Heavy-quark dynamics in an anisotropic hot QCD medium

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Outline

- Introduction and motivation
- Heavy quark dynamics in hot QCD medium
- Effective transport equation and non-equilibrium distribution functions
- Heavy-quark drag and diffusion coefficients
- Summary and conclusions

Introduction

- RHIC/LHC has already created the hot and dense form of the matter which is more like a near perfect fluid–the liquid QGP
- Hydro (second order dissipative) works reasonably well to understand the collective flow and particle spectra at RHIC/LHC
- While modelling the QGP hydrodynamically, the EoS, the first order transport coefficients (η/S, ζ/S, thermal conductivity) and a bunch of second order transport coefficients (τ_π, τ_Π, and others (vorticity etc.)) are the inputs quantities
- These quantities need to be computed from the microscopic theories
- In HIC, one estimate them from the flow observables (v_2, v_3)

- The dissipative effects (such as viscosities) need understanding of away from equilibrium
- For e.g., shear viscosity accounts for the entropy production during anisotropic expansion of the medium at constant volume
- We shall concentrate on the momentum anisotropy and exploit it to model the near equilibrium distribution functions of the QGP

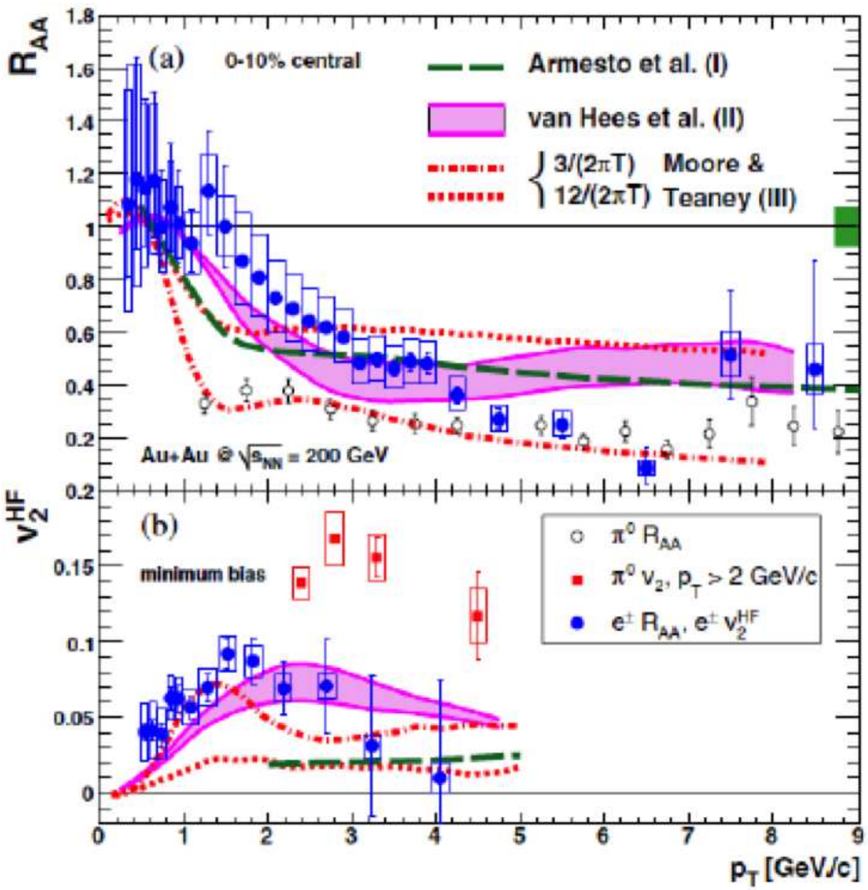
Momentum Anisotropy in HIC

- In the initial stages of the heavy ion collisions, we have spatial anisotropy
- During the hydrodynamical evolution it converts to the momentum anisotropy
- We can think of modelling the QGP in terms of near equilibrated (local) anisotropic momentum distributions
- The presence of momentum anisotropy induces Weibel-instability in ED plasma: this causes exponential growth of electromagnetic fields in the plasma which help restore momentum space isotropy Weibel, 1959, Fried, 1959.
- The non-abelian plasmas—ChromoWeibel instability Abe and Niu, 1980, Mrowczynski, PRC (1994), Strickland (2007), Arnold and Moore (2006)
  It is similar to Weibel instability in the linear reason
- In HIC, Turbulent color fields, which can arise in the early and late stages , may contribute significantly to the transport processes in the matter created in these collisions

