TRANSPORT PROPERTIES OF HOT AND DENSE QUARK MATTER

Hiranmaya Mishra

Theory Division

Physical Research Laboratory, Ahmedabad

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Based on

(Guru Prasad Kadam, Paramita Deb,HM arXiv:xxxxxx[hep-ph]

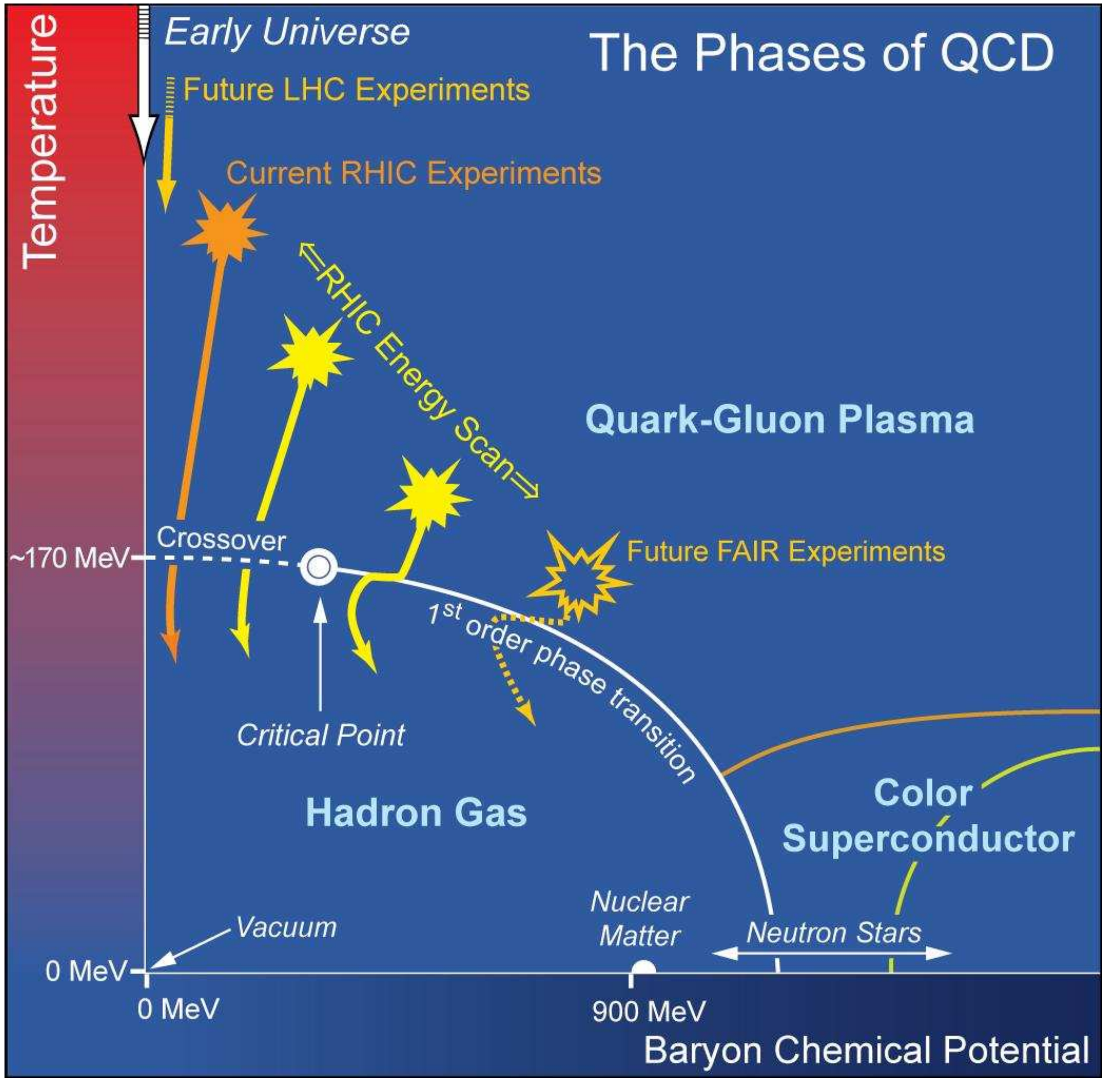
OUTLINE

- **Introduction**
- **Boltzmann equation and transport coefficients**
- **Nambu-JonaLasinio model, Thermodynamics and relaxation time estimation**
- **Results**
- **Summary and Outlook**

