A finite volume study of the thermodynamic properties of strongly interacting matter using PNJL model

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Plan of Talk

- Motivation.
- Polyakov-Loop Extended Nambu Jona-lasinio (PNJL) Model.
- Effect Of Finite System Volume On Thermodynamic Properties.
- Finite Volume Effects On Fluctuations.
- Conclusion

Motivation behind PNJL Model

- Nambu-Jona-Lasinio (NJL) model : Originally proposed for studying hadronic d.o.f. Later extended to quark d.o.f.
  Reproduces chiral symmetry breaking of QCD successfully through a non-vanishing chiral condensate.
- Polyakov loop model : Originally proposed for pure gauge system. Reproduces confinement-deconfinement transition of QCD.
- Polyakov loop-Nambu-Jona-Lasinio (PNJL) model tied together these two aspects of QCD.

Motivation for finite Volume

- Experiments in heavy-ion collisions are trying to produce the quark-gluon plasma (QGP) phase of matter by colliding nuclei at ultra-relativistic energies. The matter formed due to the energy deposition of colliding particles has a finite volume. The volume of the system thus created would depend on nature of colliding neuclei, the center of mass energy and centrality of collision. Once created the system expands until it reaches freeze-out.
- It is important to understand the effect of finite volume on the thermodynamic properties of matter created in the experiments.

Polyakov Loop potential I

The Polyakov line is represented as,

$$L(\bar{x}) = \mathcal{P}\mathrm{exp}[i\int_0^\beta d\tau A_4(\bar{x},\tau)]$$

where $A_4 = iA_0$ is the temporal component of Eucledian gauge field $(\bar{A}, A_4)$, $\beta = \frac{1}{T}$, and $\mathcal{P}$ denotes path ordering. The Polyakov loop is then given by

$\Phi = (Tr_c L)/N_c$, $\bar{\Phi} = (Tr_c L^\dagger)/N_c$.

For the Polyakov loop part we have,

$$\frac{\mathcal{U}'(\Phi,\bar{\Phi},T)}{T^4} = \frac{\mathcal{U}(\Phi,\bar{\Phi},T)}{T^4} - \kappa \ln[J(\Phi,\bar{\Phi})]$$

S. K. Ghosh *et. al.* Phys. Rev. D 77, 094024 (2008).

$$\frac{\mathcal{U}(\Phi,\bar{\Phi},T)}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}(\Phi^3+\bar{\Phi}^3) + \frac{b_4}{4}(\bar{\Phi}\Phi)^2$$

Polyakov Loop potential II

where

$$b_2(T) = a_0 + a_1(\frac{T_0}{T}) + a_2(\frac{T_0}{T})^2 + a_3(\frac{T_0}{T})^3,$$

and,

$$J[\Phi, \bar{\Phi}] = (27/24\pi^2)(1 - 6\Phi\bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi\bar{\Phi})^2)$$

$J(\Phi, \bar{\Phi}) \Longrightarrow$ VdM determinant.

$$a_0 = 6.75,\ a_1 = -1.95,\ a_2 = 2.625,\ a_3 = -7.44,\ b_3 = 0.75,\ b_4 = 7.5,$$
$$T_0 = 190\,\mathrm{MeV},\ \kappa = 0.2 \text{ (for 2 flavor)},\ \kappa = 0.13 \text{ (for 2+1 flavor)}$$

PNJL model Lagrangian

2 flavor version of PNJL model is described by the Lagrangian,

$$\mathcal{L} = \sum_{f=u,d} \bar{\psi}_f \gamma_\mu i D^\mu \psi_f - \sum_f m_f \bar{\psi}_f \psi_f + \sum_f \mu_f \gamma_0 \bar{\psi}_f \psi_f + \frac{g_S}{2} \sum_{a=1,2,3} [(\bar{\psi}\tau^a\psi)^2 + (\bar{\psi} i\gamma_5 \tau^a \psi)^2] - \mathcal{U}'(\Phi[A], \bar{\Phi}[A], T)$$

For 2+1 flavor the Lagrangian may be written as,

$$\mathcal{L} = \sum_{f=u,d,s} \bar{\psi}_f \gamma_\mu i D^\mu \psi_f - \sum_f m_f \bar{\psi}_f \psi_f + \sum_f \mu_f \gamma_0 \bar{\psi}_f \psi_f + \frac{g_S}{2} \sum_{a=0,\ldots,8} [(\bar{\psi}\lambda^a\psi)^2 - g_D[det\bar{\psi}_f P_L \psi_{f'} + det\bar{\psi}_f P_R \psi_{f'}] - \mathcal{U}'(\Phi[A], \bar{\Phi}[A], T)$$

where f denotes the flavors u or d or s respectively.

