Constraining Heavy Quark Energy Loss Using B and D Mesons in Heavy Ion Collision

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Outline

Motivation

- 2 Heavy Quark Energy Loss
- 3 Energy Loss Formalism
- 4 QGP Evolution Model
- **5** Calculation of dE/dx
- 6 Calculation of R_{AA}

7 Summary

- QGP is formed at high temperature or at high density. It consists of deconfined quarks and gluons.
- Heavy quarks ($M_c = 1.3$ GeV and $M_b = 4.7$ GeV) production can be treated using pQCD.
- Heavy quarks interact with the medium and do not become part of the medium and thus probe the opacity of the QGP medium.

- When heavy quark traverse the QGP medium, they loose energy either due to the elastic collisions with the plasma partons or by radiating a gluon or both.
- Collisional energy loss comes from the processes which have the same number of incoming and outgoing particles.
- Radiative energy loss comes from the processes in which there are more outgoing than incoming particles.
- Many formalisms have been proposed for the collisional as well as the radiative energy loss of heavy quarks. We used PP (Peigne-Peshier) to calculate the collisional energy loss while DGLV (Djordjevic, Gyulassy, Levai and Vitev) and generalized dead cone approach AJMS (Abir, Jamil, Mustafa and Srivastava) to calculate the radiative energy loss.

- The rate of energy loss (dE/dx) of heavy quark due to elastic collision given by Braaten and Thoma in the limit $q \ll E$ which is good for $E \ll M^2/T$.
- Peigne and Peshier extended this calculation in the domain $E \gg M^2/T$.

$$\frac{dE}{dx} = \frac{4\pi\alpha_s^2 T^2}{3} \left[\left(1 + \frac{N_f}{6} \right) \log \frac{E}{\mu_g^2} + \frac{2}{9} \log \frac{E}{M^2} + c(N_f) \right]$$
(1)
Here $\mu_g = \sqrt{4\pi \ \alpha_s \ T^2 \ \left(1 + \frac{N_f}{6} \right)}$ is the Debye screening mass and $c(N_f) = 0.146N_f + 0.05.$
Phys. Rev. D77, 114017 (2008).

DGLV Formalism

• DGLV used reaction operator formalism to obtain the energy loss of quark jets by gluon radiation.

- Analytical expression is obtained for energy loss in powers of gluon opacity (L/λ) . This formalism was extended to obtain the energy loss of heavy quark.
- Wicks simplified this formalism for the first order of opacity expansion.

$$\frac{\Delta E}{L} = E \frac{C_F \alpha_s}{\pi} \frac{1}{\lambda} \int_{\frac{m_g}{E+\rho}}^{1-\frac{M}{E+\rho}} dx \int_0^\infty \frac{4 \,\mu_g^2 \,q^3 \,dq}{\left(\frac{4E_x}{L}\right)^2 + (q^2 + \beta^2)^2} \times (A \log B + C) \quad ,$$
(2)
where $\beta^2 = m_g^2 (1-x) + M^2 x^2, \quad \lambda^{-1} = \rho_g \sigma_{Qg} + \rho_q \sigma_{Qq},$
 $m_g = \frac{\mu_g}{\sqrt{2}}, \quad \rho_g = 16^3 \frac{1.202}{\pi^2}, \quad \lambda^{-1} = \rho_g \sigma_{Qg} + \rho_q \sigma_{Qq},$
(2)
$$\rho_q = 9 N_f T^3 \frac{1.202}{\pi^2}, \quad \sigma_{Qq} = \frac{9 \pi \alpha_s^2}{2 \mu_g^2}, \quad \sigma_{Qg} = \frac{4}{9} \sigma_{Qq}.$$
Nucl. Phys. A784, 426 (2007)

DGLV Continue.....

The function A, B and C are given as

$$A = \frac{2\beta^2}{f_\beta^3} \Big(\beta^2 + q^2\Big), \tag{3}$$

$$B = \frac{(\beta^2 + K)(\beta^2 Q_{\mu}^- + Q_{\mu}^+ Q_{\mu}^+ + Q_{\mu}^+ f_{\beta})}{\beta^2 (\beta^2 (Q_{\mu}^- - K) - Q_{\mu}^- K + Q_{\mu}^+ Q_{\mu}^+ + f_{\beta} f_{\mu})},$$
(4)