INTRODUCTION

- Transport properties of hot/dense matter are important for heavy ion collision (HIC), cosmology and important for near equillibrium evolution of any thermodynamic system
- The most studied transport coefficient is perhaps shear viscosity η. In HIC spatial anisotrpy of colliding nuclei gets converted to momentum anisotropy throgh a hydro evoln. The equllibriation is decided by η. ($\frac{\eta}{s} \sim \frac{1}{4\pi}$, the KSS bound)
- The bulk viscosity ζ - thought earlier to be not important for HIC hydro evolution. Argument: $\zeta \sim (\epsilon - 3p)$ that vanishes for ideal gas. However, lattice simulation $\Rightarrow$ large $\epsilon - 3p$ near T_c. This,in turn, can give rise to different physical effects (Cavitation).
- The temperature and chemicalpotential dependence of transport coefficients may reveal the location of phase transition
- Most calculations are performed at zero baryon density ρ_B. Including finite density effects are relevent for upcoming HIC experiments, BES(Brookhaven), CBM at (GSI, Darmstadt), (NICA at Dubna).

QCD PHASE DIAGRAM AND HIC



Boltzmann Equation

Boltzmann equation describes the evolution of particle distribution function

$$\frac{df_a}{dt} = \frac{\partial f^a}{\partial t} + \frac{p^i}{E^a}\frac{\partial f^a}{\partial x^i} - \frac{\partial E_a}{\partial x^i}\frac{\partial f^a}{\partial p^i} = C^a$$

The equllibrium distribution function

$$f_a^0 = \frac{1}{\exp\beta(u_\alpha p^\alpha \mp \mu) + 1}$$

To estimate viscosity coefficients, consider small departure from equllibrium

$$\frac{df_a}{dt} = \frac{p^\mu}{E_a}\frac{\partial f_a^0}{\partial x^\mu} - \frac{M}{E_a}\frac{\partial M}{\partial x^i}\frac{\partial f_a^0}{\partial p^i} = -\frac{\delta f^a}{\tau^a}$$

$$\partial_\mu f_0^a = -f_0^a(1 \mp f_0^a)\partial_\mu\left(\beta(E^a - \mu - \mathbf{p}\cdot\mathbf{u}\right)$$

Boltzmann Eq. relates non equllibrium part of distribution function to variation in fluid velocity and temperature and chemical potential

$T^{\mu\nu}$, J_μ and transport coefficients

Distribution function is related to the energy momentum tensor

$$T^{\mu\nu} = \sum_a \int d\Gamma^a p^\mu p^\nu f^a + g^{\mu\nu} U(\sigma); \quad d\Gamma^a = \nu^a \frac{d\mathbf{p}}{(2\pi)^3}$$

$$J^\mu = \sum_a t_a \int d\Gamma_a \frac{p^\mu}{E_a} f_a$$

Change in nonequllibrium part $\Rightarrow$

$$\delta T^{ij} = \sum_a \int d\Gamma^a \frac{p^i p^j}{T E_a} \tau_a f_a (1 - f_a) q_a(p, \beta, \mu)$$

$$\delta J^i = \sum_a t_a \int d\Gamma_a \frac{p^i}{E_a} \tau_a f_a (1 - f_a) \left(t_a - \frac{n E_a}{\epsilon + p} \right) p^j \partial_j (\frac{\mu}{T})$$

ζ, η,λ contd.$\cdots$

The non equllibrium contribution related to the velocity gradients can be reorganised as

$$q^a = Q^a \partial_i u_i - \frac{p^i p^j}{2E_a} W_{ij}$$

;

$$W_{ij} = \partial_i u_j + \partial_j - \frac{2}{3}\delta_{ij}\partial_k u_k$$

Shear and bulk viscosities are defined through the dissipative part

$$\Delta T^{ij} = -\zeta \delta^{ij} \partial_k u_k - \eta W_{ij}$$

Thermal conductivity is defined through the dissipative part of the current

$$\Delta J_i = \lambda \left(\frac{nT}{w}\right)^2 \partial_i \left(\frac{\mu}{T}\right)$$

