Fluctuations and Correlations in Heavy Ion Collisions

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- Fluctuations in hadronic medium

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- Fluctuations in quark medium

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- Conclusion

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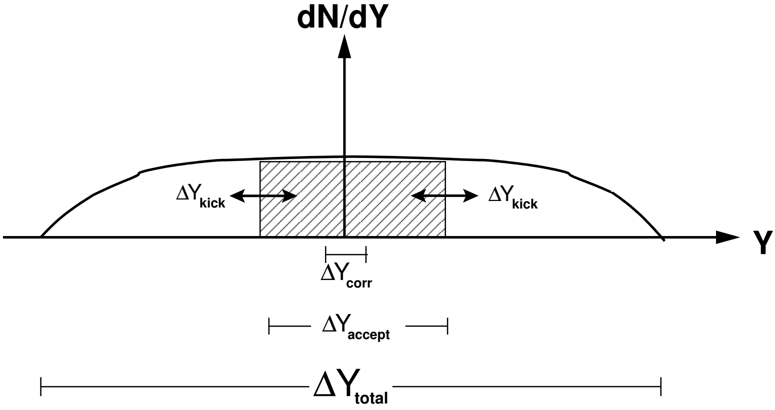
- Fluctuations and correlations are important characteristics of any physical system. They provide essential information about the effective degrees of freedom and their possible quasi-particle nature.
- Fluctuations are closely related to phase transitions.
- The most efficient way to address fluctuations of a system created in a heavy-ion collision is via the study of event-by-event fluctuations.
- In addition, the study of fluctuations may reveal information beyond its thermodynamic properties.



Charge fluctuations will be able to tell us about the properties of the early stage of the system, the QGP, if the following criteria are met:

$$\Delta Y_{accept} \gg \Delta Y_{corr} \quad \text{and} \quad \Delta \mathrm{Y}_{\mathrm{total}} \gg \Delta \mathrm{Y}_{\mathrm{accept}} \gg \Delta \mathrm{Y}_{\mathrm{kick}}$$

V. Koch, arXiv : 0810.2520

Taylor Expansion of Pressure I

$$P(T,\mu_B,\mu_Q,\mu_S) = -\Omega(T,\mu_B,\mu_Q,\mu_S),$$

$$\frac{P(T,\mu_B,\mu_Q,\mu_S)}{T^4} = \sum_{n=i+j+k} c_{i,j,k}^{B,Q,S}(T)(\frac{\mu_B}{T})^i(\frac{\mu_Q}{T})^j(\frac{\mu_S}{T})^k$$

where,

$$c_{i,j,k}^{B,Q,S}(T) = \frac{1}{i!j!k!}\frac{\partial^i}{\partial(\frac{\mu_B}{T})^i}\frac{\partial^j}{\partial(\frac{\mu_Q}{T})^j}\frac{\partial^k(P/T^4)}{\partial(\frac{\mu_S}{T})^k}\Big|_{\mu_{B,Q,S}=0}$$

$$\mu_u = \mu_q + \frac{2}{3}\mu_Q, \quad \mu_d = \mu_q - \frac{1}{3}\mu_Q, \quad \mu_s = \mu_q - \frac{1}{3}\mu_Q - \mu_S$$

Taylor Expansion of Pressure II

For diagonal Taylor coefficients we have used,

$$c_n^X = \frac{1}{n!} \frac{\partial^n \left(P/T^4\right)}{\partial \left(\frac{\mu_X}{T}\right)^n}; \qquad n = i + j$$

For off-diagonal Taylor coefficients we have used,

$$c_{i,j}^{X,Y} = \frac{1}{i!j!} \frac{\partial^{i+j} \left(P/T^4\right)}{\partial \left(\frac{\mu_X}{T}\right)^i \partial \left(\frac{\mu_Y}{T}\right)^j}$$

Diagonal and off-diagonal susceptibilities are respectively defined as,

$$\chi_{XY} = \frac{\partial^2 (P/T^4)}{\partial(\mu_X/T)\partial(\mu_Y/T)} \qquad \chi_{XX} = \frac{\partial^2 (P/T^4)}{\partial(\mu_X/T)^2}$$

Taylor Expansion of Pressure III

- Pressure consists of two parts; one regular part and one non-analytic part.
$$P(T,\mu_u,\mu_d) = P_r(T,\mu_u,\mu_d) + P_s(\bar{t},\bar{\mu}_u,\bar{\mu}_d)$$
with $\bar{t} = (T - T_C)/T_C$ and $\bar{\mu}_{u,d} = \mu_{u,d}/T$.
- $$t \equiv \bar{t} + A\mu_q^2 + B\mu_I^2$$
- From universal scaling behaviour;
$$P_s(\bar{t},\bar{\mu}_u,\bar{\mu}_d) \sim t^{2-\alpha}$$
Then second and forth cumulant get contribution like;
$$(\partial^2 P_s/\partial\mu_X^2) \sim t^{1-\alpha} + \text{regular} \quad \text{and} \quad (\partial^4 P_s/\partial\mu_X^4) \sim t^{-\alpha} + \text{regular}$$

S. Ejiri *et. al.* Phys. Lett. B 633 (2006) 275.

Hadron Resonance Gas Model

$$\ln Z^{id} = \sum_i \ln Z_i^{id},$$

$$\ln Z_i^{id} = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 \, dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)],$$

$$P_i^{id} = \frac{T}{V} \ln Z_i^{id} = \pm \frac{g_i T}{2\pi^2} \int_0^\infty p^2 \, dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)],$$

EVHRG

In excluded volume EVHRG model pressure can be written as

$$P(T, \mu_1, \mu_2, ..) = \sum_i P_i^{id}(T, \hat{\mu}_1, \hat{\mu}_2, ..),$$

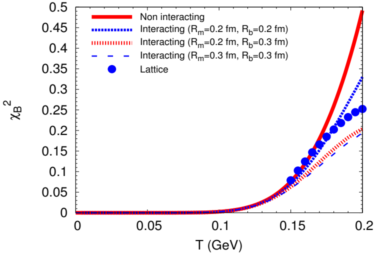
where for i th particle chemical potential is