$$C = \frac{1}{2q^2 f_{\beta}^2 f_{\mu}} \Big[\beta^2 \mu_g^2 (2q^2 - \mu_g^2) + \beta^2 (\beta^2 - \mu_g^2) K + Q_{\mu}^+ (\beta^4 - (5)) \Big] + \beta^2 (\beta^2 - \mu_g^2) K + Q_{\mu}^+ (\beta^4 - (5)) \Big] + \beta^2 (\beta^2 - \mu_g^2) K + Q_{\mu}^+ (\beta^4 - (5)) \Big] + \beta^2 (\beta^2 - \mu_g^2) K + Q_{\mu}^+ (\beta^4 - (5)) \Big] + \beta^2 (\beta^2 - \mu_g^2) K + Q_{\mu}^+ (\beta^4 - (5)) \Big] + \beta^2 (\beta^2 - \mu_g^2) K + Q_{\mu}^+ (\beta^4 - (5)) \Big] + \beta^2 (\beta^2 - \mu_g^2) K + Q_{\mu}^+ (\beta^4 - (5)) \Big] + \beta^2 (\beta^2 - \mu_g^2) K + Q_{\mu}^+ (\beta^4 - (5)) \Big] + \beta^2 (\beta^2 - \mu_g^2) K + Q_{\mu}^+ (\beta^4 - (5)) \Big] + \beta^2 (\beta^2 - \mu_g^2) K + Q_{\mu}^+ (\beta^4 - (5)) \Big] + \beta^2 (\beta^2 - \mu_g^2) K + Q_{\mu}^+ (\beta^4 - (5)) \Big] + \beta^2 (\beta^2 - \mu_g^2) K + Q_{\mu}^+ (\beta^4 - (5)) \Big] + \beta^2 (\beta^2 - \mu_g^2) K + Q_{\mu}^+ (\beta^4 - (5)) \Big] + \beta^2 (\beta^2 - \mu_g^2) K + Q_{\mu}^+ (\beta^4 - (5)) \Big] + \beta^2 (\beta^2 - \mu_g^2) K + Q_{\mu}^+ (\beta^4 - (5)) \Big] + \beta^2 (\beta^2 - \mu_g^2) K + Q_{\mu}^+ (\beta^4 - (5)) \Big] + \beta^2 (\beta^2 - \mu_g^2) K + Q_{\mu}^+ (\beta^4 - (5)) \Big] + \beta^2 (\beta^2 - \mu_g^2) K + Q_{\mu}^+ (\beta^4 - (5)) \Big] + \beta^2 (\beta^2 - \mu_g^2) K + Q_{\mu}^+ (\beta^4 - (5)) \Big] + \beta^2 (\beta^2 - \mu_g^2) K + \beta^2 (\beta^2 - \mu_g$$

$$2q^{2}Q_{\mu}^{+}) + f_{\mu}\left(\beta^{2}(-\beta^{2} - 3q^{2} + \mu_{g}^{2}) + 2q^{2}Q_{\mu}^{+}\right) + 3\beta^{2}q^{2}Q_{k}^{-}\right].(6)$$

Here

$$K = 2px(1-x), \ Q_{\mu}^{\pm} = q^2 \pm \mu_g^2, \ \ Q_{\mu}^{\pm} = q^2 + K,$$
 (7)

$$f_{\beta} = f(\beta, Q_{\mu}^{-}, Q_{\mu}^{+}), \quad f_{\mu} = f(\mu_{g}, Q_{k}^{+}, Q_{k}^{-}),$$
 (8)

$$f(x, y, z) = \sqrt{x^4 + 2x^2y + z^2} .$$
 (9)

Generalized Dead Cone Approach (AJMS)

The rate of radiative energy loss of heavy quark

$$\frac{dE}{dx} = \frac{\langle \omega \rangle}{\lambda},\tag{10}$$

here $<\omega>$ is the mean energy of the emitted gluon and λ is the mean free path of the traversing quark.

$$<\omega> = \frac{\int d\omega \int \mathcal{D} d\eta}{\int \frac{1}{\omega} d\omega \int \mathcal{D} d\eta},$$
 (11)

$$\mathcal{D} = \left(1 + \frac{M^2}{s}e^{2\eta}\right)^{-2}, \ \eta = -\ln\tan\left(\frac{\theta}{2}\right), \tag{12}$$

$$\lambda^{-1} = \left(\rho_q + \frac{9}{4}\rho_g\right)\sigma \quad , \tag{13}$$

$$\sigma = 4C_A \alpha_s^3 \int \frac{1}{(q_\perp^2)^2} dq_\perp^2 \right)^2 \int \frac{1}{\omega} d\omega \int \mathcal{D} d\eta.$$
(14)

arXiv:1109.5539, Phys. Rev. C48, 1275 (1993).