ζ, η,λ contd.$\cdots$

$$\eta = \frac{1}{15T}\sum_a \int d\Gamma_a \frac{\mathbf{p}^4}{E_a}\left(\tau_a f_a^0(1-f_a^0) + \bar{\tau}_a \bar{f}_a^0(1-\bar{f}_a^0)\right)$$

$$\zeta = -\frac{1}{3T}\sum_a \int d\Gamma_a \frac{\mathbf{p}_a^2}{E_a}\left(\tau_a f_a^0(1-f_a^0)Q_a + \bar{\tau}_a \bar{f}_a^0(1-\bar{f}_a^0)\bar{Q}_a\right)$$

$$\lambda = \frac{1}{3}\left(\frac{w}{nT}\right)^2 \sum_a t_a \int d\Gamma_a \frac{\mathbf{p}^2}{E_a^2} f_a(1-f_a)\tau_a(t^a - \frac{nE_a}{w})$$

In the bulk viscosity coefficient, the coefficient Q^a depends upon the equation of state

$$Q_a = -\left[\frac{\mathbf{p}_a^2}{3E_a} + \left(\frac{\partial P}{\partial n}\right)_\epsilon \left(\frac{\partial E}{\partial \mu} - 1\right) - \left(\frac{\partial P}{\partial \epsilon}\right)_n \left(E_a - T\frac{\partial E^a}{\partial T} - \mu\frac{\partial E_a}{\partial \mu}\right).\right]$$

ζ contd.

However, Q_a has to be supplemented by the conditions $u_\mu \delta J^\mu = 0$ and $u_\mu \delta T^{\mu\nu} u_\nu = 0$ corresponding to baryon number and energy momentum conservation. Within the relaxation time approximation, these Landau-Lifshitz conditions reduce to

$$\sum_a t_a \langle \tau_a Q_a \rangle = 0, \quad \sum_a \langle \tau_a E_a Q_a \rangle = 0$$

$$\langle \phi_a(p) \rangle = \int d\Gamma_a [\phi_a(p) f_a^0 (1 - f_a^0)]$$

If Landau Lifshitz conditions are not satisfied, replace

$$\tau_a Q_a \to \tau_a Q_a + \alpha t_a + \beta E_a$$

The unknown coefficients to be determined from the baryon number and energy momentum conservation equation. The expression for bulk viscosity consistent with the Landau Lifshitz condition is then given as

$$\zeta = -\frac{1}{T} \sum_a \langle (\tau_a Q_a + \alpha t_a + \beta E_a) \, \frac{\mathbf{p}^2}{3E_a} \rangle$$

η,ζ,λ contd.

The expressions for the transport coefficients become simpler when one realises that for ideal hydrodynamics the entropy per baryon (σ) is constant.

$$\eta = \frac{1}{15}\sum_a \int d\Gamma_a \frac{\mathbf{p}^4}{E_a^2}\tau_a f_a^0(1-f_a^0)$$

$$\zeta = \frac{1}{9T}\sum_a \int d\Gamma_a \frac{\tau_a f_a^0(1-f_a^0)}{E_a^2}\left[\mathbf{p}^2 + 3v_n^2 T^2 E_a \frac{\partial}{\partial T}\left(\frac{E_a-\mu_a}{T}\right)_\sigma\right]^2$$

$$\lambda = \frac{1}{3}\left(\frac{w}{nT}\right)^2 \sum_a \int d\Gamma_a \frac{\mathbf{p}^2}{E_a^2}\tau_a f_a(1-f_a)\left(t_a - \frac{nE_a}{w}\right)^2$$

- Transport coefficients are nonnegative as they must be.
- It is important to include the Landa-Liftshitz conditions to obtain the above results.

