Magnetic field and Neutron stars: A comprehensive study

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- Recent observation emphasized the need to study the compact astrophysical objects to have a better understanding of the properties of matter under extreme conditions.
- The extremely dense matter ($\sim 10^{15}$ gm/cc) and strong magnetic fields ($10^{8} - 10^{12} G$) of NS profoundly affects their observable manifestations like thermal evolution, emission and rotation dynamics.
- Over the last few years, a number of observational discoveries have brought Magnetars (ultra-magnetized isolated neutron stars) to the forefront of researchers attention.

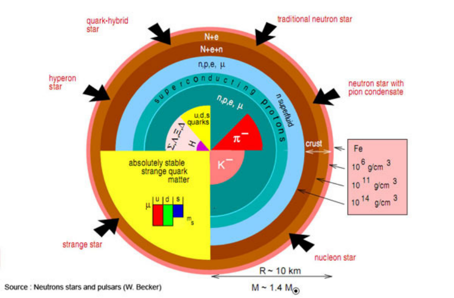


Figure : Pictorial description of a model NS (Source: F. Weber.)

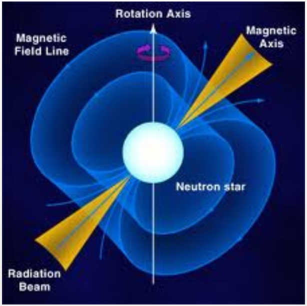


Figure : Magnetic field of neutron star

- These extreme objects comprise the Anomalous X-ray Pulsars (AXPs; 10 objects) and the Soft Gamma-ray Repeaters (SGRs; 5 objects), which are observationally very similar in many aspects:

$$
\begin{aligned}
P &\sim 2 - 12\, s \\
B_s &\sim 10^{14} - 10^{15} G
\end{aligned}
$$

- The surface magnetic field is calculated from spin down and the rate of change of spin down (assuming the profile to be dipolar).
- Max field Strength: General relativistic calculations show it to lie between $10^{18} - 10^{19}$ G. Similar values are also obtained from Virial theorem estimates employing a classical calculation (Pothekin & Yakovlev PRC 85, 2012).

- Star Deformation
- The general relativistic approach by Bonazolla & Gourgoulhon (A&A 278, 1993), Bocquet et al.(A&A 301, 1995) and Cardall et al.(ApJ 554, 2001).
  Solve the coupled matter and electromagnetic equation numerically. Starting with an current function and obtain either a poloidal or a toroidal field. Deformation of the order of $10-15\%$.
- An analytic discussion was done by Konno et al. (A&A 352, 1999), lacked a discussion involving a realistic EoS. They solve the field equations perterbatively, expanding the metric potentials upto quadrupole terms.
- Recent calculation by Ciolfi et al. (MNRAS 397, 2009) also solves for the star deformation due to magnetic field following a perterbative approach. They assume the magnetic field to be of the form known as "twisted torus".

- The effect of magnetic field in the EoS of the neutron star. In most of these calculations (Bandyopadhyay et al. PRL 79, 1997) the charged particles are Landau quantized perpendicular to the magnetic field.

$$E_i = \sqrt{{p_i}^2 + {m_i}^2 + |q_i|B(2n + s + 1)}.$$

- The number density and energy density changes for the charged particles, which changes the EoS. However, the most significant effect comes from the magnetic stress and pressure which adds to the matter energy and pressure.
- Although the magnetic pressure is anisotropic, they usually add or subtract the magnetic pressure contribution from the matter pressure. General 2D treatment is needed.