Gap Equation

The gap equation for the constituent quark masses are,

$$M_f = m_f - g_S \sigma_f + g_D \sigma_{f+1} \sigma_{f+2},$$

where $\sigma_f = \langle \bar{\psi}_f \psi_f \rangle$ denotes chiral condensate of the quark with flavor f. The expression for σ_f at zero temperature ($T = 0$) and chemical potential ($\mu_f = 0$) may be written as,

$$\sigma_f = -\frac{3M_f}{\pi^2} \int^{\Lambda} \frac{p^2}{\sqrt{p^2 + {M_f}^2}} dp,$$

Λ being the three-momentum cut-off. This cut-off have been used to regulate the model because it contains dimensionful couplings rendering the model to be non-renormalizable.

Finite volume constraint

- Idealy one should choose proper boundary condition. This would lead to infinite sum over discrete momentum values $p_i = \pi n_i / R$. This implies a lower momentum cut-off $p_{min} = \pi / R = \lambda$(say).
- Here we integrate over continuous values of momenta with lower momentum cut-off λ.
- We shall not use any modifications to the mean-field parameters due to finite size effects. Our philosophy had been to hold the known physics at zero T, zero μ and infinite V fixed. That means we treat V as a thermodynamic variable in the same footing as T and μ. Therefore any variation due to change in either of these thermodynamic parameters were translated into the changes in the effective fields of σ_f, Φ etc.

Thermodynamic Potential

$$
\begin{aligned}
\Omega' &= \mathcal{U}'[\Phi, \bar{\Phi}, T] + 2g_S \sum_{f=u,d,s} \sigma_f^2 - \frac{g_D}{2}\sigma_u\sigma_d\sigma_s - 6\sum_f \int_\lambda^\Lambda \frac{d^3p}{(2\pi)^3} E_{p_f} \Theta(\Lambda - |\vec{p}|) \\
&- 2T \sum_{f=u,d,s} \int_\lambda^\infty \frac{d^3p}{(2\pi)^3} \ln\left[1 + 3\Phi e^{-\frac{(E_f-\mu_f)}{T}} + 3\bar{\Phi} e^{-2\frac{(E_f-\mu_f)}{T}} + e^{-3\frac{(E_f-\mu_f)}{T}}\right] \\
&- 2T \sum_{f=u,d,s} \int_\lambda^\infty \frac{d^3p}{(2\pi)^3} \ln\left[1 + 3\bar{\Phi} e^{-\frac{(E_f+\mu_f)}{T}} + \Phi e^{-2\frac{(E_f+\mu_f)}{T}} + e^{-3\frac{(E_f+\mu_f)}{T}}\right] \\
&= \Omega - \kappa T^4 \ln J[\Phi, \bar{\Phi}]
\end{aligned}
$$

where $E_{p_f} = \sqrt{p^2 + M_f^2}$ is the single quasiparticle energy. In the last line Ω contains all the terms of Ω' except the Vandermonde term.

Transition Temperature

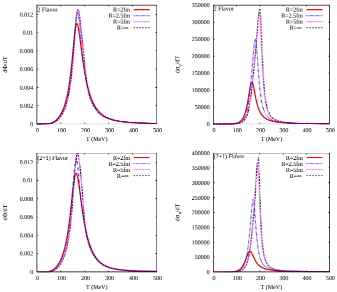

Figure : Derivatives of order parameters for different system sizes

	R	2 fm	2.5 fm	3 fm	5 fm	∞
T_c	2 flavor	167	171	180	184	186
(MeV)	2+1 flavor	160	167	174	180	181

Table : Transition temperatures for different system sizes.

Constituent mass

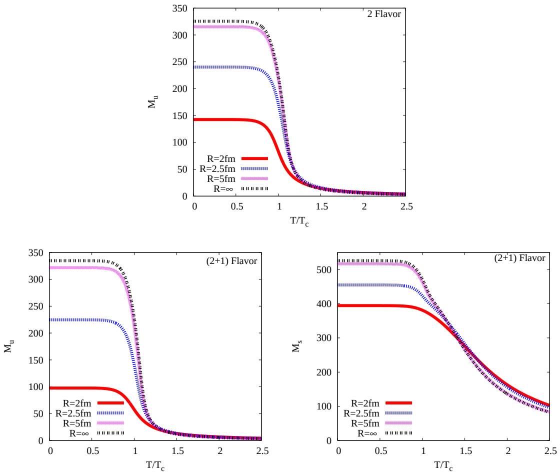

Figure : Constituent masses of quarks as a function of temperature for different system sizes.