Knowing the equation of state and other thermodynamic quantities like velocity of sound etc. and the relaxation time one can estimate the viscosity coefficient.

This thermodynamics and estimation of relaxation time is done within the Nambu Jonalasinio model.

Nambu JonaLasinio model : Thermodynamics

$$\mathcal{L} = \bar{\psi}(i\gamma_\mu \partial^\mu - m_0)\psi - G\left((\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5 \mathbf{t}^a \psi)^2\right)$$

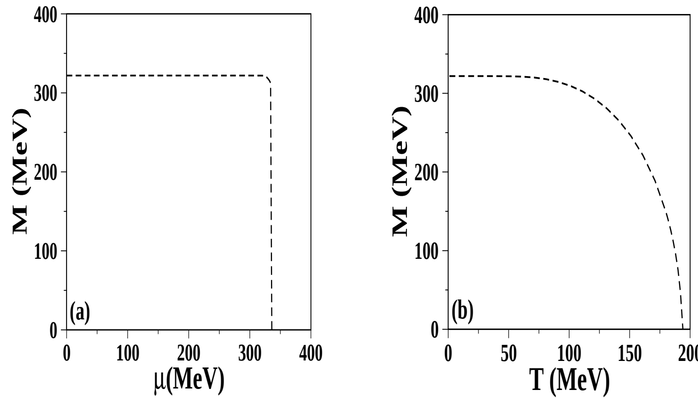
The thermodynamic potential (negative of pressure):

$$\Omega(\beta,\mu) = -\frac{\gamma}{(2\pi)^3}\int E(\mathbf{k})d\mathbf{k} - \frac{\gamma}{(2\pi)^3\beta}\int d\mathbf{k}\left(\ln(1+\exp(-\beta(E-\mu)) + \mu \to -\mu\right) + \frac{(M-m_0)^2}{4G}$$

$\gamma = 2N_cN_f$ (degeneracy); $E(\mathbf{k}) = \sqrt{\mathbf{k}^2 + M^2}$,$M$: Constituent quark mass get determined self consistently solving the mass gap equation

$$M = m_0 - 2G\langle\bar{\psi}\psi\rangle\rho_s = m_0 + \frac{\gamma}{(2\pi)^3}\int \frac{M}{E(\mathbf{k})}\left(1 - f_-(\mathbf{k},\beta,\mu) - f_+(\mathbf{k},\beta,\mu)\right)d\mathbf{k}$$

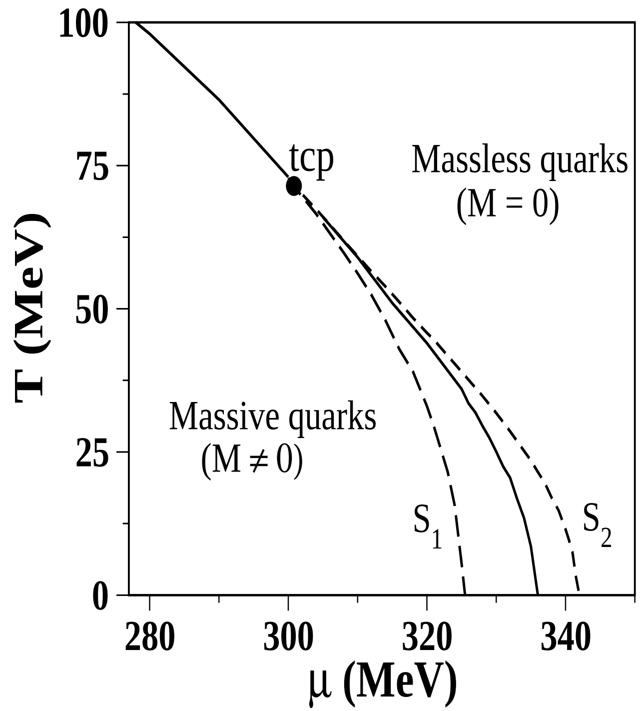
PHASE DIAGRAM; NJL MODEL



a b

Mass$\sim G\langle\bar{\psi}\psi\rangle$ as a function of μ for T=0 (Fig a) and as a function of T for $\mu = 0$ (Fig b)