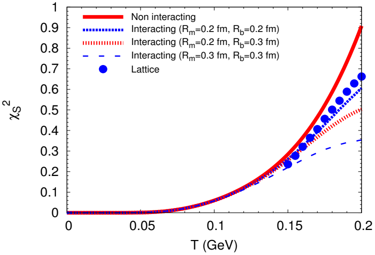
$$\hat{\mu}_i = \mu_i - V_{ev,i} P(T, \mu_1, \mu_2, ..).$$

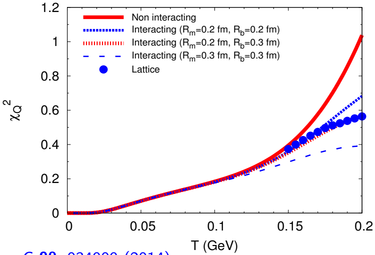
$P(T, \mu_1, \mu_2, ..)$ is suppressed compared to the P^{id} because of suppression of effective chemical potential. Particles number density, entropy density and energy density are suppressed by a factor

$$\frac{1}{1 + \sum_k V_{ev,k} n_k^{id}(T, \hat{\mu}_k)}.$$

Second order susceptibilities

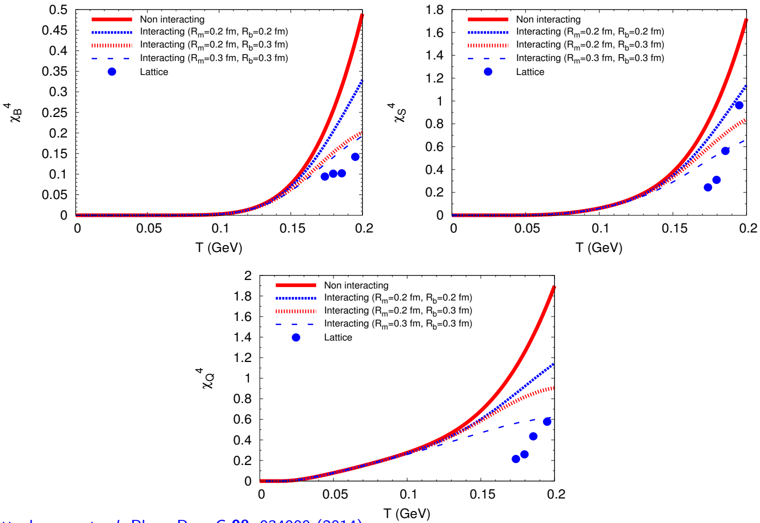






Ref : Bhattacharyya *et. al.* Phys. Rev. C **90**, 034909 (2014).

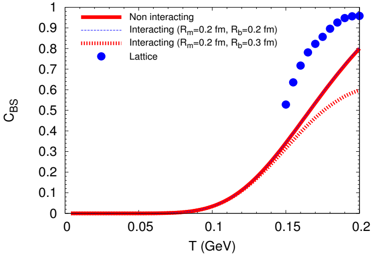
Fourth order susceptibilities

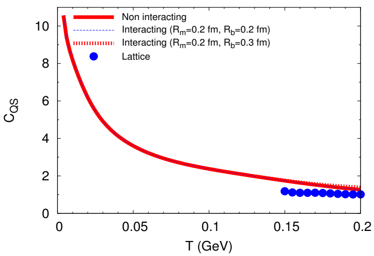


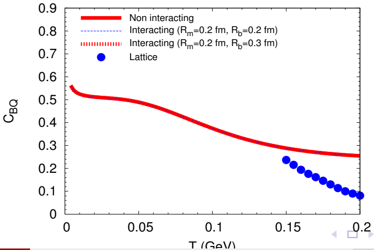
Ref : Bhattacharyya *et. al.* Phys. Rev. C **90**, 034909 (2014).

Offdiagonal susceptibilities

$$C_{BS} = -3\frac{\chi^{11}_{BS}}{\chi^{2}_{S}}, \qquad C_{QS} = 3\frac{\chi^{11}_{QS}}{\chi^{2}_{S}}, \qquad C_{BQ} = \frac{\chi^{11}_{BQ}}{\chi^{2}_{B}} \tag{1}$$





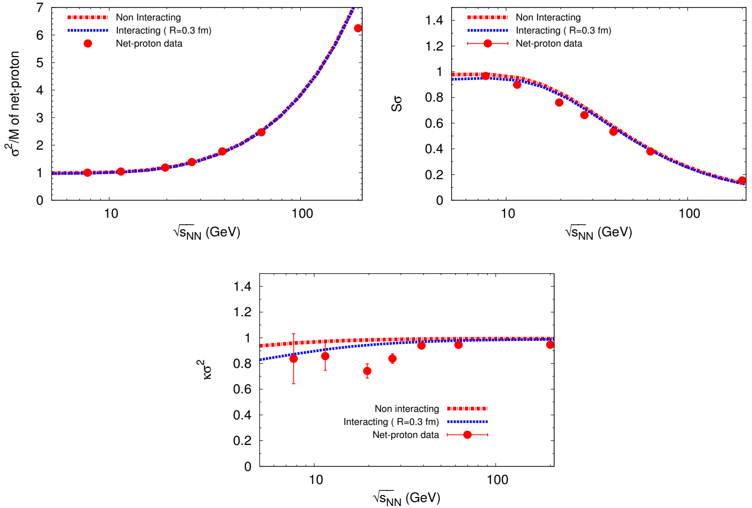


Experimental Observables

$$\begin{aligned}\sigma &= \sqrt{\langle (N - \langle N \rangle)^2 \rangle} \\ s &= \frac{\langle (N - \langle N \rangle)^3 \rangle}{\sigma^3} \\ \kappa &= \frac{\langle (N - \langle N \rangle)^4 \rangle}{\sigma^4}\end{aligned} \tag{2}$$

$$\frac{\chi_X^2}{\chi_X^1} = \frac{\sigma_X^2}{M_X}, \quad \frac{\chi_X^3}{\chi_X^2} = S_X \sigma_X, \quad \frac{\chi_X^4}{\chi_X^2} = \kappa_q \sigma_X^2. \tag{3}$$