Heavy-quarks ar RHIC, LHC

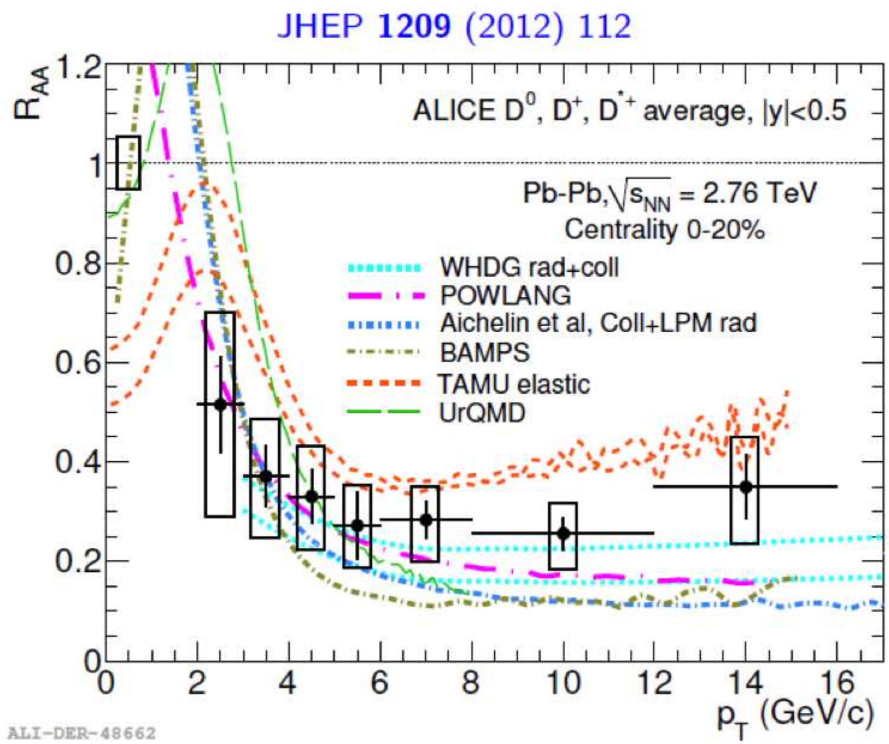
- HQs play crucial role in characterizing QGP as they are produced in the early stages of the heavy-ion collisions and remain extant throughout the evolution and hence can capture the information of the entire evolution of the system.
- The HQs thermalizes much later as compare to light quarks and gluons, Moore and Teaney, 2005, Hees and Ralf Rapp, 2005
- The HQ mass is higher than temperature , so their thermal production is substantially suppressed ($\exp(-M_Q/T)$)
- At RHIC and LHC, their v_2 and R_AA is comparable to that of the light quarks