Phase diagram; NJL model contd. · · ·

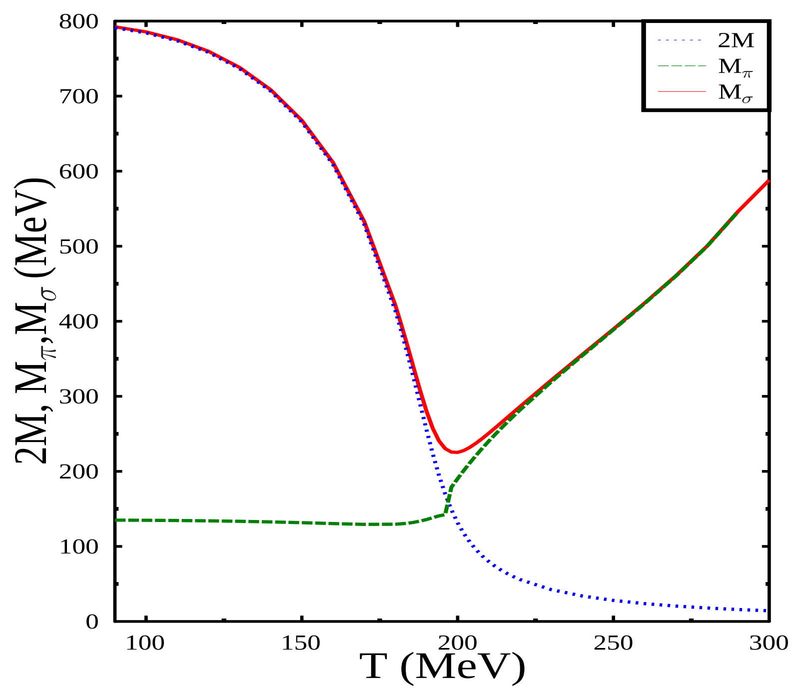


Phase diagram of the Nambu-Jona-Lasinio model in the (μ, T)-plane for zero current quark mass. A line of first-order transitions (I) meets a line of second-order transitions (II) at the tricritical point (tcp). $(\mu_{\text{tcp}}, T_{\text{tcp}}) \simeq (282.58, 78)$ MeV. The dot-dashed lines S_1 and S_2 denote the spinodals or metastability limits for the first-order transitions.

MASSES ; NJL MODEL CONTD.$\cdots$

Meson propagators: $D = \frac{2iG}{1-2G\Pi_{\sigma/\pi}}$ Mass of the meson determined by pole position of the meson propagator:

$$1 - 2G\Pi_M(m_M, \mathbf{0}) = 0$$



ESTIMATING THE AVERAGE RELAXATION TIME

Avg. relaxation time

$$\tau_a^{-1} = \sum_b n_b \bar{W}_{ab}$$

Thermally averaged transition rate

$$\bar{W}_{a,b} = \frac{1}{n_a n_b} \int f_a f_b W_{ab} d\pi_a d\pi_b$$

$$W_{ab}(s) = \frac{2\sqrt{s(s-4m^2)}}{1+\delta_{ab}} \int_{t_{min}}^{0} dt(\frac{d\sigma}{dt})(1-f_c(\frac{\sqrt{s}}{2},\mu))(1-f_d(\frac{\sqrt{s}}{2},\mu))$$

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s}\frac{1}{p_{ab}}|M|^2$$

ESTIMATING THE AVERAGE RELAXATION TIME

- For two flavors we consider the following possible scatterings.

$$u\bar{u} \to u\bar{u}, \quad u\bar{d} \to u\bar{d}, \quad u\bar{u} \to d\bar{d},$$

