Energy loss of a heavy quark in a hot QCD plasma

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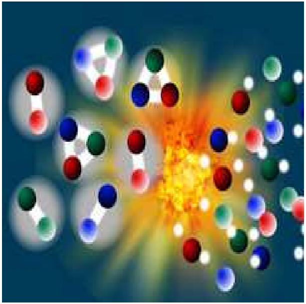
Today's headlines

- Our motivations.
- Formalism for theoretical calculations.
- The soft contribution to the energy loss of the heavy quark.
- The contribution to the hard sector of heavy quark energy loss.
- Effect of the running coupling on the energy loss.
- Energy loss of heavy quark in presence of light partons.
- Our findings..
- What we have learnt?
- Scope of these calculations...

Our goals....

- In this work we have studied the collisional energy loss of a heavy quark propagating through a high temperature Quark Gluon Plasma (QGP) to leading order in the Quantum Chromodynamics (QCD) coupling constant.
- While calculating charm quark energy loss, apart from the heavy-light scattering, we have also considered the contribution from the scattering of the test particle (heavy) with thermal charm quark.
- We have also studied the energy loss of the incident heavy quark incorporating the running coupling.

[*This work has been done in collaboration with Mahatsab Mandal, sreemoyee Sarkar, Pradip K. Roy and Sukalyan Chattopadhyay.*]



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Formalism

- One of the excellent probes of the quark gluon plasma (QGP) are the jets formed in relativistic heavy ion collisions. Due to the formation of such jets, the partonic energy loss in a QCD plasma need to be studied in detail.
- To investigate the process, one of the basic quantities is the evaluation of the rate of energy loss per unit distance of a parton produced in a large sized medium.

$$\left(-\frac{dE}{dx}\right)_{QQ\to QQ} = \frac{1}{E_p}\int_{p'}\frac{1}{2E_{p'}}\int_k\frac{1}{2E_k}\int_{k'}\frac{1}{2E_{k'}}(2\pi)^4\delta^4(P+K-P'-K')\frac{1}{2}\sum_{\text{spins}}|\mathcal{M}|^2\frac{\omega}{v}\Theta(q-q^\star)[PSF], \quad (1)$$

- The following points:
  - The factor (ω) makes $\frac{dE}{dx}$ calculable with presently available methods for resumming perturbation theory.
  - The interaction rate Γ calculated with tree level diagrams of scattering process has quadratic IR divergence.
  - Factor ω in $\frac{dE}{dx}$ makes IR divergence $\Rightarrow$ Logarithmic divergence.
  - Use of effective propagator for exchanged photon softens the divergences, so that Γ is only logarithmic divergent.
  - Thus, $\frac{dE}{dx}$ is infrared finite $\Rightarrow$ divergences are screened by plasma effects at scale gT.

Formalism (II)

- In case of fermionic initial $(n_F(E_k))$ and final states $(n_F(E'_k), n_F(E'_p))$, the phase space factor (PSF) is given as,
- Here $N(q_0) = (exp(q_0/T) - 1)^{-1}$ and $q_0 = \omega$.

$$
\begin{aligned}
PSF = \quad & n_F(E_k)(1 - n_F(E'_k))(1 - n_F(E'_p)) + (1 - n_F(E_k))n_F(E'_k)n_F(E'_p) \\
& = (n_F(E_k) - n_F(E'_k)) \left[1 + N(q_0) - n_F(E'_p)\right] \qquad (2) \\
& \simeq -\frac{dn_F(E_k)}{dE_k} q_0 \left[\frac{T}{q_0} - \frac{1}{2}\right] \qquad (3)
\end{aligned}
$$

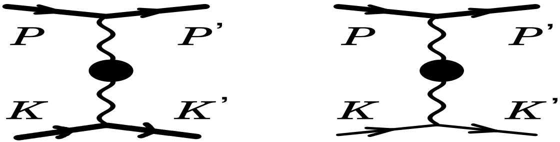

Figure: Feynman diagram for the scattering process with effective gluon propagator