Phase diagram

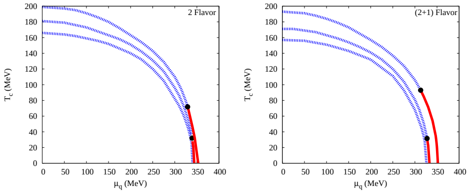

Figure : Phase diagram for different system sizes. The inside curve is for $R = 2$ *fm*, the next curve is for $R = 2.5$ *fm* and the outermost curve is for $R = \infty$.

	$R = 2$ *fm*	$R = 2.5$ *fm*	$R = 3$ *fm*	$R = 5$ *fm*	$R = \infty$
T_c, μ_{q_c} (2)	No CEP	32, 339	52, 335	69, 330	72, 329
T_c, μ_{q_c} (2+1)	No CEP	32, 328	60, 324	86, 316	93, 313

Table : Location of chiral CEP for different system sizes.

Pressure and energy density

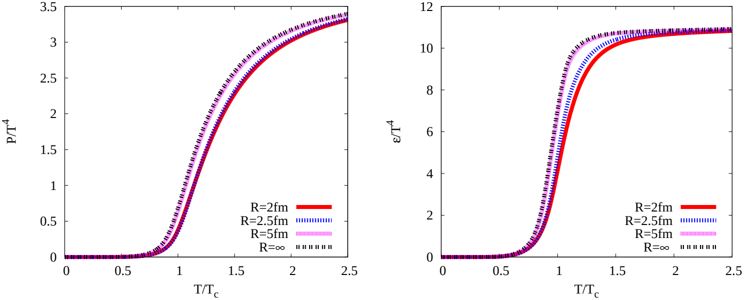

Figure : Pressure and energy density as a function of temperature for 2 flavor.

Specific Heat

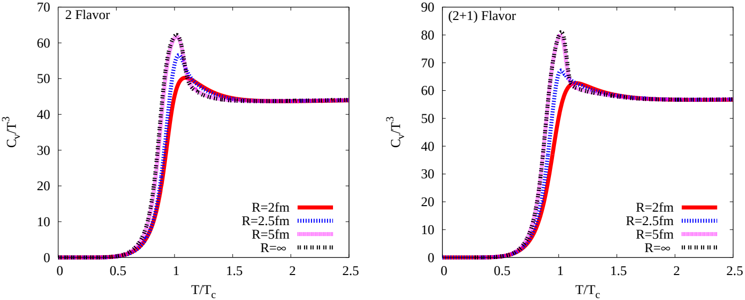

Figure : Variation of specific heat with temperature for different system sizes.

$$C_V = \frac{\partial \epsilon}{\partial T}\bigg|_V = -\,T\frac{\partial^2 \Omega}{\partial T^2}\bigg|_V \quad .$$

Speed of sound

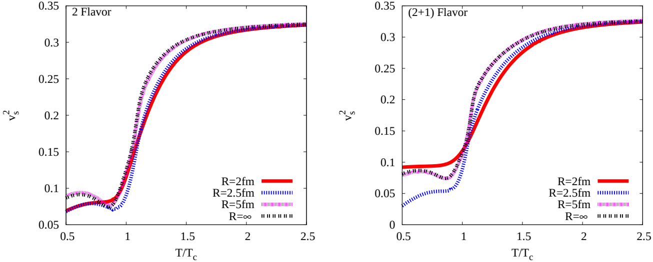

Figure : Variation of squared speed of sound with temperature for different system sizes.

The square of velocity of sound at constant entropy S is given by,

$$v_s^2 = \left.\frac{\partial P}{\partial \epsilon}\right|_S = \left.\frac{\partial P}{\partial T}\right|_V \Big/ \left.\frac{\partial \epsilon}{\partial T}\right|_V = \left.\frac{\partial \Omega}{\partial T}\right|_V \Big/ \left. T\frac{\partial^2 \Omega}{\partial T^2}\right|_V .$$

Meson mass

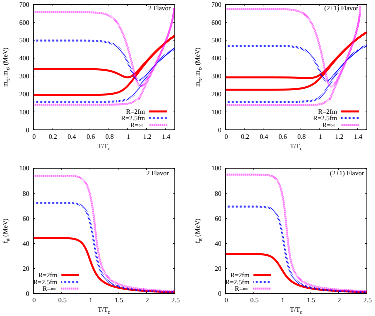

Figure : Variation of meson masses (upper panel) and pion decay constant (lower panel) with temperature for different system sizes. In the upper panel the three monotonously rising curves are for m_π and the other three are for m_σ.