Net proton



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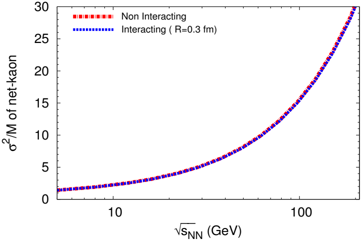
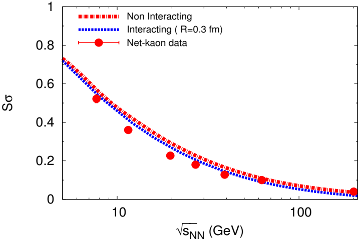
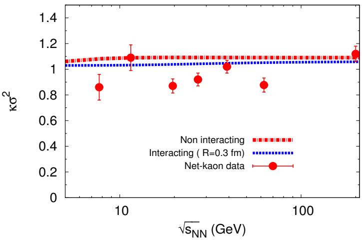
Net kaon

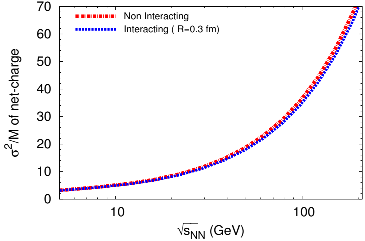


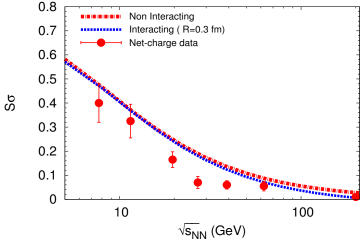


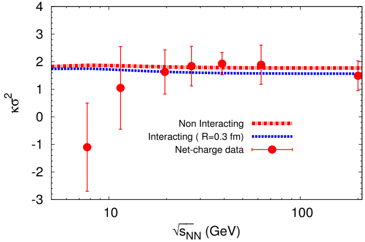


Ref : Bhattacharyya *et. al.* Phys. Rev. C **90**, 034909 (2014).

Net charge







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Model Study

$$\mathcal{L} = \sum_{f=u,d} \left(\bar{\psi}_f \gamma_\mu i\partial^\mu \psi_f - m_f \bar{\psi}_f \psi_f + \mu\gamma_0 \bar{\psi}_f \psi_f\right) + \frac{g_S}{2}\sum_a [(\bar{\psi}\tau^a\psi)^2 + (\bar{\psi}i\gamma_5\tau^a\psi)^2] \quad (4)$$

Symmetries : $\underbrace{SU(2)_V \otimes SU(2)_A \otimes U(1)_B \otimes U(1)_A}$

At the mean-field level : $\bar{\psi}\psi \rightarrow <\bar{\psi}\psi>; \quad \bar{\psi}i\gamma^5\psi \rightarrow <\bar{\psi}i\gamma^5\psi>$

Gap equation : $M_f = m_f - g_s \sum_f <\bar{\psi}_f \psi_f>$

PNJL-model

$$\mathcal{L} = \sum_{f=u,d} \left(\bar{\psi}_f \gamma_\mu i D^\mu \psi_f - m_f \bar{\psi}_f \psi_f + \mu \gamma_0 \bar{\psi}_f \psi_f\right) + \frac{g_S}{2} \sum_a [(\bar{\psi}\tau^a\psi)^2 + (\bar{\psi} i\gamma_5 \tau^a \psi)^2] - \mathcal{U}(\Phi[A], \bar{\Phi}[A], T) \quad (5)$$

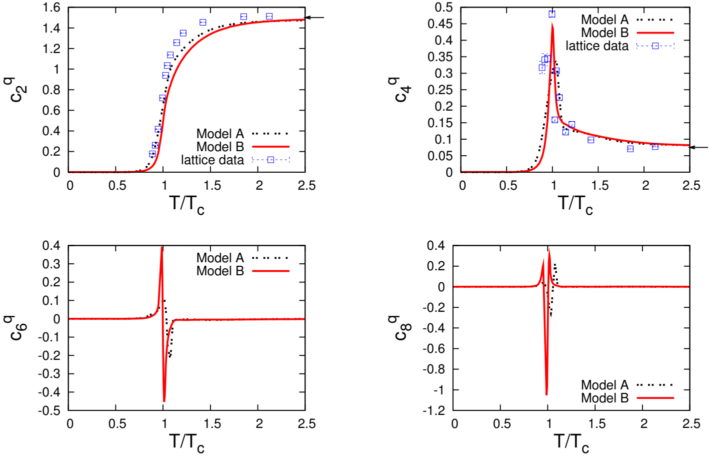
$$L(\bar{x}) = \mathcal{P} exp[i \int_0^\beta d\tau A_4(\bar{x}, \tau)] \quad (6)$$