Braaten and Yuan prescription

- Introduction of an arbitrary scale of momentum $(q*)$ $\Rightarrow$ separates hard momentum transfer $(q \sim T)$ from soft momentum transfer $(q \sim gT)$.
- Thus, the separation scale is $(gT << q* << T)$ valid for weak coupling limit.
- HARD $\looparrowright$

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$$A_{hard} + B \ln(T/q*).$$

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- SOFT $\looparrowright$ Effective photon propagator $\curvearrowright$

$$A_{soft} + B \ln(q*/gT).$$

- The result is independent on the arbitrary scale.
- $\ln(1/g) \Longrightarrow$ Contributions from all momentum scales from (T) to (gT).

Number density estimations

- In Table the variation of the number densities with the temperature of the medium can be observed. From the table it is evident that at temperatures relevant to LHC energies heavy quark density is quite significant along with light quarks and gluons.
- The number density of charm quarks, light quarks and gluons present in the medium has been estimated by the following formula

$$n_i = \frac{g_i}{(2\pi)^3} \int_0^\infty \frac{d^3p}{\mathrm{e}^{\mathrm{E_i/T}} \pm 1}$$

Temperature (GeV)	$n_q + n_{\bar{q}}(fm^{-3})$	$n_g(fm^{-3})$	$n_Q + n_{\bar{Q}}(fm^{-3})$
0.3	7.70374	6.8478	0.5795
0.4	18.2607	16.2317	2.5511
0.5	35.6655	31.7026	7.3500
0.6	61.6299	54.7822	16.0630

Table: Variation of number density of heavy quarks, light quarks and gluons with temperature

The soft contribution to the energy loss

- The soft contribution to the energy loss is evaluated in the region where the momentum q of the gluon line is small and is of the order of gT.
- Thus, in this kinematical region, the gluon propagator has has to be modified using the hard thermal loop (HTL) resummation method devoloped by Braaten and Pisarski to incorporate the in-medium modifications ($m^2 = m_d^2/2$ and $x = \omega/q$).

$$\begin{aligned} \frac{1}{2}\Sigma_{spins} \mid \mathcal{M} \mid^2 &= 32g^4 E_p^2 \Big\{ \mid \Delta_L(Q) \mid^2 E_k^2 + 2E_k \mid \vec{k} \mid [(\vec{v}_p.\hat{k}) - (\hat{q}.\hat{k})(\hat{q}.\vec{v}_p)] Re[\Delta_L(Q)\Delta_T(Q)^*] \\ &+ \mid \Delta_T(Q) \mid^2 \mid \vec{k} \mid^2 [(\vec{v}_p.\hat{k}) - (\hat{q}.\hat{k})(\hat{q}.\vec{v}_p)]^2 \Big\} \qquad (4) \end{aligned}$$

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&+ \mid \Delta_T(Q) \mid^2 \mid \vec{k} \mid^2 [(\vec{v}_p.\hat{k}) - (\hat{q}.\hat{k})(\hat{q}.\vec{v}_p)]^2 \Big\} \qquad (4)
\end{aligned}
$$

HTL propagators

$$
\Delta_L(Q) = \frac{-1}{q^2 + m_d^2(1 - \frac{x}{2}\log(\frac{x+1}{x-1}))}
$$

$$
\Delta_T(Q) = \frac{-1}{q_0^2 - q^2 - \frac{m_d^2}{2}\left(x^2 + \frac{x(1-x^2)}{2}\log(\frac{x+1}{x-1})\right)} \qquad (5)
$$

The soft contribution to the energy loss (II)