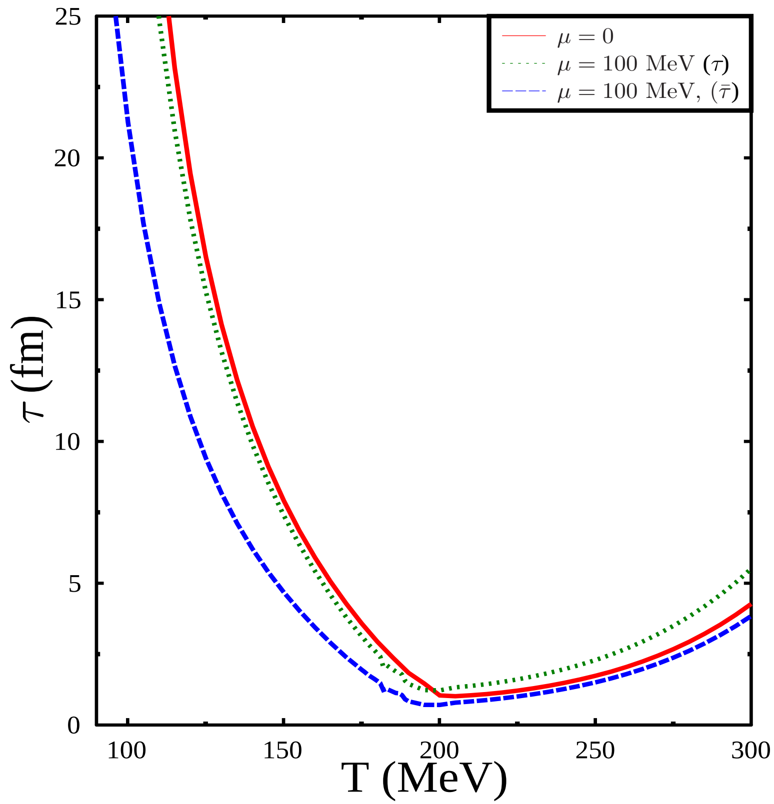
$$uu \to uu, \quad ud \to ud, \quad \bar{u}\bar{u} \to \bar{u}\bar{u},$$

$$\bar{u}\bar{d} \to \bar{u}\bar{d}, \quad d\bar{d} \to d\bar{d}, \quad d\bar{d} \to u\bar{u},$$

$$d\bar{u} \to d\bar{u}, \quad dd \to dd, \quad \bar{d}\bar{d} \to \bar{d}\bar{d},$$

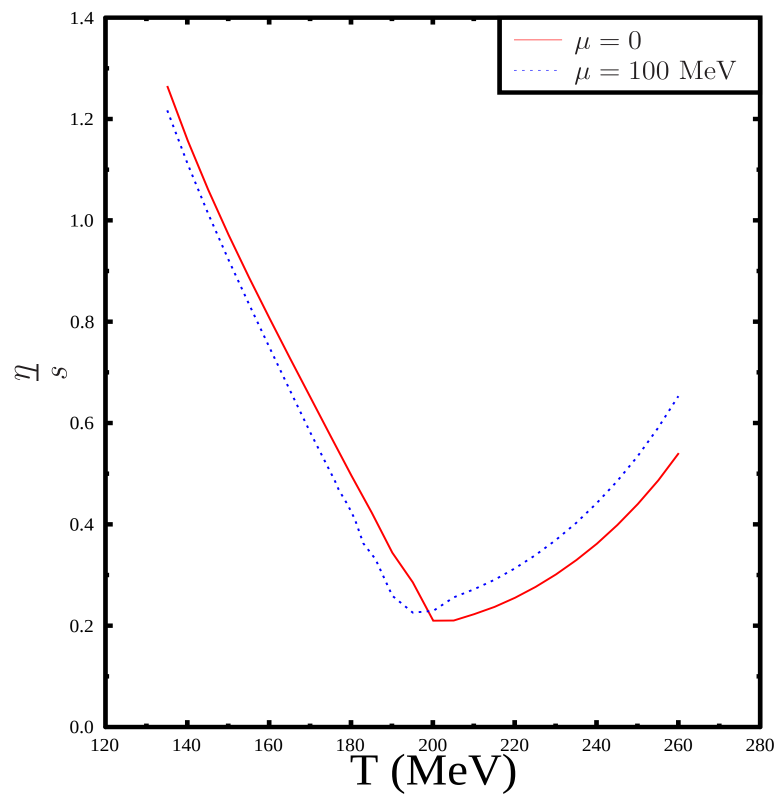
- Using i-spin symmetry, charge conjugation symmetry as well as the crossing symmetry to relate the matrix element square for the above 12 processes reduce to evaluating only two independent matrix elements $u\bar{u} \to u\bar{u}$ and $u\bar{d} \to u\bar{d}$
- Dominant contribution comes from propagation of pion and sigma mode in the s-channel.
- The temperature dependence of π and σ modes play an important role in these cross section evaluation.

