Study of finite size effects from Polyakov--Nambu--Jona-Lasinio model

KINKAR SAHA

CAPSS, BOSE INSTITUTE

KOLKATA

COLLABORATORS

Dr. Rajarshi Ray Prof. Sibaji Raha Dr. Abhijit Bhattacharyya

Prof. Sanjay K. Ghosh Sudipa Upadhaya

VECC

Scheme for the next 20 minutes

1

[1]A. Bhattacharyya *et.al* PRD 87, 054009 (2013), 91, 051501 (2015), arXiv : 1507.08795v1 [hep-ph].

Scheme for the next 20 minutes

1

- Prelude

[1] A. Bhattacharyya *et.al* PRD 87, 054009 (2013), 91, 051501 (2015), arXiv : 1507.08795v1 [hep-ph].

Scheme for the next 20 minutes

[1]

- Prelude
- Few important facts to recollect

[1] A. Bhattacharyya *et.al* PRD 87, 054009 (2013), 91, 051501 (2015), arXiv : 1507.08795v1 [hep-ph].

Scheme for the next 20 minutes

[1]

- Prelude
- Few important facts to recollect
- Motivation

[1] A. Bhattacharyya *et.al* PRD 87, 054009 (2013), 91, 051501 (2015), arXiv : 1507.08795v1 [hep-ph].

Scheme for the next 20 minutes

[1]

- Prelude
- Few important facts to recollect
- Motivation
- Formalism

[1] A. Bhattacharyya *et.al* PRD 87, 054009 (2013), 91, 051501 (2015), arXiv : 1507.08795v1 [hep-ph].

Scheme for the next 20 minutes

[1]

- Prelude
- Few important facts to recollect
- Motivation
- Formalism
- Results and Discussions

[1] A. Bhattacharyya *et.al* PRD 87, 054009 (2013), 91, 051501 (2015), arXiv : 1507.08795v1 [hep-ph].

Scheme for the next 20 minutes

[1]

- Prelude
- Few important facts to recollect
- Motivation
- Formalism
- Results and Discussions
- Wrap-up

[1] A. Bhattacharyya *et.al* PRD 87, 054009 (2013), 91, 051501 (2015), arXiv : 1507.08795v1 [hep-ph].

Prelude

- The hot and dense matter formed in ultra-relativistic heavy ion collisions possesses a rich phase structure.
- In the region of low baryon chemical potential there is a smooth crossover for the matter while for higher baryonic chemical potential there lies a first order transition belt.
- A critical end point thus seems to naturally occur in such a situation and establishing this scenario forms an integral part of exploration in the international collaborative experiments at CERN and BNL and the upcoming experiments at GSI.
- Here we will be discussing effects of finiteness of the system sizes on such a matter within the framework of 2+1-flavor Polyakov–Nambu–Jona-Lasinio model with special emphasis to check the volume scaling of thermodynamic observables for various temperatures and chemical potentials.

In this regard...

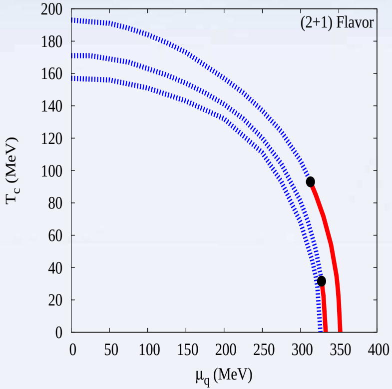
- Matter generically divided into phases:
  color confined states $\rightarrow$ hadronic phase.
  exotic state with color degrees of freedom $\rightarrow$ Quark-gluon plasma.
- Such exotic state in experimental scenario produced by ultra relativistic heavy-ion collisions and thus system's volumes depend on
  $\rightarrow$ nature of colliding nuclei,
  $\rightarrow$ center of mass energy $\sqrt{s}$,
  $\rightarrow$ centrality of collisions.
- Efforts made to estimate the system's size :
  $\rightarrow$ HBT radii,
  $\rightarrow$ volume of homogeneity from UrQMD etc.

In this regard...

- Given the freeze out volumes, one can trace back to the initial equilibration time and expect an even smaller system size.
- In fact the whole fireball can not be considered as the system and therefore choice of proper rapidity interval becomes essential.
- So it becomes important to analyse the behavioral nature of thermodynamic quantities under these circumstances.
- Specifically the finiteness of the system sizes lead to smoothening of singularities appearing at a phase transition.

