Dynamical Restoration of Z_N Symmetry in $SU(N)$+Higgs Theories

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Introduction.

- In fundamental gauge interactions such as QCD, EW the matter fields are in the fundamental representation.The pure gauge sector of these theories have Z_N symmetry.
- There are many unresolved issues regarding the effect of the matter fields on this symmetry.
- However some studies suggest the symmetry leads to unphysical consequences, others suggest the symmetry is explicitly broken.
- In this work we study the Z_N symmetry in $SU(N)+$Higgs theory for $N = 2, 3$ using lattice Monte Carlo simulations.

Pure $SU(N)$ Gauge Theory.

- The Euclidean action of the non-abelian pure gauge theory is given by

$$S_E = \int_V d^3x \int_0^\beta d\tau \left\{ \frac{1}{2} Tr\left(F^{\mu\nu} F_{\mu\nu}\right) \right\} \tag{1}$$

- where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu], \qquad A_\mu = A_\mu^a T^a. \tag{2}$$

- At finite temperatures, the gauge fields A_μ^a are periodic in the temporal direction,i.e

$$A_\mu^a(\vec{x}, 0) = A_\mu^a(\vec{x}, \beta). \tag{3}$$

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- Under Gauge transformation, The gauge field transforms

$$A_\mu \longrightarrow VA_\mu V^{-1} + \frac{1}{g}\left(\partial_\mu V\right) V^{-1} \tag{4}$$

- Where the gauge transformation $V(\vec{x},\tau) \in SU(N)$. Because of Equation(3), the gauge transformations which keep the action invariant are

$$V(\vec{x},\tau = 0) = zV(\vec{x},\tau = \beta). \tag{5}$$

- Where $z \in Z_N$,with $z = exp(2\pi in/N)$, $n = 0, 1, 2...N - 1$, Z_N is the center of the gauge group $SU(N)$.
- TheZ_N symmetry originates from Equation(5).
- Since the action S_E is invariant under the Z_N group.Its a symmetry of the theory.

Z_N Symmetry.

- Polyakov loop(L) is defined as:

$$L(\vec{\mathbf{x}}) = \frac{1}{N} Tr \left\{ Pe^{\left(-ig \int_0^{\beta} A_0 d\tau\right)} \right\} \tag{6}$$

- Polyakov loop(L) transforms under above gauge transformation $L \longrightarrow zL$.
- The Polyakov loop behaves like a Z_N spin and plays the role of an order parameter for the pure gauge Confinement-Deconfinement(C-D) transition.
- In the deconfined phase the Polyakov loop acquires a non-zero expectation value which leads to spontaneous breaking of the Z_N symmetry. On the other hand in the confined phase it has zero expectation value so the Z_N symmetry is restored.

$Z_{(}N)$ Symmetry in Presence of fundamental Higgs field.

- The modified action which describes the interaction of the gauge fields and the Higgs field Φ is given in the fundamental representation,

$$S_E = \int_V d^3x \int_0^\beta d\tau \left\{ \frac{1}{2} Tr\left(F^{\mu\nu} F_{\mu\nu}\right) + \frac{1}{2}|D_\mu \Phi|^2 + \frac{m^2}{2}\Phi^\dagger \Phi + \frac{\bar{\lambda}}{4!}(\Phi^\dagger \Phi)\right. \qquad (7$$

- The covariant derivative $D_\mu \Phi$ is defined as $D_\mu \Phi = \partial_\mu \Phi + igA_\mu \Phi$.
- Under a gauge transformation $V(\vec{\mathbf{x}}, \tau)$ the Φ field transforms as, $\Phi' = V\Phi$.
- But being a bosonic field, Φ satisfies periodic boundary condition in the temporal direction, i.e

$$\Phi(\vec{x}, 0) = \Phi(\vec{x}, \beta). \qquad (8)$$

- The action in equation(7) is not invariant under Z_N symmetry.

$SU(N)+$ Higgs Theory .

- Even though the classical action is not invariant under Z_N symmetry, the partition function may exhibit this symmetry because it include thermal fluctuations.
- The discretized lattice action in 4-dimensional Euclidean space is given by,

$$S = \beta \sum_p \mathrm{Tr}(1 - \mathrm{U_p} - \mathrm{U_p^\dagger}) - \kappa \sum_\mu \mathrm{Re}\left[\mathrm{Tr}(\phi^\dagger_{\mathrm{n}+\mu} \mathrm{U}_{\mathrm{n},\mu} \phi_\mathrm{n})\right]$$
$$+\frac{1}{2}\mathrm{Tr}\left(\phi^\dagger_\mathrm{n}\phi_\mathrm{n}\right) + \lambda\left(\frac{1}{2}\mathrm{Tr}\left(\phi^\dagger_\mathrm{n}\phi_\mathrm{n}\right) - 1\right)^2 .$$

- Where Higgs and gauge fields are defined on the discrete set of points $n = (n_1, n_2, n_3, n_4)$,with n_i's are integers.
- The gauge fields A_μ in the action are replaced by the link variables $U_\mu = exp(-iagA_\mu)$.

Numerical Simulations.