Relaxation time: T behavior

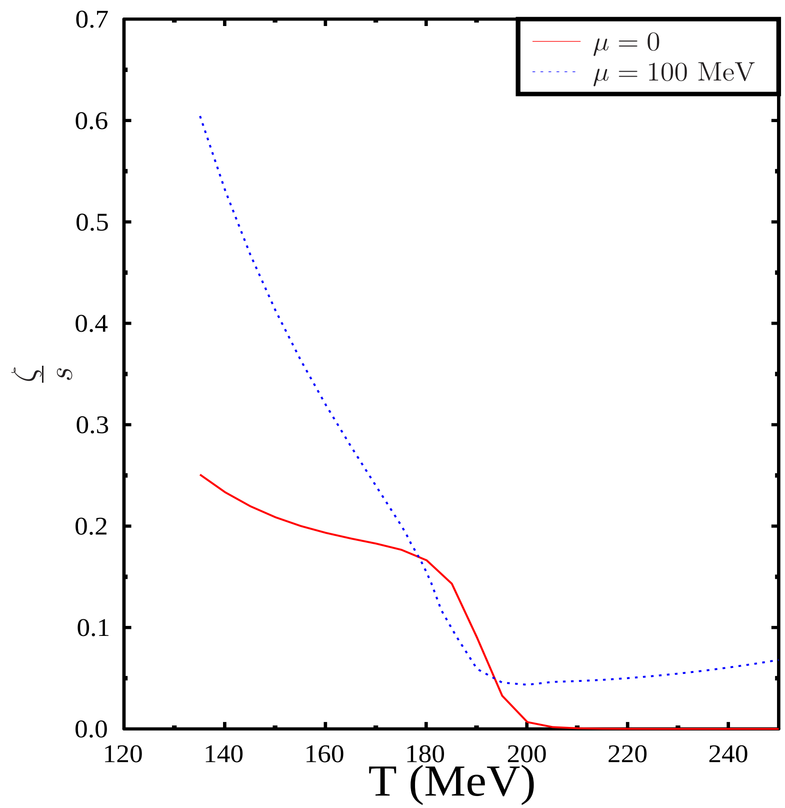


Shear viscosity: T behavior

$$\eta = \frac{1}{15}\sum_a \int d\Gamma_a \frac{\mathbf{p}^4}{E_a^2}\tau_a f_a^0(1-f_a^0)$$