Motivation

- Chiral symmetry gradually gets restored for smaller and smaller volumes.
- CEP gradually shifts towards higher μ_q and lower T and finally disappears.
- Encouraging fact for critical point search in heavy ion collision experiments.



♣ Fluctuations of conserved charges in this regard, are sensitive indicators of the transitions from hadronic matter to partonic state.

♣ Alongside the existence of CEP may be signalled by their diverging behavior.

Formalism

♣ Simplifications adopted ::

- Choice of the proper boundary conditions–periodic for bosons and anti-periodic for fermions –would lead to infinite sum over discrete momentum values $p_i = \pi n_i/R$, with R being the lateral size of a cubic volume.
  $\Rightarrow$ a lower momentum cut-off, $p_{min} = \pi/R = \lambda(say)$. Here the infinite sum is considered as an integration over continuous variation of momentum albeit with a lower cut-off.
- The surface and curvature effects are neglected.
- Modifications to the mean field parameters due to finite size effects are not considered. Philosophy has been to hold the known physics at zero T, zero μ and infinite V fixed, thus to treat V as a thermodynamic variable in the same footing as T and μ.

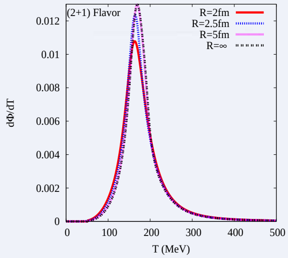
Thermodynamic potential

$$
\begin{aligned}
\Omega \quad &= \mathcal{U}'[\Phi, \bar{\Phi}, T] + 2g_S \textstyle\sum_f \sigma_f^2 \; - \; \frac{g_D}{2}\sigma_u\sigma_d\sigma_s - 6\textstyle\sum_f \int_\lambda^\Lambda \frac{d^3p}{(2\pi)^3} E_f \Theta(\Lambda - |\vec{p}|) \\
&-2T\textstyle\sum_f \int_\lambda^\infty \frac{d^3p}{(2\pi)^3} \ln\left[1 + 3(\Phi + \bar{\Phi}e^{-\frac{(E_f-\mu_f)}{T}})e^{-\frac{(E_f-\mu_f)}{T}} + e^{-3\frac{(E_f-\mu_f)}{T}}\right] \\
&-2T\textstyle\sum_f \int_\lambda^\infty \frac{d^3p}{(2\pi)^3} \ln\left[1 + 3(\bar{\Phi} + \Phi e^{-\frac{(E_f+\mu_f)}{T}})e^{-\frac{(E_f+\mu_f)}{T}} + e^{-3\frac{(E_f+\mu_f)}{T}}\right]
\end{aligned}
$$

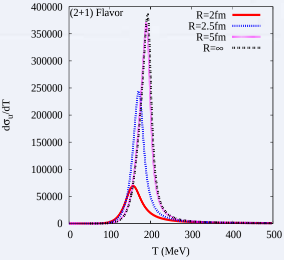
We opt to use the saddle point approximation to study the scenario.

What is done..

Search for the saddle point of the thermodynamic potential gives the T and μ dependence of the fields. The crossover temperature is identified to be the point of inflection of the corresponding order parameters of the transitions.



The shift in T_c is mainly due to the shift in transition temperature of the chiral phase transition.



Similar procedure can be applied for non-zero μ as well and CEP can be seen to gradually vanish with decreasing system size

Results and Discussions

- The analysis here has been done under the mean field approximation and the result for the transition temperature corresponding to different system sizes is ::