- In the rest frame of the fluid the magnetic field is aligned along the z-axis

$$\epsilon = \epsilon_m + \frac{B^2}{8\pi}$$
$$P_\perp = P_m - MB + \frac{B^2}{8\pi}$$
$$P_\parallel = P_m - \frac{B^2}{8\pi}.$$

- The magnetization becomes significant at ultra large strong fields ($> 10^{19}$ G) when the star itself becomes unstable.

$$P_\perp = P_m + \frac{B^2}{8\pi}$$
$$P_\parallel = P_m - \frac{B^2}{8\pi}.$$

- The pressure (Mallick & Schramm, PRC 89, 045805, 2014)

$$P = P_m \pm P_B$$
$$P = P_m + \frac{B^2}{8\pi}(1 - 2cos^2\theta).$$

- Total pressure as an expansion in spherical harmonics

$$P = P_m + \frac{B^2}{8\pi}[\frac{1}{3} - \frac{4}{3}P_2(cos\theta)]$$
$$P = P_m + [p_0 + p_2 P_2(cos\theta)].$$

- Static spherically symmetric object can be written in Schwarzschild coordinates

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2)$$

- Multipole expansion of metric

$$\begin{aligned} ds^2 = & -e^{\nu(r)}[1 + 2(h_0(r) + h_2(r)P_2(cos\theta))]dt^2 \\ & + e^{\lambda(r)}[1 + \frac{e^{\lambda(r)}}{r}(m_0(r) + m_2(r)P_2(cos\theta))]dr^2 \\ & + r^2[1 + 2k_2(r)P_2(cos\theta)](d\theta^2 + sin^2\theta d\phi^2) \end{aligned}$$

- Solving the Einstein equations ($G_{\mu\nu} = 8\pi G T_{\mu\nu}$)

$$\frac{dm_0}{dr} = 4\pi r^2 p_0,$$

$$\frac{dh_0}{dr} = 4\pi r e^{\lambda} p_0 + \frac{1}{r}\frac{d\nu}{dr} e^{\lambda} m_0 + \frac{1}{r^2} e^{\lambda} m_0,$$

$$\frac{dh_2}{dr} + \frac{dk_2}{dr} = h_2\left(\frac{1}{r} - \frac{\frac{d\nu}{dr}}{2}\right) + \frac{e^{\lambda}}{r} m_2\left(\frac{1}{r} + \frac{\frac{d\nu}{dr}}{2}\right),$$

$$h_2 + \frac{e^{\lambda}}{r} m_2 = 0,$$

$$\frac{dh_2}{dr} + \frac{dk_2}{dr} + \frac{1}{2} r \frac{d\nu}{dr}\frac{dk_2}{dr} = 4\pi r e^{\lambda} p_2 + \frac{1}{r^2} e^{\lambda} m_2 + \frac{1}{r}\frac{d\nu}{dr} e^{\lambda} m_2 + \frac{3/r}{e}^{\lambda} h_2 + \frac{2}{r} e^{\lambda} k_2.$$

- The assumed magnetic profile of the star is density dependent

$$B(n_b) = B_s + B_0 \left\{ 1 - e^{-\alpha \left(\frac{n_b}{n_0} \right)^{\gamma}} \right\}.$$

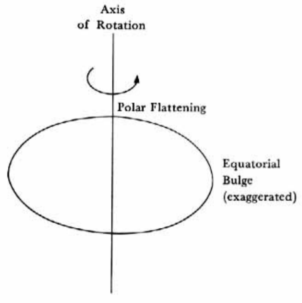
- For $B_0 = 4 \times 10^{18}$ G, the magnetic field at the center (max 8 times nuclear density) is less than $B_c = 1.75 \times 10^{18}$ G. The ratio of magnetic pressure (P_B) to that of matter pressure (P_m) is always less than 0.5, and the ratio of total magnetic energy to gravitational energy is less than 0.1.
- The mass change

$$M = M_0 + \delta M$$

where, $\delta M = m_0(R)$.