Fluctuations

- Fluctuations are important characteristics of any physical system. They provide essential information about the effective degrees of freedom and their possible quasi-particle nature.
- Fluctuations are closely related to phase transitions.
- The most efficient way to address fluctuations of a system created in a heavy-ion collision is via the study of event-by-event fluctuations.

Suceptibilities

Fluctuations of conserved quantum numbers are related to the respective susceptibilities via the fluctuation-dissipation theorem.
Various susceptibilities for quark number and iso-spin number are defined as

$$c_n(T) = \frac{1}{n!} \frac{\partial^n \left(\Omega(T, \mu_q, \mu_I)/T^4\right)}{\partial \left(\frac{\mu_X}{T}\right)^n}\Bigg|_{\mu_X=0} \quad ,$$

where $\mu_X = \mu_q$ or μ_I.

Quark number susceptibilities

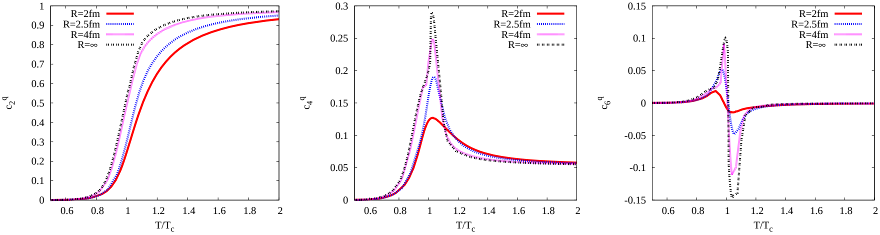

Figure : Variation of quark number susceptibilities with temperature for different system sizes.

Iso-spin number susceptibilities

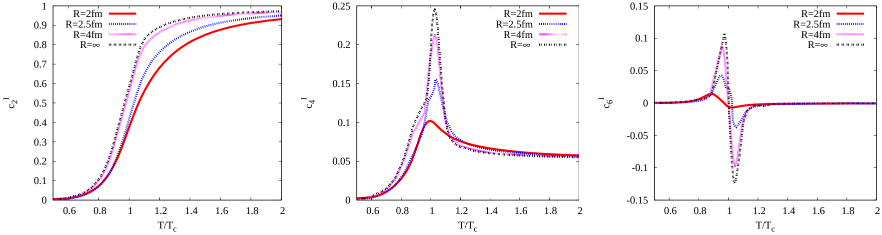

Figure : Variation of iso-spin number susceptibilities with temperature for different system sizes.

Ratios of susceptibilities

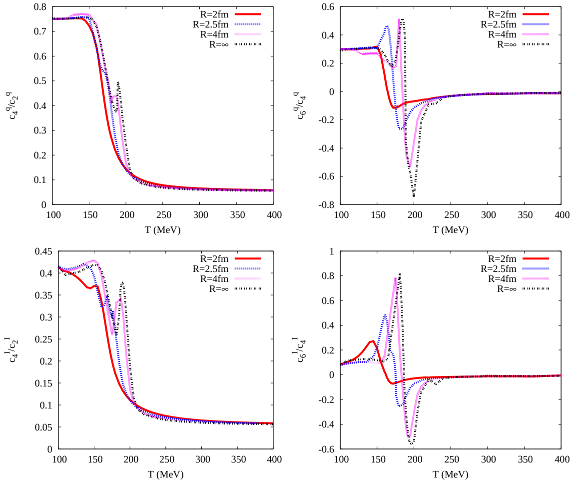

Figure : Variation of ratios of susceptibilities with temperature for different system sizes.

Conclusion

- We have studied the thermodynamics and fluctuations of strongly interacting matter inside finite volume using PNJL model.
- The critical temperature T_c has a strong dependence on system volume.
- Spontaneously broken chiral symmetry may be restored at much lower temperature in small volume.
- With the increase of volume CEP shifts towards high T and low μ. For $R = 2fm$ there is no CEP.
- Changes in the equation of state and speed of sound may have important consequences in the flow properties of the exotic medium created in the experiments.
- Significant volume dependence in quark number and isospin number susceptibilities.

Collaborators

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Back-up

Due to the dynamical breaking of chiral symmetry, $N_f^2 - 1$ Goldstone bosons appear. These are the pions and kaons whose masses, decay widths etc. from experimental observations are utilized to fix the NJL model parameters.

Model	m_u MeV	m_s MeV	Λ MeV	$g_S\Lambda^2$	$g_D\Lambda^5$
2 flavor	5.5	0	651	4.27	0
2+1 flavor	5.5	134.76	631	3.67	9.33

Table : Parameters of the Fermionic part of the model.