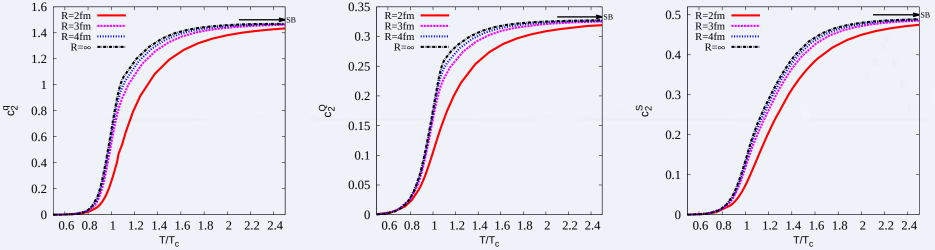
R(fm)	2	3	4	∞
T_c(MeV)	160	174	178	181

- The globally conserved charges are quark number q, the electric charge Q and the strangeness, S.
- Their fluctuations are obtained as the moments of different orders from corresponding μ-dependence of the free energy as,

$$c_n^X(T) = \frac{1}{n!}\frac{\partial^n(-\Omega(T,\mu_X)/T^4)}{\partial(\frac{\mu_X}{T})^n}\Big|_{\mu_X=0}$$

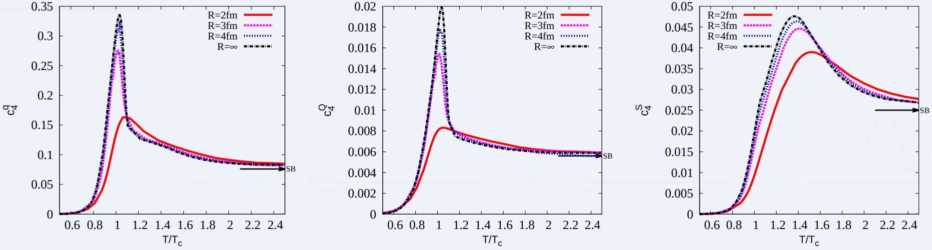
Susceptibilities...

- 2nd and 4th order susceptibilities for all conserved charges ::



- Qualitative feature similar to the infinite volume case, though with significant quantitative size dependence in the range of $0.8T_c < T < 2T_c$.

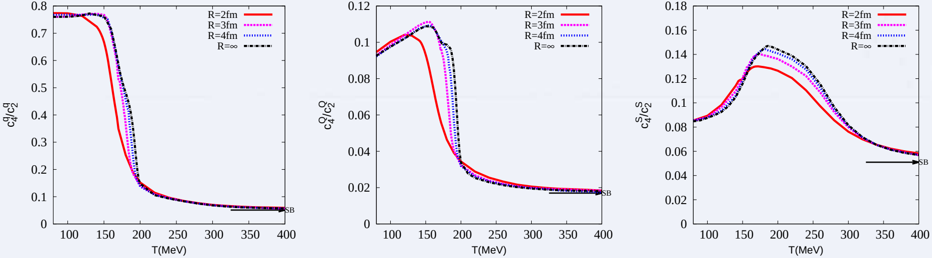
Susceptibilities...



- Strongest volume dependence in the range of $0.8T_c < T < 1.2T_c$.
- If the experimentally produced fireball thermalizes near T_c, consideration of volume becomes important alongwith the other parameters.

Ratios...

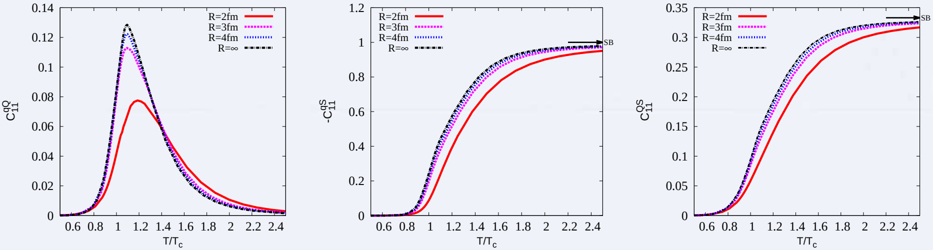
- Helps in situation where volume is not directly measured with free energy being assumed to be proportional to volume.



- Scaling of the ratios with the system volume holds good away from the crossover region with violation of volume scaling happening in a finite T window around T_c.