- The magnetic pressure gets added in the equatorial direction and subtracted along the polar direction (Oblate spheroid). The polar and equatorial radius of the deformed star is defined as

$$R_e = R + \xi_0(R) - \frac{1}{2}(\xi_2(R) + Rk_2),$$
$$R_p = R + \xi_0(R) + (\xi_2(R) + Rk_2)$$



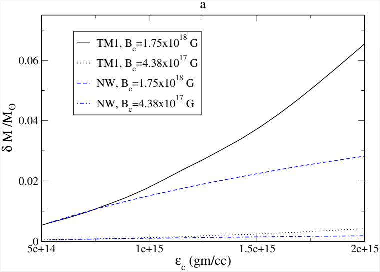


Figure : Mass change vs central energy density

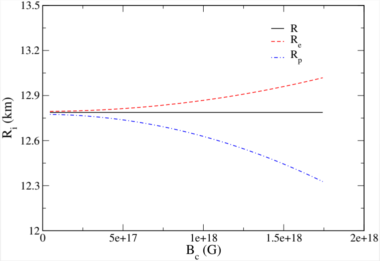


Figure : Radial change due to B

- Mass or the deformation is about $5-6\%$, much less than previous 1D calculation.

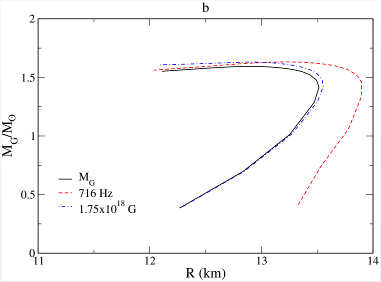


Figure : mass-radius diagram comparing magnetic and rotational effect

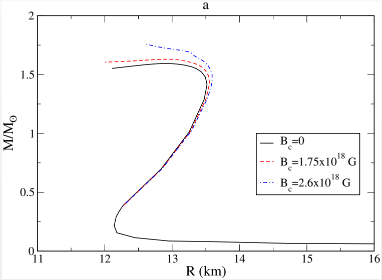


Figure : mass-radius diagram for the limiting magnetic field value

- A rotating star which deviates from axissymetry emits GW signals.
- If we assume that the magnetic axis and the rotation axis are not aligned, as it is the case in observable pulsars.
- The amplitude of the GW signal is given by

$$h_0 = \frac{4G}{dc^4}\Omega^2 I\epsilon \sin\alpha$$

- SGR 1900+14 $h_0 \sim 10^{-28}$
- SGR 10501 + 4516 $h_0 \sim 10^{-29}$
- Capabilities of the VIRGO detector, minimum amplitude detectable $h_{min} \sim 10^{-26}$.
- the situation may drastically change if we have a magnetar much closer to us (around 2 kpc), and particularly with a significantly reduced rotational period (100 ms) $h_0 \sim 10^{-23} - 10^{-24}$.

- Magnetic field can have other important consequence like cooling of the NS and formation of condensate.
- Formation of condensate in NS is not new. Earliest work done by Migdal (JETP 61, 1971) in respect to pion condensate.
- Kaon condensate also can occur in NS (Kaplan & Nelson, PLB 175, (1986)).
- Electron chemical potential is an increasing function of density, whereas meson mass is an decreasing function of it.
- When the meson energy becomes less than the electron chemical potential, it is favorable for neutrons to decay to protons and meson(negative), than protons and electrons, giving rise to a mesonic condensate.

- The energy squared of the lowest state for a charged rho-meson corresponding to $p = 0, n = 0$ and $S_x = 1$ is

$$\epsilon^2_{0,1}(0) = m^{*2}_{B\rho^-} = m^2_{\rho^-} - eB.$$

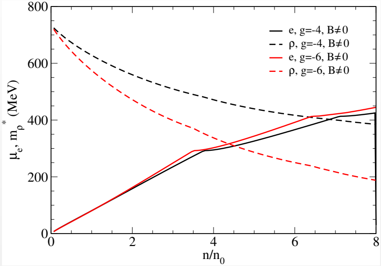
- To produce rho condensation solely due to a magnetic field, very high magnetic field are required ($3 - 4 \times 10^{19}$ G). Difficult, if not impossible to realize in NS.
- What if the effective mass is further reduced due to in-medium effect.
- For simplicity, we adopt a linear dependence of the rho mass on the scalar field (Mallick et al. ArXiv:1408.0139, 2014).