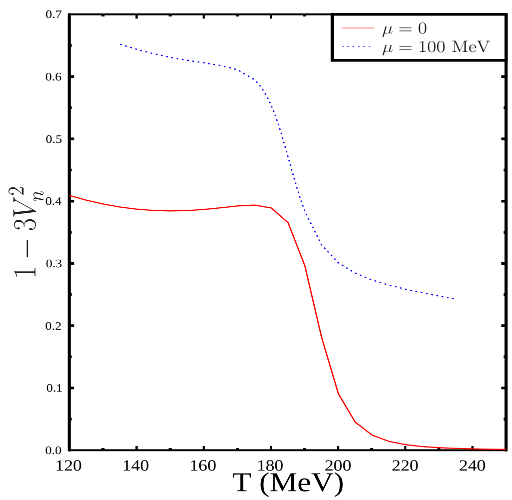
Bulk viscosity: T behavior

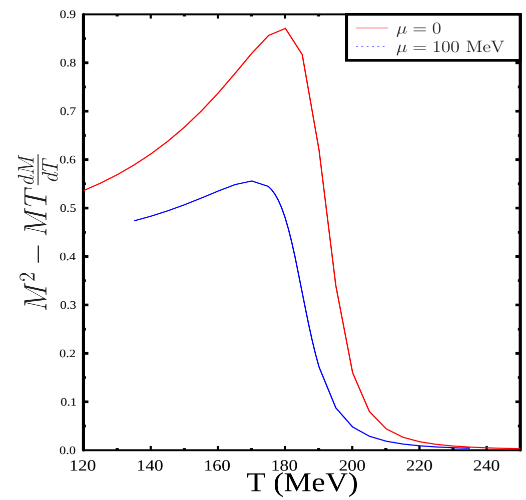


Bulk viscosity: T behavior

For zero chemical potential

$$\zeta = \frac{1}{9T}\sum_a d\Gamma^a \frac{\tau_a}{E_a^2}\left[\mathbf{p}^2(1-3v^2) - 3v^2(M^2 - TM\frac{dM}{dT}\right]^2$$

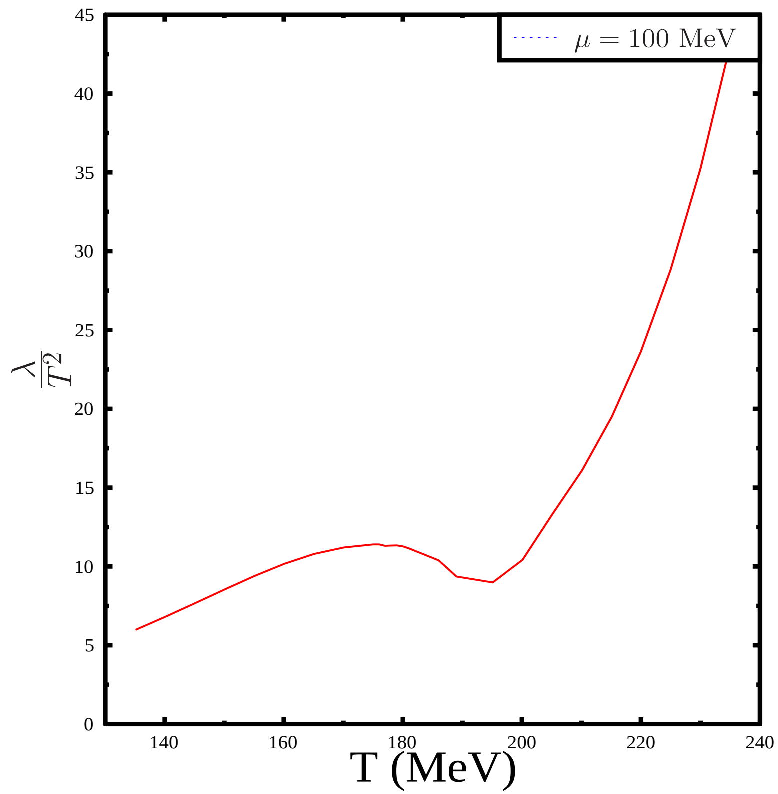




$$v_n^2 = (\frac{\partial P}{\partial \epsilon})_n = \frac{s\chi_{\mu\mu} - n\chi_{\mu T}}{T(\chi_{TT}\chi_{\mu\mu} - \chi_{\mu T}^2)}$$

$\lambda(T)$

$$\lambda = \frac{1}{3}\left(\frac{w}{nT}\right)^2 \sum_a \int d\Gamma_a \, \frac{\mathbf{p}^2}{E_a^2} \tau_a f_a (1 - f_a) \left(t_a - \frac{nE_a}{w}\right)^2$$



SUMMARY,CONCLUSIONS AND OUTLOOK

- We tried to derive the viscosity coefficients using Boltzmann kinetic equation withing relaxation time approximation within NJL model.
- While η depends only on the behaviour of relaxation time and the medium dependent masses, ζ depends on other thermodynamic quantities and the equation of state.
- The deviation from equllibrium should be consistent the Landau Lifshitz conditions.
- The thermodynamics of hot and dense matter is estimated within NJL model.
- The transport coefficients are non negative in the relaxation time approximation which is a consequence of Landau-Liftshitz conditions of fit.