$A_4 = iA_0$ is the temporal component of Eucledian gauge field $(\bar{A}, A_4)$, $\beta = \frac{1}{T}$.
$D^\mu = \partial^\mu - iA^\mu$ and $A^\mu = \delta^{\mu 0} A_0$

$$\Phi = (Tr_c L)/N_c, \qquad \bar{\Phi} = (Tr_c L^\dagger)/N_c$$

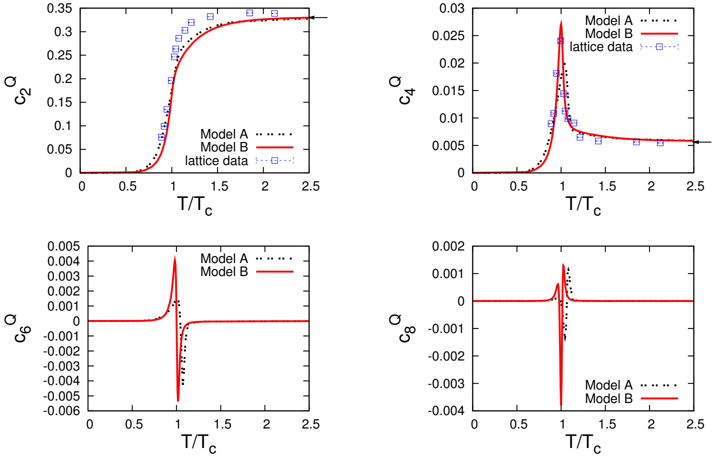
$$\frac{\mathcal{U}\left(\Phi, \bar{\Phi}, T\right)}{T^4} = -\frac{b_2\left(T\right)}{2} \bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3 + \bar{\Phi}^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2 \quad (7)$$

Taylor Coefficents for μ_q.....



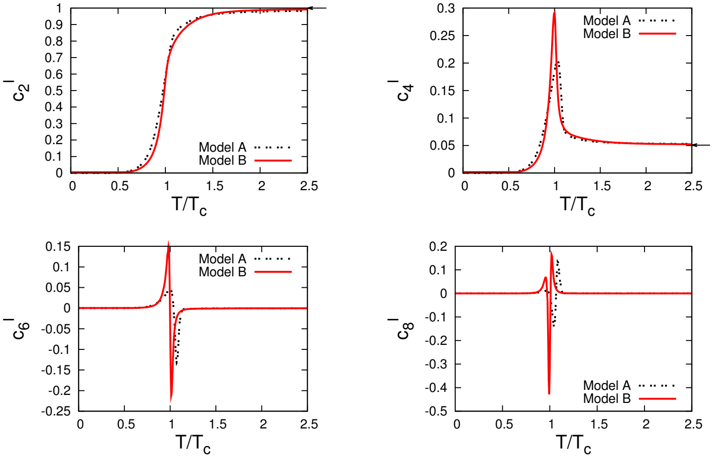
Ref : Bhattacharyya *et. al.* Phys. Rev. D **82**, 114028 (2010).

Taylor Coefficents for μ_Q.....



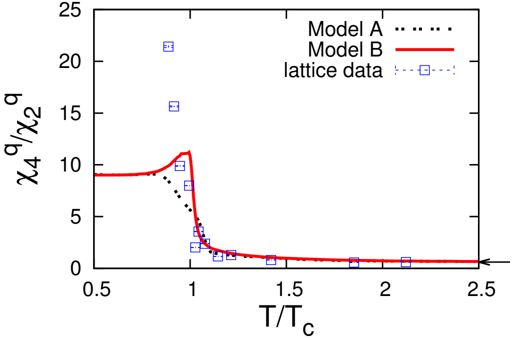
Ref : Bhattacharyya *et. al.* Phys. Rev. D **82**, 114028 (2010).

Taylor Coefficents for μ_I.....



Ref : Bhattacharyya *et. al.* Phys. Rev. D **82**, 114028 (2010).

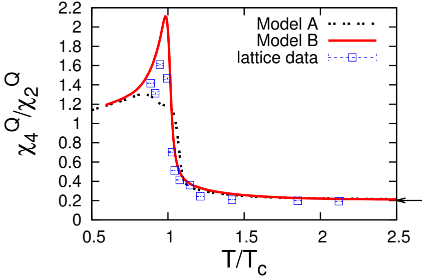
Kurtosis I

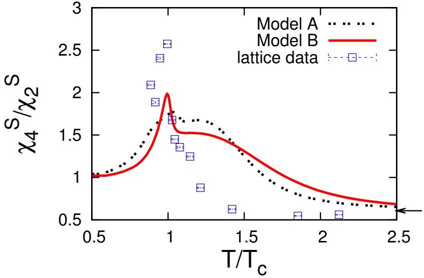


At low T kurtosis $R_q = (N_c B)^2 = 9$ and at high T it becomes unity in classical consideration and if corrected by quantum statistics $R_q = (6/\pi^2)$.

Ref : Bhattacharyya *et. al.* Phys. Rev. D **82**, 114028 (2010).

Kurtosis II

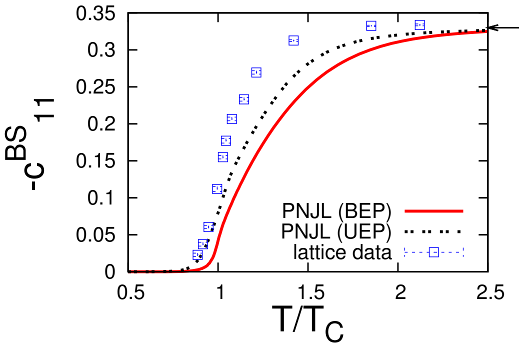




At low T, R_Q is dominated by charge fluctuations in pion sector resulting $R_Q = 1$. At high T, $R_Q = 2/\pi^2$ which is its SB limit.
Kurtosis for strange sector shows a peak at T_c. Model shows enhanced fluctuations after T_c and then converges to its SB limit.

Ref : Bhattacharyya *et. al.* Phys. Rev. D **82**, 114028 (2010).