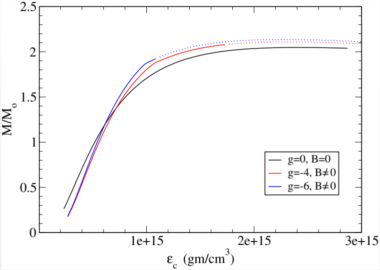
$$m^{\sigma *}_{\rho} = m_\rho - g\sigma$$

g effectively introduces a coupling term of the vector and scalar fields.

- Finally, we have

$$\epsilon^2 = m^{*2}_{\rho^-} = (m_\rho - g\sigma)^2 - eB.$$





- The electron chemical potential (solid lines) and effective ρ^- mass (dashed lines) are plotted as functions of normalized density. The points of intersection mark the density at which the *rho*-meson condensation appears.
- Sequence of stars where mass is shown as function of the central energy density. The dotted lines indicate stars that contain some amount of rho condensate.

- The magnetic effect is not strong enough to have any big effect which on the existing observable signatures coming from NS. Both for the mass change and stars deformation (distinguishing different stars).
- If the magnetic field is very high, in the M-R curve there no maximum mass and the mass goes on increasing. This may indicate some type of instability (star going towards a black hole) or to limit the maximum magnetic field of the star.
- No current isolated magnetars would provide enough strong GW to detected in the near future.
  However, NS mergers would emit strong GW signals to be detected by them.
- Whether there is a condensate or not? Only magnetic field is not sufficient but requires rho-meson mass modification in the dense medium.
- **The origin of such strong magnetic field.** Many existing models like flux freezing and dynamo mechanism but none are satisfactory.

Thank You

- Origin of such high magnetic field? Duncan & Thompson, ApJ 392 (1992)
- The "magnetar" theory of SGRs originated from the study to explain the origin of magnetic fields in radio pulsars.
- Star born with fast enough rotation: the combined effects of rotation and convection, drag field lines through the star, build up the star's overall magnetic field, via a complicated process known as "dynamo action."
- The dynamo works in a hot, newborn neutron star to generate a field of about 10^{16} G. As the star cools, convection and dynamo action cease. The field remains trapped in the heavy, stratified liquid of neutrons and protons.
- However, the problem still remains unsolved.

- TOV equation (differential equation for pressure and mass) for spherically symmetric star

$$\frac{dP(r)}{dr} = -\frac{Gm(r)\varepsilon(r)}{r^2}\frac{[1+P(r)/\varepsilon(r)]\left[1+4\pi r^3 P(r)/m(r)\right]}{1-2Gm(r)/r},$$

$$\frac{dm(r)}{dr} = 4\pi r^2\varepsilon(r).$$

- Starting with a given central density (for a known EoS) we solve the TOV equation from the center to the surface.
- We stop at the surface (defined when the pressure is zero).
- A sequence of mass-radius curve is obtained when we carry this process for different central densities.
- In this work first we solve the TOV equation and then the perturbed equation.

- It seems, increasing value of g we can make the rho-condensate to appear whereas we like.
- No so?
- lower bound to the ρ-meson mass which is provided by the symmetry energy slope L. Expected to be in the range between $40 - 115$ MeV at nuclear saturation density.
- n_c saturates at about $4n_0$ for large L. Increasing L (meaning increasing g) further does not affect n_c significantly.
- However large we make g, the condensate does not appear before the density of the star reaches 4 times nuclear saturation density for reasonable nuclear matter properties.

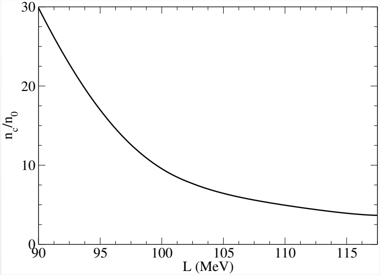


Figure : n_c/n_0 is shown as a function of the slope (L) parameter. The L parameter is dependent on the scalar coupling g.