- To study the Z_N symmetry we compute the distribution of the Polyakov loop and other properties using Monte-Carlo simulations.
- In the Monte Carlo simulations an initial configuration of Φ_n and $U_{\mu,n}$ is repeatedly updated to generate a Monte Carlo history.
- In an update a new configuration is generated from an old one according to the Boltzmann probability factor e^{-S} and the principle of detailed balance.
- These conditions are implemented using pseudo heat-bath algorithm for the Φ field and the standard heat-bath algorithm for the link variables U_μ's.
- We also use over-relaxation methods to reduce the autocorrelations between adjacent configurations along the Monte Carlo trajectory.

Results.

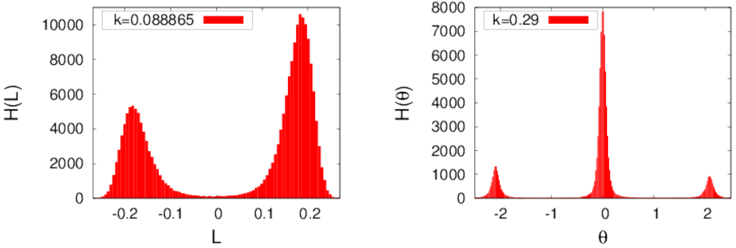

Figure: Distribution of Polyakov loop in the Higgs broken phase for (a) $SU(2)$, $16^3 \times 4$ lattice and (b) $SU(3)$, $8^3 \times 4$ lattice.

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- Figure(a) shows that one global maximum gives rise to explicit symmetry breaking and the local maximum corresponds to metastable state of the system. Similarly for $SU(3)$ there are two local maximas.
- The asymmetry of the distribution increases when κ is increased further.
- The distribution becomes symmetric for some lower κ(non zero) values.

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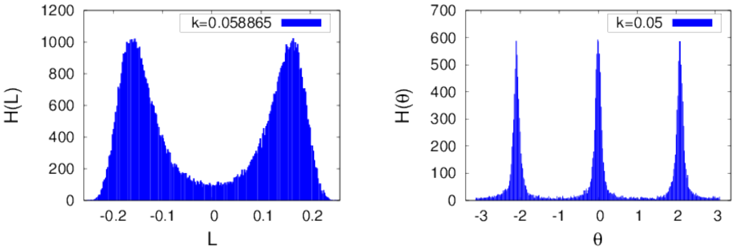

Figure: Distribution of Polyakov loop in Higgs symmetric phase for (a) $SU(2)$, $16^3 \times 4$ lattice and (b) $SU(3)$, $8^3 \times 4$ lattice.

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- The Histogram of the polyakov loop in the two sectors are in perfect agreement when one is Z_2 rotated. Similarly in $SU(3)$.
- For further clarification of restoration of Z_N symmetry, We need to calculate the corresponding free energies for different sectors.
- We show the average values of the action for both $+ve$ and $-ve$ sectors agrees quite well for $N = 2$ .So we can argue that the two sectors have same free energies.

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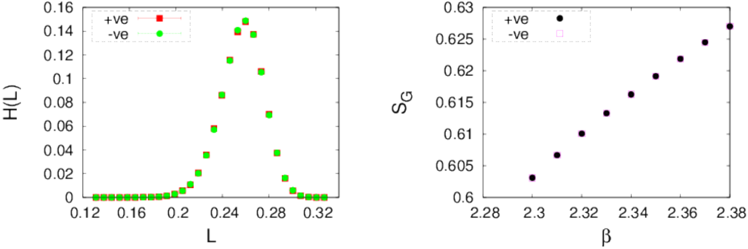

Figure: (a)Polyakov loop distribution in the two vacua ($-ve$ is Z_2 rotated). (b)Average gauge action for $+ve$ and $-ve$ sectors.

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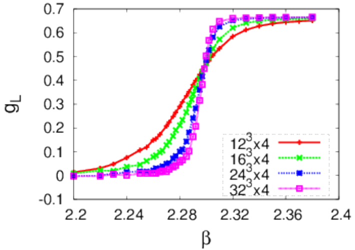

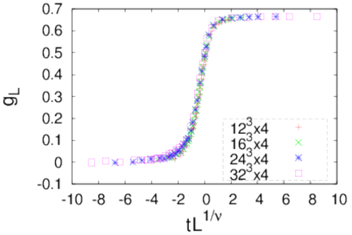

Figure: (a) Binder cumulant and (b) its scaling for $SU(2)$.

- we carry out the finite size scaling analysis of the Binder cumulant.
- The value of the Binder cumulant at the crossing point corresponds to the universality class of the 3$-$D Ising model.
- The scaling of Binder cumulant also consistent with 3$-$D Ising model.
- These results clearly show that the C-D transition is second order even for finite but small κ.

Summary.

- Our results show that the Z_N symmetry is restored in the Higgs symmetric phase (when the condensate is expected to be zero). This suggests may be the condensate plays the role of the external field.
- The C-D transition was thought to be a crossover for non-zero κ.But we show that there will be a line of second order C-D transition in the $\beta - \kappa$ plane extending from the point $(\beta_c, \kappa = 0)$.

Conclusions.

- The Z_N restoration at non-zero κ is in contradiction with effective potential calculations which show that the Z_N symmetry will be restored only when the Higgs mass is infinite.
- In these calculations only the zero mode of the Polyakov loop is coupled to the matter fields. We expect that taking care of the higher modes of the Polyakov loop will reduce the discrepancy between the non-perturbative and analytic approaches.
- Spontaneous symmetry breaking of the Z_N symmetry will lead to time independent topological defects solutions such as domain walls, strings etc.