The angular integrations required for evaluation of the soft part are,

$$\int \frac{d\Omega}{4\pi}\delta(\omega-\vec{v}_k.\vec{q}) = \frac{1}{2v_k q};$$

$$\int \frac{d\Omega}{4\pi}\delta(\omega-\vec{v}_k.\vec{q})\left[\vec{v}_p.\hat{k}-\frac{\omega}{v_k q}(v_k\hat{q}).\hat{k}\right] = 0;$$

$$\int \frac{d\Omega}{4\pi}\delta(\omega-\vec{v}_k.\vec{q})\left[\vec{v}_p.\hat{k}-\frac{\omega}{v_k q}(v_k\hat{q}).\hat{k}\right]^2 = \frac{1}{4v_k q}\left[1-\frac{\omega^2}{(v_k q)^2}\right]\left[v_p^2-\frac{\omega^2}{(v_k q)^2}\right].$$

The soft contribution to the heavy quark energy loss can be written as,

$$\left(-\frac{dE}{dx}\right)^{soft}_{QQ\to QQ} = \frac{g^4 C_F}{v_p^2\pi^3}\int qdq\int k^2dk\frac{1}{E_k^2}\int_{-v_p q}^{v_p q}\frac{\omega^2}{2}(-\frac{E_k}{k})n_F'(E_k)d\omega$$
$$\times\left\{\mid\Delta_L(Q)\mid^2\frac{E_k^2}{2v_k q}+\mid\Delta_T(Q)\mid^2 k^2\frac{1}{4v_k q}\left[1-\frac{\omega^2}{(v_k q)^2}\right]\left[v_p^2-\frac{\omega^2}{(v_k q)^2}\right]\right\}$$

- The ω,q and k integration cannot be performed analytically and have to be solved numerically.

The hard contribution to the energy loss

- The hard contribution to the energy loss can be obtained easily by setting $m_d = 0$ and setting the limit of q integration from q^* to $\sqrt{4E_p T}$. This is only valid in the momentum transfer region of $q > q*$.

The total result is obtained by adding the soft and hard contributions to the energy loss

$$\Big(-\frac{dE}{dx}\Big)_{QQ\to QQ} = \Big(-\frac{dE}{dx}\Big)_{soft} + \Big(-\frac{dE}{dx}\Big)_{hard} \quad (7)$$

[1]

[1] E. Braaten and M.H. Thoma, Phys. Rev. D **44**, 1298(1991).

Energy loss in presence of light bath particles

Collisional energy loss of heavy quarks with light quarks

$$\Big(-\frac{dE}{dx}\Big)^{Qq\rightarrow Qq}_{soft} = \frac{g^4 T^2 N_f}{6\pi}\Big(\ln\frac{q*}{m_g} - 0.843\Big)$$

$$\Big(-\frac{dE}{dx}\Big)^{Qq\rightarrow Qq}_{hard} = \frac{g^4 T^2 N_f}{12\pi^6}\Big(\ln\frac{2TE}{(q*)^2} + \frac{8}{3} - \gamma_E + \frac{\zeta'(2)}{\zeta(2)}\Big)$$

Collisional energy loss of heavy quarks with gluons

$$\Big(-\frac{dE}{dx}\Big)^{Qg\rightarrow Qg}_{soft} = \frac{g^4 T^2}{6\pi}\Big(\ln\frac{q*}{m_g} - 0.843\Big)$$

$$\Big(-\frac{dE}{dx}\Big)^{Qg\rightarrow Qg}_{hard} = \frac{g^4 T^2}{12\pi}\Big(\ln\frac{TE}{(q*)^2} + \frac{8}{3} - \gamma_E + \frac{\zeta'(2)}{\zeta(2)}\Big)$$

[2] [3]

[2] E. Braaten and M.H. Thoma, Phys. Rev. D **44**, R2625(1991).

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Implementation of the running coupling

- In the previous treatment we have kept α_s fixed. To study the effect of the running coupling, we use,

$$\alpha_{eff}(Q^2) = \frac{4\pi}{\beta_0} L_-^{-1} \tag{8}$$

with $Q^2 = \omega^2 - q^2$, $\beta_0 = 11 - \frac{2}{3}\, n_f$ and $L_- = \ln(-Q^2/\Lambda^2)$ with $\Lambda = 0.263$ GeV.