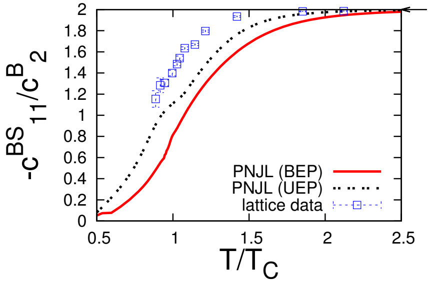
B-S Correlation I

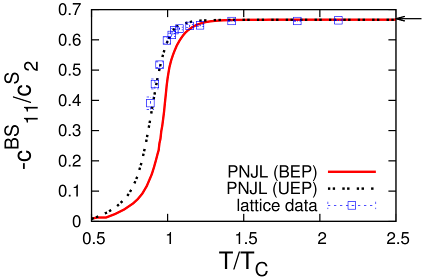


In the high T phase B and S is highly correlated resulting high value of c_{11}^{BS}. At low T the lowest lying hadrons do not carry strangeness.

Ref : Bhattacharyya *et. al.* Phys. Rev. D **83**, 014011 (2011).

B-S Correlation II



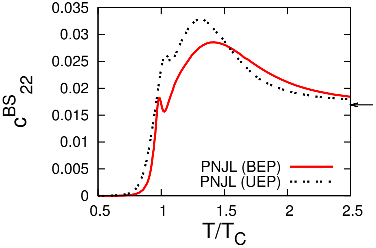


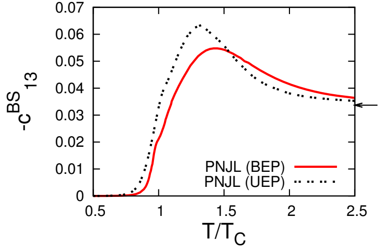
$$\mathrm{C_{SB}} = -\frac{3}{2}\frac{c_{11}^{BS}}{c_2^B} = -3\frac{\chi_{BS}}{\chi_{BB}} = -3\frac{\langle BS\rangle - \langle B\rangle\langle S\rangle}{\langle B^2\rangle - \langle B\rangle^2}$$

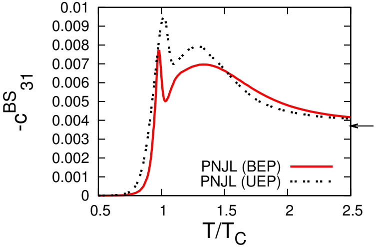
$$\mathrm{C_{BS}} = -\frac{3}{2}\frac{c_{11}^{BS}}{c_2^S} = -3\frac{\chi_{BS}}{\chi_{SS}} = -3\frac{\langle BS\rangle - \langle B\rangle\langle S\rangle}{\langle S^2\rangle - \langle S\rangle^2}$$

Ref : Bhattacharyya *et. al.* Phys. Rev. D **83**, 014011 (2011).

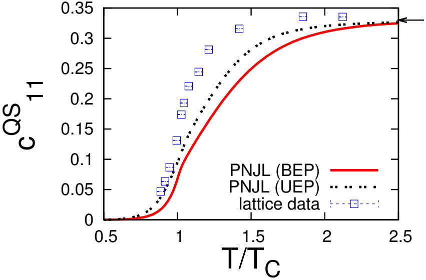
B-S Correlation III







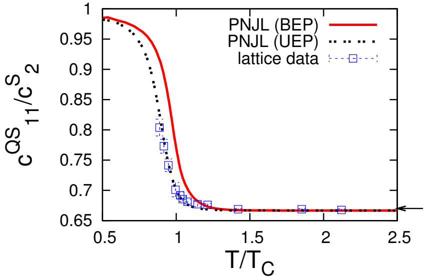
Q-S Correlation I

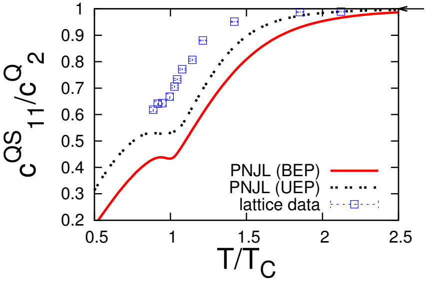


At high T, Q and S are related by strange quasiparticle which leads to high value of c_{11}^{QS}.

Ref : Bhattacharyya *et. al.* Phys. Rev. D **83**, 014011 (2011).

Q-S Correlation II

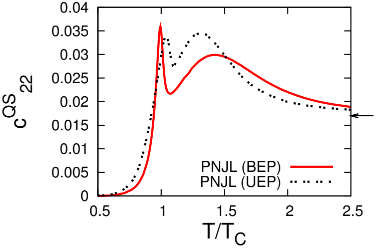


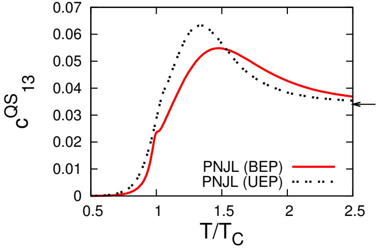


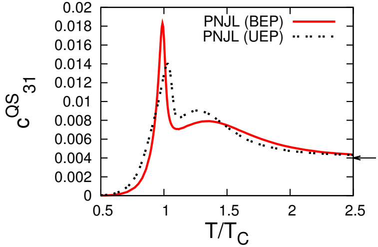
$$\mathrm{C_{QS}} = -3\frac{\langle QS\rangle - \langle Q\rangle\langle S\rangle}{\langle S^2\rangle - \langle S\rangle^2} = 3\frac{\chi_{QS}}{\chi_{SS}} = \frac{3}{2}\frac{c_{11}^{QS}}{c_2^S} \qquad \mathrm{C_{SQ}} = -3\frac{\langle QS\rangle - \langle Q\rangle\langle S\rangle}{\langle Q^2\rangle - \langle Q\rangle^2} = 3\frac{\chi_{QS}}{\chi_{QQ}} = \frac{3}{2}\frac{c_{11}^{QS}}{c_2^Q}$$

Ref : Bhattacharyya *et. al.* Phys. Rev. D **83**, 014011 (2011).