HQs at RHIC

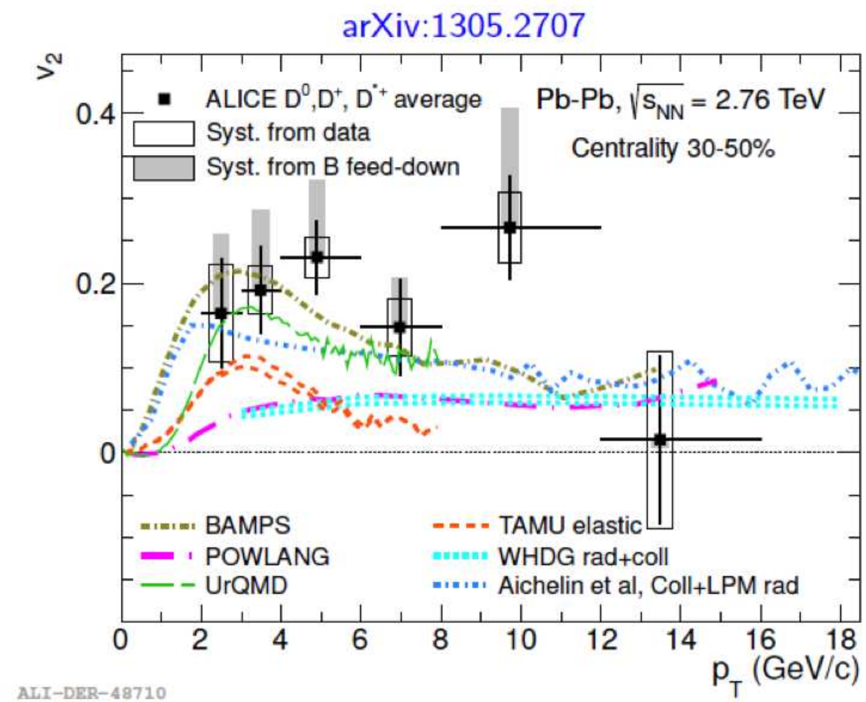


R_AA
(a)
0-10% central
Armesto et al. (I)
van Hees et al. (II)
3/(2πT) Moore &
12/(2πT) Teaney (III)
Au+Au @ √s_NN = 200 GeV
v_2^HF
(b)
minimum bias
π^0 R_AA
π^0 v_2, p_T > 2 GeV/c
e^± R_AA, e^± v_2^HF
p_T [GeV/c]

HQs at LHC



JHEP 1209 (2012) 112
ALICE D^0, D^+, D^*+ average, |y|<0.5
Pb-Pb, √s_NN = 2.76 TeV
Centrality 0-20%
WHDG rad+coll
POWLANG
Aichelin et al, Coll+LPM rad
BAMPS
TAMU elastic
UrQMD
R_AA
p_T (GeV/c)
ALI-DER-48662



arXiv:1305.2707
ALICE D^0,D^+, D^*+ average
Syst. from data
Syst. from B feed-down
Pb-Pb, √s_NN = 2.76 TeV
Centrality 30-50%
BAMPS
POWLANG
UrQMD
TAMU elastic
WHDG rad+coll
Aichelin et al, Coll+LPM rad
v_2
p_T (GeV/c)
ALI-DER-48710

- The dynamics of HQs while traveling in the QGP medium can be understood in terms of the drag and diffusion coefficients following Landau's prescription.
- Let us consider the elastic interaction experienced by HQs while traversing in to the hot QCD medium. Next, we consider the process $c(p) + l(q) \rightarrow c(p') + l(q')$ (l stands for gluon and light quarks and anti quarks).
- The the drag coefficient, γ can be calculated by using the following expression [**?**]:

$$\gamma = p_i A_i / p^2 \tag{1}$$

Svetitsky, PRD (1988).

- The coefficients, A_i is given by

$$A_i = \frac{1}{2E_p}\int\frac{d^3q}{(2\pi)^3E_q}\int\frac{d^3p'}{(2\pi)^3E'_p}\int\frac{d^3q'}{(2\pi)^3E'_q} \frac{1}{g_Q}\sum\overline{|M|^2}(2\pi)^4\delta^4(p+q-p'-q') f(q)(1\pm f(q'))[(p-p')_i] \equiv \langle\langle(p-p')\rangle\rangle \quad (2)$$

g_Q being the statistical degeneracy of the HQ propagating through QGP.

- The above expression indicates that the drag coefficient is the measure of the thermal average of the momentum transfer, $p-p'$ due to interaction of the heavy quarks with the bath particle weighted by the square of the invariant amplitude, $\overline{\mid M \mid^2}$.
- Here, $f(q)$ will involve three types of thermal phase space distribution functions corresponding to the gluons (g), light-quarks ($q \equiv$ up and down) and the strange quarks (s) and corresponding anti-quarks. Hence, $f(q)$ jointly denote these three phase space distribution as,

$$f(q) \equiv \{f_g, f_q, f_s\}. \quad (3)$$

- In the presence of momentum anisotropy, we need to model them appropriately by first setting up the transport equation and then solving it either analytically or numerically.
- In the present work, we consider the linearized transport equation and capture all the effects coming from the anisotropy as the first order modification to the equilibrium distribution functions for quark-antiquark and gluons.
- we consider the following decomposition for the f(q) in three sectors

$$\begin{aligned} f_g &= f_0^g(p) + f_1^g(\vec{p}, \vec{r}) \\ f_q &= f_0^q(p) + f_1^q(\vec{p}, \vec{r}) \\ f_s &= f_0^s(p) + f_1^s(\vec{p}, \vec{r}). \end{aligned} \quad (4)$$

Where, $p = |\vec{p}|$.