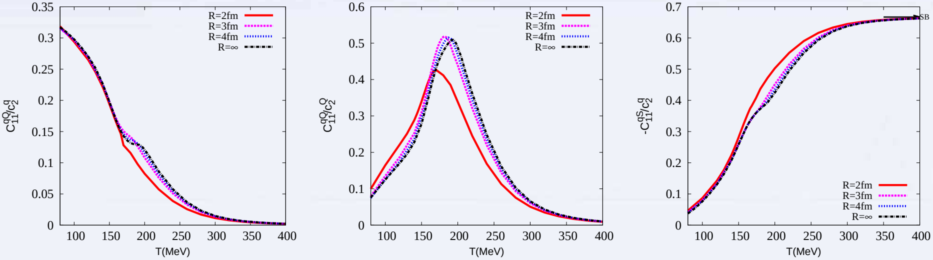
Correlations

$$C_{i,j}^{X,Y} = \frac{1}{i!j!} \frac{\partial^{i+j}(-\Omega/T^4)}{(\partial(\frac{\mu_X}{T})^i)(\partial(\frac{\mu_Y}{T})^j)}$$



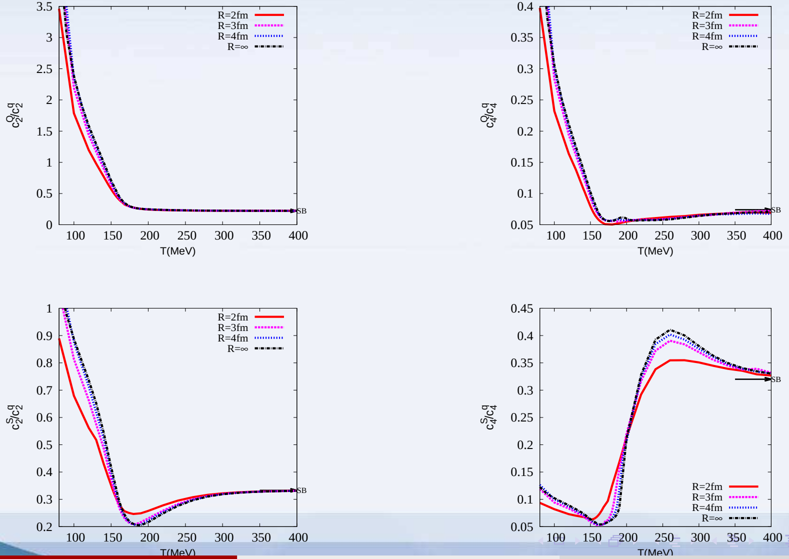
- The quark-charge correlation is most significant near the transition and has the most prominent volume dependence.
- The quark-strangeness correlation shows order parameter like behavior and takes up quite different values in two phases.

contd...



- The volume scaling violation is quite small for all the sectors.
- Important issue to note is that both of them are same order derivatives of free energy.
- Motivates to check what happens for ratios of similar order fluctuations of different conserved charges.

Back to fluctuations...



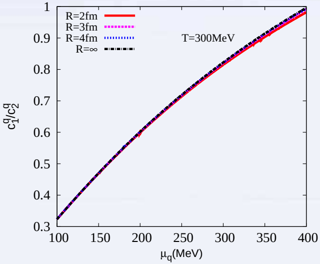
contd...

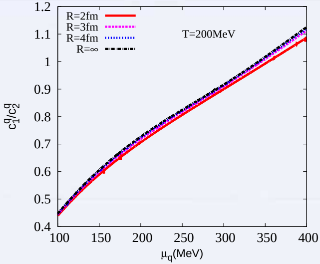
- Ratios of fluctuations of different charges for the 2nd and 4th orders show very small scaling violation.
- However 4th order fluctuation ratios have somewhat more violation than that of 2nd order.
- Large distance behavior of fluctuations change with the order of fluctuations.
- Higher order moments are found to be more sensitive to the correlation length.

Non-zero chemical potentials...

- As discussed, the free energy can be expanded in series in baryon (or quark) chemical potential around $\mu = 0$ unless there is a singularity.
- The coefficients provide the measure of fluctuations around $\mu = 0$.
- As these fluctuations show different volume dependences, the free energy is also expected to show violation in volume scaling in the non vanishing chemical potential domain.
- We have considered ratios of net number (c_1^q) and fluctuation (c_2^q), which are simplest to analyze in experimental scenario as well.
- The temperatures are specifically chosen in order to cover the whole phase diagram.