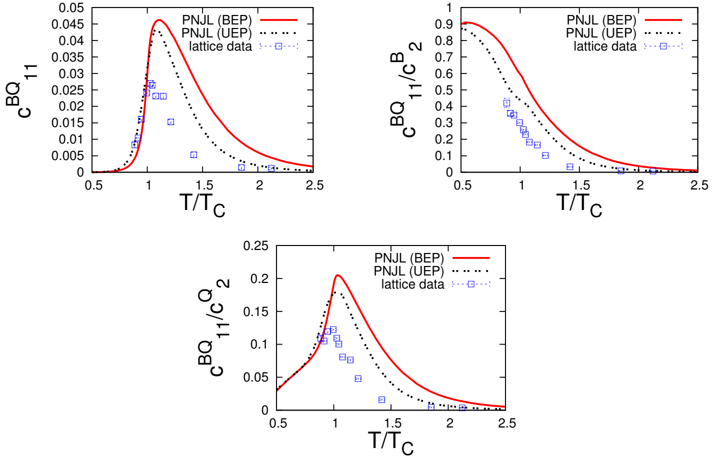
Q-S Correlation III



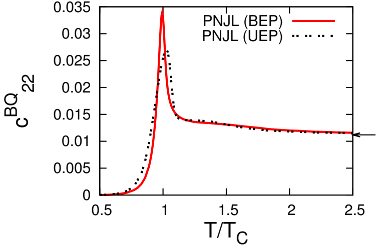


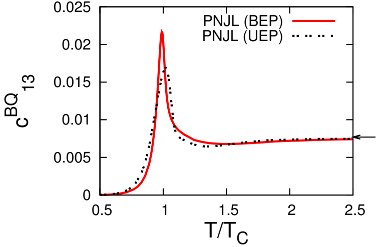


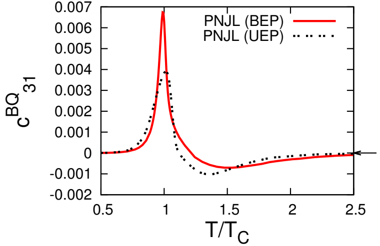
B-Q Correlation I



B-Q Correlation II



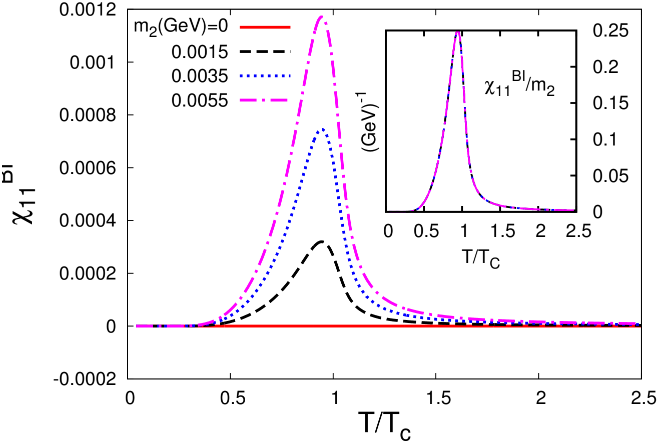




Mass splitting

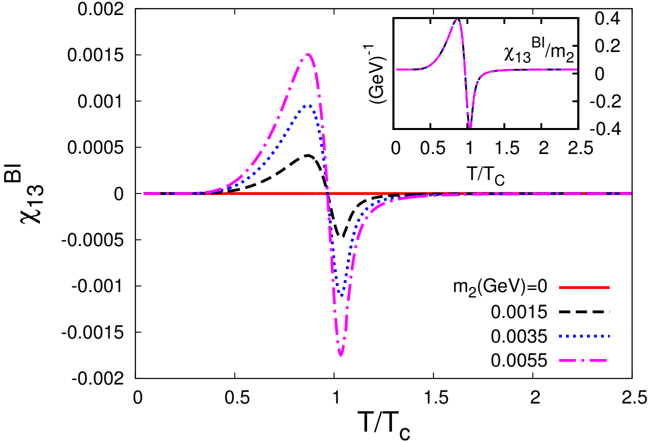
$$\hat{m} \equiv m_1 \mathbb{1}_{2\times 2} - m_2 \tau_3$$
$$= \begin{pmatrix} m_1 - m_2 & 0 \\ 0 & m_1 + m_2 \end{pmatrix} = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}.$$

2^{nd} order $B - I$ correlation along T



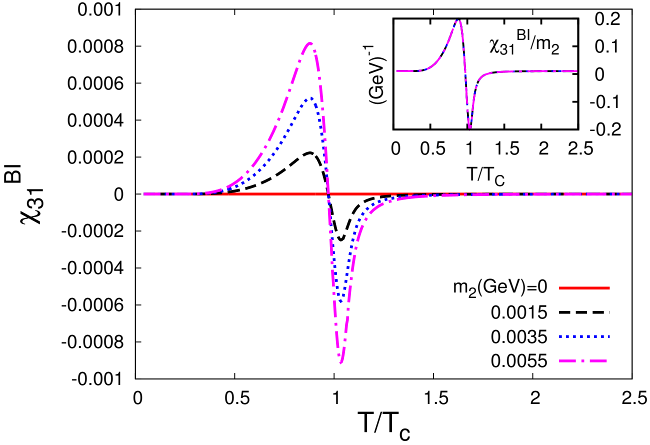
Ref : Bhattacharyya *et. al.* Phys. Rev. C **89**, 064905 (2014).

4^{th} order $B - I$ correlation along T



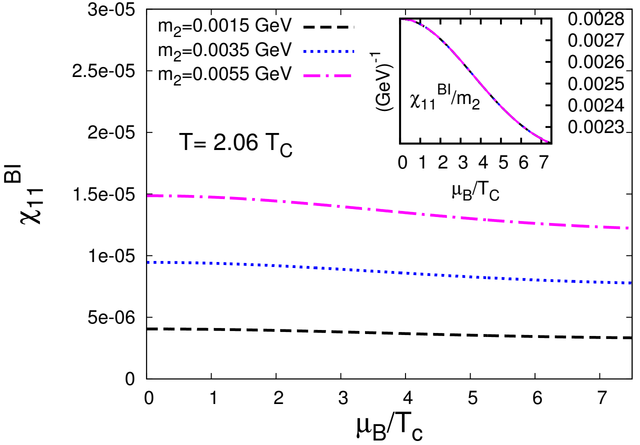
Ref : Bhattacharyya *et. al.* Phys. Rev. C **89**, 064905 (2014).