- Similar to the HQ diffusion coefficient, B_0 can be evaluated as:

$$B_0 = \frac{1}{4}\left[\langle\langle p\prime^2\rangle\rangle - \frac{\langle\langle (p.p\prime)^2\rangle\rangle}{p^2}\right]. \quad (5)$$

- With an appropriate choice of $\mathcal{F}(p')$ both the drag and diffusion coefficients can be evaluated from a single expression as follows:

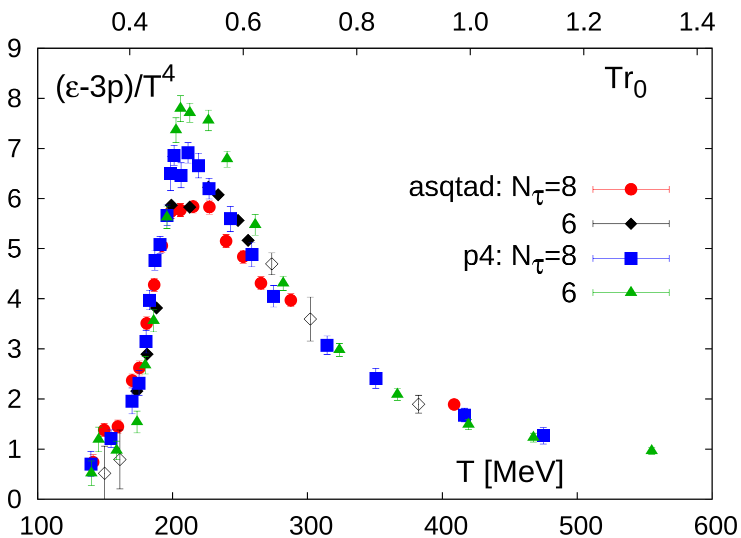
$$<< \mathcal{F}(p) >>= \frac{1}{512\pi^4}\frac{1}{E_p}\int_0^\infty \frac{q^2 dq d(cos\chi)}{E_q}\hat{f}(q)$$
$$\frac{w^{1/2}(s, m_Q^2, m_p^2)}{\sqrt{s}}\int_1^{-1} d(cos\theta_{c.m.})$$
$$\frac{1}{g_Q}\sum\overline{|M|^2}\int_0^{2\pi} d\phi_{c.m.}\mathcal{F}(p') \quad (6)$$

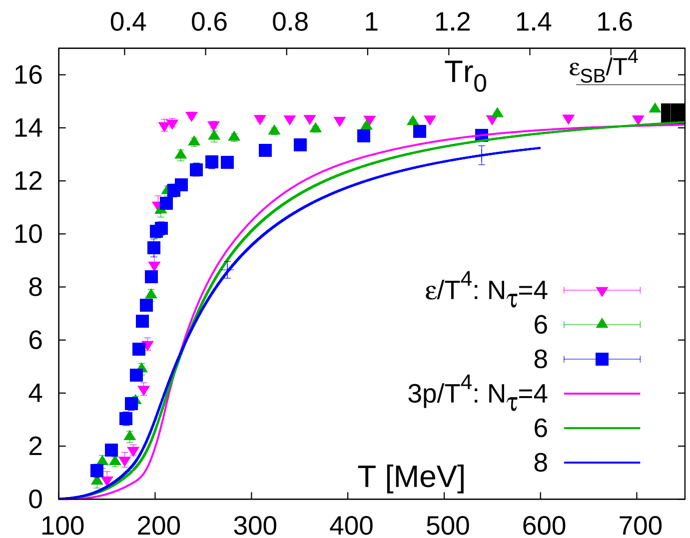
- Where, s is the Mandelstam variable and $w(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ac$ is the triangular function. In the next section we will present modelling of non-equilibrium distribution functions for a rapidly expanding plasma in the presence of small momentum anisotropy.

our approach

- Model for isotropic/equilibrium distributions: based on LEoS (2+1 Flavor)
- Momentum anisotropy present during the hydro evolution of the QGP
- Set- up effective transport equation with an effective vlasov term
- Solve linearized transport equation and model near equilibrium distribution functions
- Employ the distribution function to investigate heavy quark dynamics

LEoS





- We will interpret/describe the LEoS in terms of a quasi-particle model

Isotropic distributions

- We adopt, effective fugacity quasi-particle model (Chandra, Ravishankar, PRD2012), where

$$f_0^{g/q} = \frac{z_{g/q}\exp[-\beta p]}{\Big(1 \mp z_{g/q}\exp[-\beta p]\Big)},$$
$$f_0^s = \frac{z_q \exp[-\beta\sqrt{p^2+m_s^2}]}{\Big(1 + z_q\exp[-\beta\sqrt{p^2+m_s^2}]\Big)}, \quad (7)$$

where $p = |\vec{p}|$, m_s denotes the mass of the strange quark(which we choose to be $0.1GeV$), and $\beta = T^{-1}$ denotes inverse of the temperature.

