vs eB



20
18
16
14
12
10
8
6
4
2
0
eB/m$_\pi^2$
0.32
0.33
0.34
0.35
0.36
0.37
μ_c (GeV)
T = 0
$\theta = \pi/2$



10
9
8
7
6
5
4
3
2
1
0
eB/m$_\pi^2$
T = 0
$\theta = \pi$
0.32
0.325
0.33
0.335
0.34
0.345
0.35
0.355
0.36
0.365
0.37
μ_c (GeV)

Summary

1. CP violation is possible within NJL model for $\theta \neq 0$.
2. The scalar and the pseudoscalar contributions behaves in a complimentary way such that the total mass remains almost constant for different values of θ.
3. Magnetic field enhances the constituent mass significantly.
4. Inverse magnetic catalysis is seen for finite μ but not for finite T.
5. CP transition with temperature is second order for $\theta = \pi$.
6. CP transition is first order with $\mu \implies$ CP odd metastable domains might be found in collider experiments.

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2. Centre for Nuclear Theory, VECC.
3. Organizers of ICPAQGP 2015.

1. The positive energy solutions are

$$U_\uparrow(x, \boldsymbol{p}_{\not{x}}, n) = N_n \begin{bmatrix} (\epsilon_n + m)\left\{\Theta(q) I_n + \Theta(-q) I_{n-1}\right\} \\ 0 \\ p_z \left\{\Theta(q) I_n + \Theta(-q) I_{n-1}\right\} \\ -i\sqrt{2n|q|B}\left\{\Theta(q) I_{n-1} + \Theta(-q) I_n\right\} \end{bmatrix}$$

$$U_\downarrow(x, \boldsymbol{p}_{\not{x}}, n) = N_n \begin{bmatrix} 0 \\ (\epsilon_n + m)\left\{\Theta(q) I_{n-1} + \Theta(-q) I_n\right\} \\ i\sqrt{2n|q|B}\left\{\Theta(q) I_n - \Theta(-q) I_{n-1}\right\} \\ -p_z \left\{\Theta(q) I_n - \Theta(-q) I_{n-1}\right\} \end{bmatrix}.$$

1. The negative energy solutions are

$$V_{\uparrow}(x, -\boldsymbol{p}_{\not x}, n) = N_n \begin{bmatrix} \sqrt{2n|q|B}\,\{\Theta(q)I_n - \Theta(-q)I_{n-1}\} \\ ip_z\,\{\Theta(q)I_{n-1} + \Theta(-q)I_n\} \\ 0 \\ i(\epsilon_n + m)\,\{\Theta(q)I_{n-1} + \Theta(-q)I_n\} \end{bmatrix},$$

$$V_{\downarrow}(x, -\boldsymbol{p}_{\not x}, n) = N_n \begin{bmatrix} ip_z\,\{\Theta(q)I_n + \Theta(-q)I_{n-1}\} \\ \sqrt{2n|q|B}\,\{\Theta(q)I_{n-1} - \Theta(-q)I_{n-1}\} \\ -i(\epsilon_n + m)\,\{\Theta(q)I_n + \Theta(-q)I_{n-1}\} \\ 0 \end{bmatrix}.$$