4^{th} order $B - I$ correlation along T



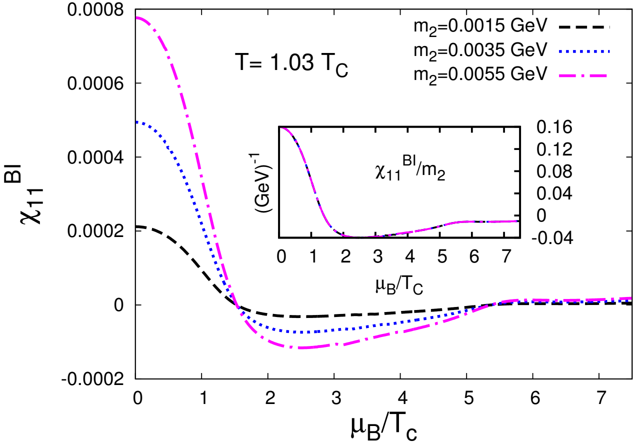
Ref : Bhattacharyya *et. al.* Phys. Rev. C **89**, 064905 (2014).

2^{nd} order $B - I$ correlation along μ_B



Ref : Bhattacharyya *et. al.* Phys. Rev. C **89**, 064905 (2014).

2^{nd} order $B - I$ correlation along μ_B



Ref : Bhattacharyya *et. al.* Phys. Rev. C **89**, 064905 (2014).

Mathematical point of view I

For second order :

$$\chi_{11}^{BI} = \frac{1}{6}(\chi_2^u - \chi_2^d)$$

For fourth order :

$$\begin{aligned}
\chi_{13}^{BI} &= \frac{1}{24}(\chi_4^u - \chi_4^d + 2\chi_{13}^{ud} - 2\chi_{31}^{ud}) \\
\chi_{31}^{BI} &= \frac{1}{54}(\chi_4^u - \chi_4^d - 2\chi_{13}^{ud} + 2\chi_{31}^{ud}) \\
\chi_{22}^{BI} &= \frac{1}{36}(\chi_4^u + \chi_4^d - 2\chi_{22}^{ud})
\end{aligned}$$

For flavor susceptibilities, we write;

$$\chi_2^f(m_u, m_d) = \sum_{n=0}^{\infty}\sum_{i=0}^{n}\frac{1}{i!j!}\left[\frac{\partial^n \chi_2^f}{\partial m_u^i \partial m_d^j}\right]_{m_u=m_d=0} \equiv \sum_{n=0}^{\infty}\sum_{i=0}^{n} a_{i,j}^f m_u^i m_d^j$$

Mathematical point of view II

with $i + j = n$ and $f \in u, d$.

Generally $a^u_{i,j} = a^d_{j,i}$ within a fixed value of $i + j = n$.

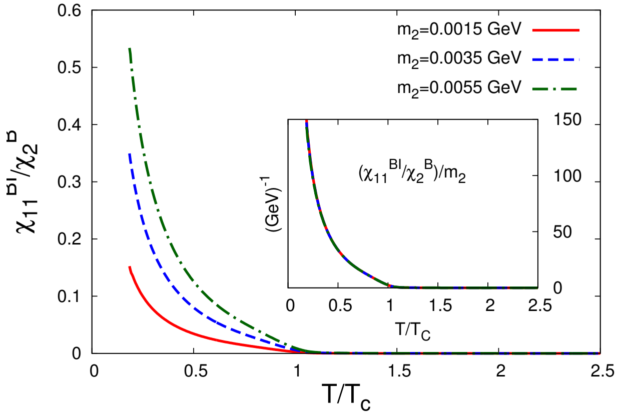
$$\chi^u_2(n^{th}\text{order}) - \chi^d_2(n^{th}\text{order}) = \sum_{i=0}^{n} \alpha_i m^i_u m^i_d (m^{n-2i}_d - m^{n-2i}_u)$$

where $\alpha_i = a^u_{i,n-i} = a^d_{n-i,i}$.

For finite μ_B :

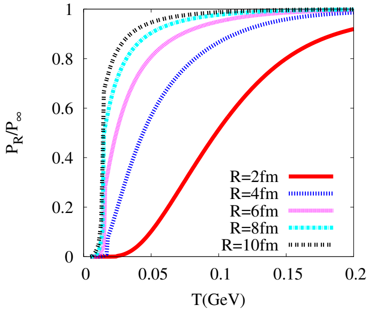
$$\chi^{BI}_{11}(\mu_B) = \chi^{BI}_{11}(0) + \frac{\mu^2_B}{2!}\chi^{BI}_{31}(0) + \frac{\mu^4_B}{4!}\chi^{BI}_{51}(0) + \cdots\cdots$$

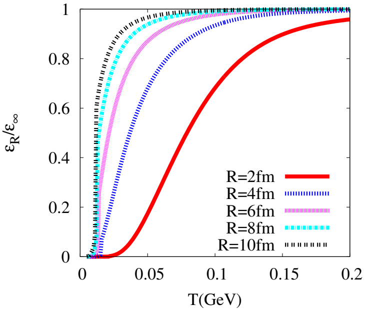
Connection to HIC experiment



Estimation of mass asymmetry : $m_2{}^{\text{expt}} = \dfrac{\mathrm{R}_2^{\text{expt}}(T, \mu_B)}{\mathrm{R}_2^{\text{PNJL}}(T, \mu_B)} \times m_2{}^{\text{PNJL}}.$