- Effective fugacities lead to non-trivial dispersion relation both in the gluonic and quark sectors as,

$$\omega_g = p + T^2\partial_T ln(z_g)$$
$$\omega_q = p + T^2\partial_T ln(z_q)$$
$$\omega_s = \sqrt{p^2+m^2} + T^2\partial_T ln(z_q), \quad (8)$$

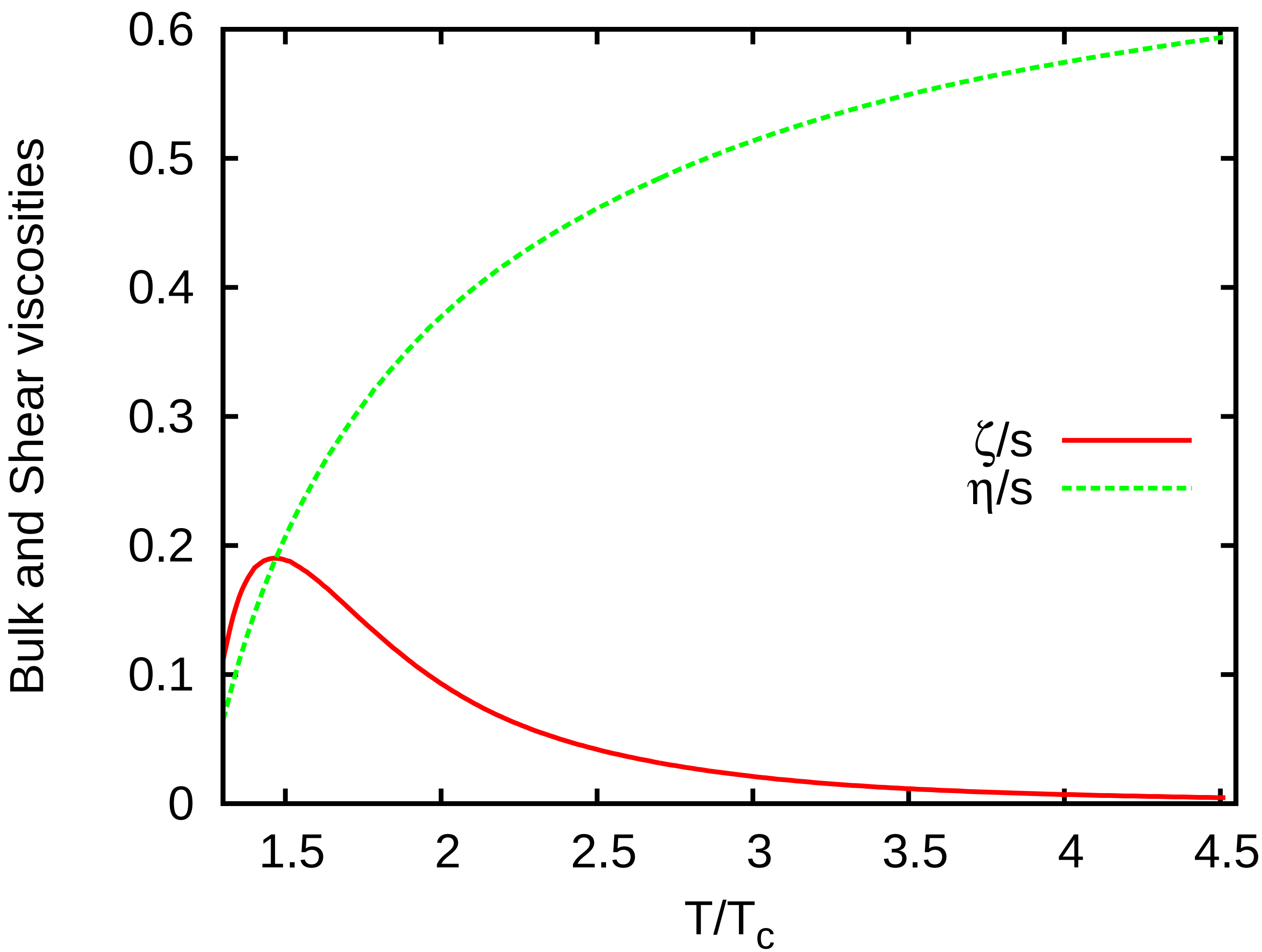
Cheng et. al , 2008

Thanks to Karsch and Saumen Datta for helping me in understanding LEOS and

Effective transport equation

- The momentum anisotropy present in quark and gluon momentum distribution functions induces instability in the Yang-Mills equations in similar way as Weibel instability in the case of Electromagnetic plasmas.
- This instability while coupled with the rapid expansion of the QGP leads to anomalous transport and modulates the transport coefficients of the plasma substantially. Dupree, Phy. of Plasmas, 1954, Asakawa, Bass, Müller, PRL 96, 252301 (2006).
- The η/S thus obtained was of the similar order that is relevant for HIC
- The η/S thus obtained can be related with the jet quenching parameter. Stronger the quenching lower will be value of η/S, Mjumder, Müller, Wang, PRL2007.
- We relook the whole analysis for the realistic QGP EoS, Chandra and Ravishankar, EPJC (2009).
- We estimated the temperature dependence of the η/S and ζ/S within the above setup, Chandra, PRD 2011, PRD 2012.

The η/S and ζ/S



Chandra, PRD 2012.

Modeling near equilibrium distributions

- We start with the following ansatz for the non-equilibrium distribution function,

$$f(\vec{p},\vec{r}) = \frac{z_{g,q}\exp(-\beta u^{\mu}p_{\mu})}{1 \pm z_{g,q}\exp\big(-\beta u^{\mu}p_{\mu} + f_1(\vec{p},\vec{r})\big)} \tag{9}$$

  $z_{g,q}$ are the effective gluon, quark fugacities coming from the isotropic modelling of the QGP in terms of lattice QCD equation of state.

- The parameter β is the temperature inverse (in units of $K_B = 1$), u^{μ} is the fluid 4-velocity considering fluid picture of the QGP medium. Here, $f_1(\vec{p},\vec{r})$ denotes the effects from the anisotropy (momentum).