Generalized Dead Cone Approach Continue.....

$$\frac{dE}{dx} = 24\alpha_{s}^{3}\left(\rho_{q} + \frac{9}{4}\rho_{g}\right) \int_{q_{\perp}^{2}|_{min}}^{q_{\perp}^{2}|_{max}} \frac{1}{(q_{\perp}^{2})^{2}} dq_{\perp}^{2} \int_{\omega_{min}}^{\omega_{max}} d\omega \int_{\eta_{min}}^{\eta_{max}} \mathcal{D} d\eta.$$
(15)
$$\frac{dE}{dx} = 24\alpha_{s}^{3}\left(\rho_{q} + \frac{9}{4}\rho_{g}\right) \frac{1}{\mu_{g}} (1 - \beta_{1}) \left(\sqrt{\frac{1}{(1 - \beta_{1})}\log\left(\frac{1}{\beta_{1}}\right)} - 1\right) \mathcal{F}(\delta)$$
(16)

where

$$\mathcal{F}(\delta) = 2\delta - \frac{1}{2} \log \left(\frac{1 + \frac{M^2}{s} e^{2\delta}}{1 + \frac{M^2}{s} e^{-2\delta}} \right) - \frac{\frac{M^2}{s} \sinh(2\delta)}{1 + 2\frac{M^2}{s} \cosh(2\delta) + \frac{M^4}{s^2}}$$
(17)
$$\delta = \frac{1}{2} \log \left[-\frac{1}{s} - \log\left(\frac{1}{s}\right) \left(1 + \sqrt{\frac{1 - \beta_1}{s}} \right)^2 \right]$$
(19)

$$\delta = \frac{1}{2} \log \left[\frac{1}{(1-\beta_1)} \log \left(\frac{1}{\beta_1} \right) \left(1 + \sqrt{1 - \frac{(1-\beta_1)}{\log \left(\frac{1}{\beta_1} \right)}} \right) \right].$$
(18)

Image: Image:

The previous expression differs with the original AJMS formalism where $\mathcal{F}(\delta)$ is given by

$$\mathcal{F}(\delta) = 2\delta - \frac{1}{2} \log \left(\frac{1 + \frac{M^2}{s} e^{2\delta}}{1 + \frac{M^2}{s} e^{-2\delta}} \right) - \frac{\frac{M^2}{s} \cosh(\delta)}{1 + 2\frac{M^2}{s} \cosh(\delta) + \frac{M^4}{s^2}}$$
(19)
$$\delta = \frac{1}{2} \log \left[\frac{1}{(1 - \beta_1)} \log \left(\frac{1}{\beta_1} \right) \left(1 + \sqrt{1 - \frac{(1 - \beta_1)^{1/2}}{\left(\log \frac{1}{\beta_1} \right)^{1/2}}} \right)^2 \right]$$
(20)

Phys. Lett. B715, 183 (2012).

QGP Evolution Model : Present

Heavy quarks are produced at a point (r, ϕ) . The distance travelled by heavy quark in the plasma is given by

$$L(\phi, r) = \sqrt{R^2 - r^2 \sin^2 \phi} - r \cos \phi.$$
(21)

We calculated average path length (< L >) considering nucleus as a sharp sphere.

Effective path length is calculated as

$$\mathcal{L}_{eff} = \min[\langle L \rangle, v_T \times \tau_F].$$
(22)

We use entropy conservation condition $s(T)V(\tau) = s(T_0)V(\tau_0)$ to obtain temperature as a function of the time. The EOS obtained by lattice QCD along with hadronic resonance gas (HRG) are used as input. The transverse size for a given centrality is obtained as $R(N_{part}) = R_A \sqrt{2A/N_{part}}$.

Vineet Kumar and Prashant Shukla, arXiv:1410.3299

Model	Exp.	Centrality (%)	$N_{ m part}({ m given})$	$dN/d\eta$ (measured)	
Present	RHIC	0-10	329	623.54	
Present	LHC	0-10	355.34	1447.26	
Present	LHC	0-20	308.38	1205.77	

Model	Exp.	Centrality	Ь	$<$ L $>$	$\tau_0(given)$	$ au_F$	T_0
		(%)	(fm)	(fm)	(fm/c)	(fm/c)	(GeV)
Present	RHIC	0-10	3.268	5.63	0.6	3.0	0.340
Present	LHC	0-10	3.32	5.73	0.3	6.0	0.488
Present	LHC	0-20	4.70	5.62	0.3	6.0	0.481
Old	RHIC	0-10	0.0	5.78	0.2	2.63	0.400
Old	LHC	0-10/20	0.0	6.14	0.2	5.90	0.525

dE/dx for charm and bottom quark at $\sqrt{s_{NN}} = 200 \text{ GeV}$



- Collisional energy loss is similar for charm and bottom quark.
- For radiative energy loss, there is large difference between DGLV and AJMS specially for bottom quark.
- At high p_T , radiative energy loss dominates.

dE/dx for charm and bottom quark at $\sqrt{s_{NN}} = 2.76$ TeV



- Collisional energy loss increases from RHIC to LHC.
- When we move from RHIC to LHC, the radiative energy loss increases.
- dE/dx for charm quark (DGLV) increases dramatically from RHIC to LHC because of explicitly *L* dependence inside the expression.