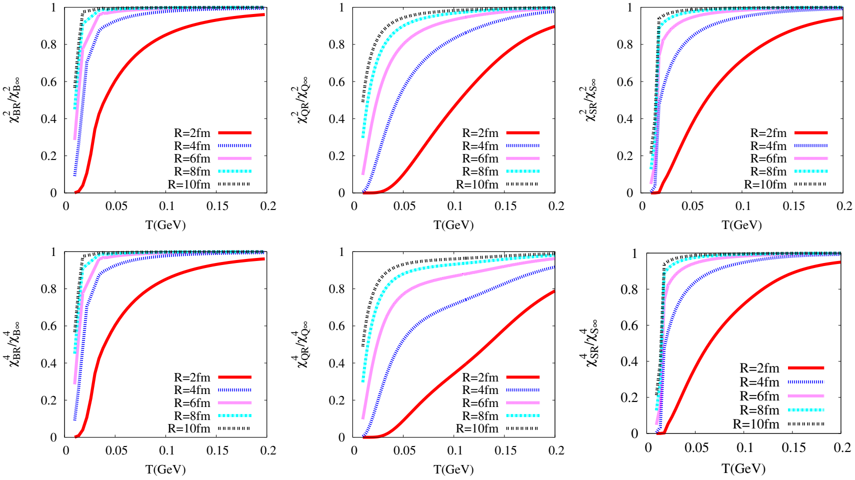
Hadronic medium I





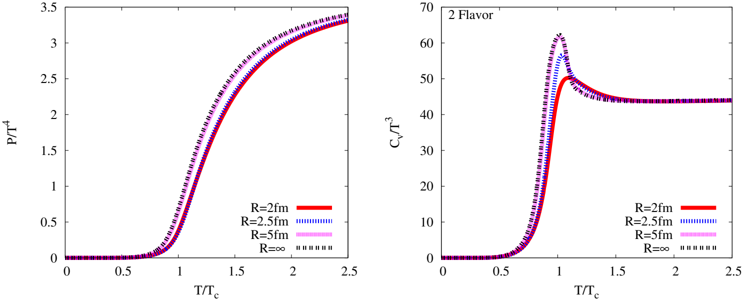
Ref : Bhattacharyya *et. al.* Phys. Rev. C **91**, 041901 (R) (2015).

Hadronic medium II



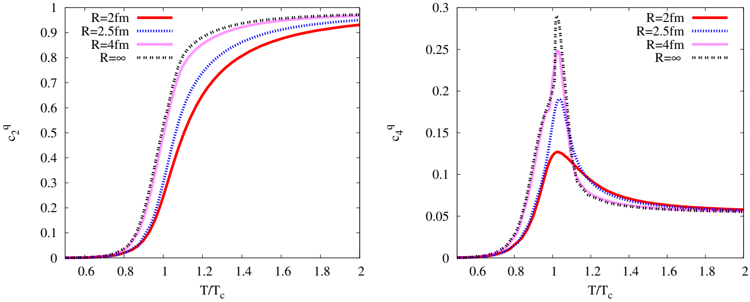
Ref : Bhattacharyya *et. al.* Phys. Rev. C **91**, 041901 (R) (2015).

Quark medium I



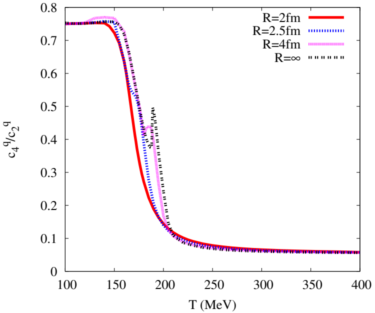
Ref : Bhattacharyya *et. al.* Phys. Rev. D **87**, 054009 (2013).

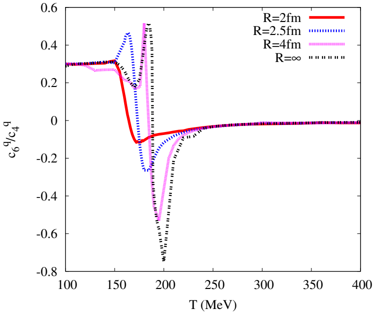
Quark medium II



Ref : Bhattacharyya *et. al.* Phys. Rev. D **91**, 051501 (R) (2015).

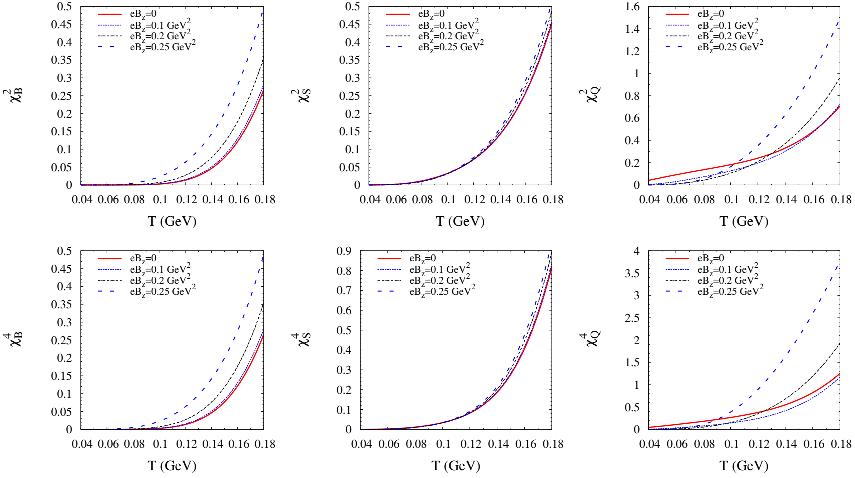
Quark medium III





Ref : Bhattacharyya *et. al.* Phys. Rev. C **91**, 051501 (R) (2015).

Hadronic matter with $B \neq 0$



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3. Explicit ISB through $m_u \neq m_d$ only.
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6. Magnetic field modifies the cumulants.

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Thank You.