- Moreover, we have also treated Debye mass (m_D) to be a function of both Q^2 and T, i.e.,

$$m_D^2 \equiv m_D^2(T, Q^2) = 4\pi\alpha_{eff}(Q^2)(1 + n_f/6)T^2$$

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Results

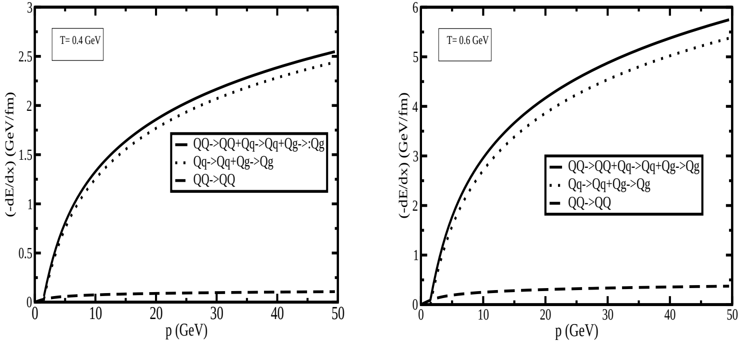

Figure: Energy loss dE/dx of a charm quark as a function of its momentum for $T = 0.4$ GeV (left panel) and $T = 0.6$ GeV (right panel).

Results (II)

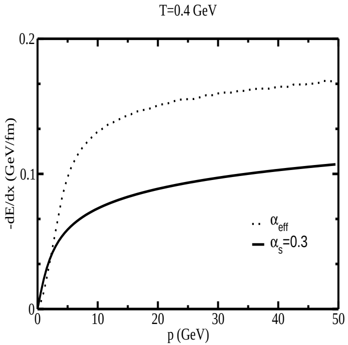

Figure: Comparison of running coupling (α_{eff}) on energy loss for $QQ \rightarrow QQ$ scatterings at 0.4 GeV (left panel) and at 0.6 GeV (right panel) with the fixed value of the coupling constant (α_s).

Summary and conclusions

- We have calculated the energy loss of a heavy quark in a medium where, in addition to the light particles, the partonic consists of thermalized heavy quarks.
- The heavy quark loses energy in the hot medium has been done via theory of collisional($qQ \to qQ$, $Qg \to Qg$, and $QQ \to QQ$) energy loss processes.
- The total contribution from these three processes are plotted and compared with energy loss when the heavy quark with the heavy quark scattering taken into account.
- In this calculation, the momentum has been scaled to an upper limit of $q_{max} = \sqrt{4E_pT}$.
- It is observed that by including the process $QQ \to QQ$ the total heavy quark energy loss increases by 5% and 8% for temperatures 400 MeV and 600 MeV respectively at momentum 25 GeV.
- We have also found that the energy loss is significantly higher when the coupling constant is replaced by running.
- These observations are consistent with the nature of number densities of plasma particles with temperature.

What next?

All suggestions are welcome...

References

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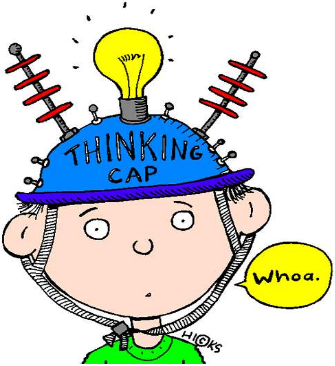

THANKS

Dynamical Screening

Longitudinal and transverse HDL propagators

$$\Delta_L(q_0,q) = \frac{-1}{q^2+2m^2\left[1-\frac{q_0}{2q}\,log\left(\frac{q_0+q}{q_0-q}\right)\right]}$$

$$\Delta_T(q_0,q) = \frac{-1}{q_0^2-q^2-m^2\,\frac{q_0^2}{q^2}\left[1+\frac{q^2-q_0^2}{2qq_0}\,log\left(\frac{q_0+q}{q_0-q}\right)\right]}$$

- For $q_0 \to 0$ longitudinal photons acquire an effective mass $m_D^2 = 2m^2 = \frac{e^2T^2}{3}$ which screens IR singularities.
- For $q_0 \to 0$ transverse (or magnetic) interactions are NOT screened; only dynamical screening.