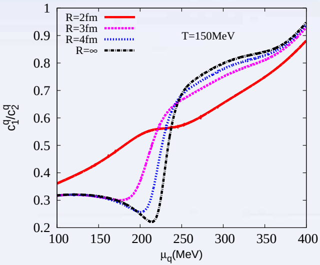
$T \geq 200 MeV \ldots$

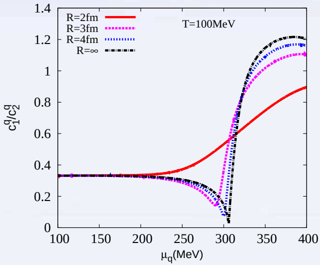




- These choices of temperature lie well above the phase transition line.
- Therefore expectedly the ratio shows no violation in volume scaling and is only a monotonically increasing function.

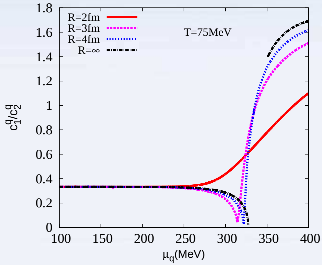
$200 MeV \geq T \geq 100 MeV \ldots$

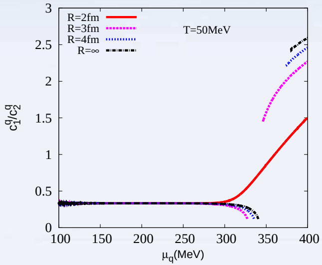




- Here a dip is observed in certain window of μ_q, which incidentally correspond to those close to the phase boundary.
- The dips occur due to the shooting up of the quark number fluctuations in these regions.

$100 MeV \geq T \geq 50 MeV \ldots$





- There is even a discontinuity in this region indicating the existence of first order line for the corresponding system size.

Violation of volume scaling if any, occur close to the transition region, where large correlation lengths come into play and lead to separate finite size behavior of the derivatives of free energy.

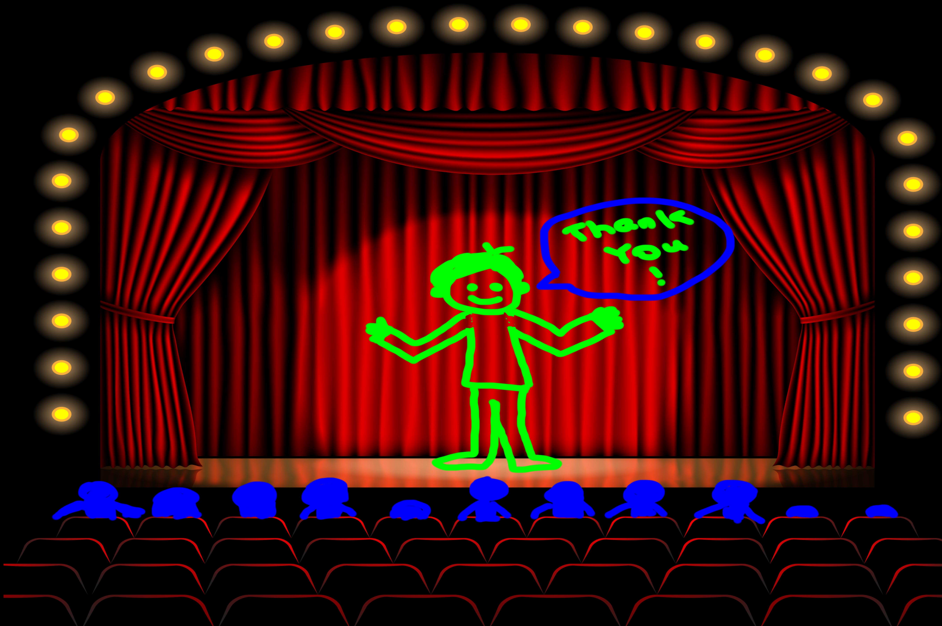
Wrapping-up...

- Study of volume dependence of the free energy density has been carried out for 2+1 flavor strongly interacting matter in terms of various second and fourth order fluctuations of the conserved charges.
- Volume scaling violations have been seen to occur mainly at two levels,

♣ The free energy density is itself volume dependent within a certain range of T and μ_q, as given by the behavior of susceptibilities.

♣ The ratio of these derivatives themselves show violation of volume scaling in a small window of T and μ_q all along the transition region in the T-μ_q plane.

Further investigation in this direction is under process with the computation of higher order moments in terms of skewness, kurtosis etc.


Thank You !