The nuclear modification factor (R_{AA}) is defined as the ratio of yield in nucleus nucleus (AA) collision to yield in pp collision scaled by a normalizing factor (T_{AA}), known as nuclear overlapping function.

$$R_{AA} = \frac{1}{T_{AA}(b)} \frac{dN^{AA}/dp_T dy}{d\sigma^{pp}/dp_T dy}$$
(23)

- If $R_{AA} = 1$, No medium modification
- If $R_{AA} > 1$, Enhancement
- If $R_{AA} < 1$, Suppression



DGLV+PP describes the data at high p_T range.
AJMS formalism reproduces the data without collisional energy loss.



AJMS and DGLV reproduce similar suppression at high p_T.
If we include collisional energy loss, there will be large suppression.



- \bullet DGLV+PP reproduces the data.
- AJMS (without collisional) reproduces the data.

R_{AA} : Dependence on the QGP evolution model



• We compare AJMS formalism with present and old QGP evolution model.

Kapil Saraswat, Prashant Shukla and Venktes

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04/02/2015 19 / 25

- \bullet We study the two different radiative energy loss formalisms (DGLV and AJMS). There is large difference between them.
- Collisional energy loss is similar for charm and bottom quark.
- If we add the radiative and collisional energy loss, it does not describe the data specially in low p_T range.
- DGLV+PP are reproducing the data at high p_T range.
- AJMS formalism (radiative only) surprisingly reproduces the data in all measurements.

Thank You

Image: A math and A

Back up slides

QGP Evolution Model : Old

In old QGP evolution model, the time formation of a quark gluon plasma is calculated as

$$\tau = L_{\rm eff}/2 \tag{24}$$

The gluon density at time τ is calculated as

$$\rho(\tau) = \frac{1}{\pi R^2 \tau} \frac{dN_g}{dy}$$
(25)

 dN_g/dy is the rapidity distribution of gluon density.

The temperature of the quark gluon plasma is calculated from the gluon density

$$T(\tau) = \left(\frac{\pi^2}{1.202} \ \frac{\rho(\tau)}{(9N_f + 16)}\right)^{1/3} \tag{26}$$

Nucl. Phys. A784, 426 (2007), Phys. Lett. B715, 183 (2012).

Integration Limits

The minimum limit of the integration

$$q_{\perp}^{2}|_{min} \approx \omega_{min}^{2} \approx k_{\perp}^{2}|_{min} \approx \mu_{g}^{2}, \qquad (27)$$

The maximum limit of the integration

$$q_{\perp}^{2}|_{max} = CET, \ C = \frac{3}{2} - \frac{M^{2}}{4ET} + \frac{M^{4}}{48E^{2}T^{2}\beta_{0}} \log \left[\frac{M^{2} + 6ET(1+\beta_{0})}{M^{2} + 6ET(1-\beta_{0})}\right]$$

$$\beta_{0} = \sqrt{1 - \frac{M^{2}}{E^{2}}}.$$
(29)

arXiv:1109.5539, arXiv:9711059 and Chin. Phys. Lett.22, 72 (2005).

$$\omega_{max}^{2} = \langle q_{\perp}^{2} \rangle = \frac{q_{\perp}^{2}|_{min} q_{\perp}^{2}|_{max}}{q_{\perp}^{2}|_{max} - q_{\perp}^{2}|_{min}} \log\left(\frac{q_{\perp}^{2}|_{max}}{q_{\perp}^{2}|_{min}}\right), \quad (30)$$

$$\omega_{max}^{2} = \frac{\mu_{g}^{2}}{(1 - \beta_{1})} \log\left(\frac{1}{\beta_{1}}\right), \quad \beta_{1} = \frac{\mu_{g}^{2}}{CET}. \quad (31)$$

arXiv:9711059 and Chin. Phys. Lett.22, 72 (2005).

Using the relation $\omega=k_{\perp}\cosh\eta,$ the finite cut on ω and k_{\perp} leads to an inequality

$$\frac{\omega_{\max}}{k_{\perp}|_{\min}} < \cosh \eta. \tag{32}$$

We can write the above expression as $|\eta|{<}\delta$, where

$$\delta = \frac{1}{2} \log \left[\frac{1}{(1-\beta_1)} \log \left(\frac{1}{\beta_1} \right) \left(1 + \sqrt{1 - \frac{(1-\beta_1)}{\log \left(\frac{1}{\beta_1} \right)}} \right)^2 \right].$$
(33)