Dynamical Screening

Retaining the leading term for $\frac{q_0}{q} \to 0$

$$\Delta_T \simeq \frac{1}{q^2 - \frac{i\pi m^2 q_0}{2q}}$$

$$q_C = \left(\frac{\pi m^2 q_0}{2q}\right)^{(1/2)}$$

- Frequency dependent screening with a frequency dependent cut-off.
- In some, but not all situations, this cut-off is able to screen IR singularities so that finite results are obtained.

The hard contribution to the energy loss.

- First, we will focus on the calculation where the heavy quark interacts with the thermal heavy quarks having mass M_2 and momentum k. The hard contribution to $(-dE/dx)$ reads as,

$$\left(-\frac{dE}{dx}\right)^{\rm hard}_{QQ\rightarrow QQ}=\frac{(2\pi)^4}{(2\pi)^9 E_p 2^3.2v_p}\int\left[\frac{E'_pE'_k}{E'_pE_kE'_kE_pE_k}\right]q^2dqd(\cos\theta_q)d\phi_q d(\cos\theta_k)d\phi_k k^2dk\times$$
$$\times[PSF]\omega\mid\mathcal{M}\mid^2\delta\left(\omega-(\vec{v}_p.\vec{q})-\frac{t}{2E_p}\right)\delta\left(\omega-(\vec{v}_k.\vec{q})+\frac{t}{2E_k}\right)d\omega \qquad (9)$$

- The squared matrix amplitude for the process $QQ\rightarrow QQ$(summed and averaged over spins) is given by, where $m^2_{eq}=M_1^2+M_2^2; s+t+u=2M_1^2+2M_2^2$,

$$\mid\mathcal{M}\mid^2=\frac{g^4}{4}N_FC_F\left\{\frac{8}{t^2}\left[(s-m^2_{eq})^2+(m^2_{eq}-u)^2+2m^2_{eq}t\right]\right\} \qquad (10)$$

The calculation of dirac delta functions

- The Delta function of the energy variable can be written without any approximation as,

$$\delta(E_p + E_k - E_p' - E_k') = \int_{\infty}^{-\infty} \delta(E_p - E_p' - \omega)\delta(\omega - E_k' + E_k) d\omega \quad (11)$$

$$\delta(E_p - E_p' - \omega) = \frac{E_p'}{E_p}\delta\Big(\omega - (\vec{v}_p.\vec{q}) - \frac{t}{2E_p}\Big) \quad (12)$$

where $\vec{q} = \vec{p} - \vec{p}' = \vec{k}' - \vec{k}$.

- In order to calculate the integrals given in previous equations, let us introduce the following identities [3],

$$\int \frac{d\Omega}{4\pi}\delta(\omega - \vec{v}_k.\vec{q}) = \frac{1}{2v_k q}; \quad (13)$$

$$\int \frac{d\Omega}{4\pi}\delta(\omega - \vec{v}_k.\vec{q})\Big[\vec{v}_p.\hat{k} - \frac{\omega}{v_k q}(v_k\hat{q}).\hat{k}\Big] = 0; \quad (14)$$

$$\int \frac{d\Omega}{4\pi}\delta(\omega - \vec{v}_k.\vec{q})\Big[\vec{v}_p.\hat{k} - \frac{\omega}{v_k q}(v_k\hat{q}).\hat{k}\Big]^2 = \frac{1}{4v_k q}\Big[1 - \frac{\omega^2}{(v_k q)^2}\Big]\Big[v_p^2 - \frac{\omega^2}{(v_k q)^2}\Big]. \quad (15)$$