- To achieve the above mentioned near equilibrium situation, f_1 must be a small perturbation. Under this condition, we obtain,

$$f(\vec{p},\vec{r}) = f_0(p) + f_0(1 \pm f_0(p))f_1(\vec{p},\vec{r}) + O(f_1(\vec{p},\vec{r})^2). \tag{10}$$

  The *plus* sign is for gluons and *minus* sign is for the quarks/antiquarks.

- Form of the perturbation f_1:

$$f_1(\vec{p},\vec{r}) \equiv f_1^{g,q} = -\frac{1}{\omega_{g,q}T^2} p_i p_j (\Delta_{g,q}(\vec{p})(\nabla u)_{ij}, \tag{11}$$

  The quantities, $\Delta_{g,q}$ denotes the strength of the momentum anisotropy for the gluons and quarks respectively.

- In the local rest frame of the fluid (LRF) $f_0 = f_{eq} = (f_0^g, f_0^q)$, and considering longitudinal boost invariance, we obtain, $\nabla \cdot \vec{u} = \frac{1}{\tau}$ and $\nabla u_{ij} = \frac{1}{3\tau} diag(-1,-1,2)$, leading to

$$f_1^{g,q} = -\frac{\Delta_{g,q}(p)}{\omega_{g,q}T^2\tau}(p_z^2 - \frac{p^2}{3}). \tag{12}$$

Effective transport equation

- The evolution of the quasi-quark and quasi-gluon momentum distribution functions in the anisotropic QGP medium can be described by the Vlasov-Boltzmann equation.
- After invoking the argument that the soft color fields are turbulent and that their action on the quasi-partons in can be described by taking an ensemble average, the Vlasov-Boltzmann equation can be replaced by Dupree's ensemble averaged, diffusive Vlasov-Boltzmann equation:

$$v^{\mu}\frac{\partial}{\partial x^{\mu}}\bar{f}-\mathcal{F}_A\bar{f}=0\,. \tag{13}$$

- Here, $\bar{f}$ denotes the ensemble averaged thermal distribution function of quasi-partons. In our case, $\bar{f}\equiv f(\vec{p},\vec{r})$ (given in Eq. (9)). Note that we are only considering the anomalous transport , the collision term is not taken in to account here.
- The force term ($\mathcal{F}_A$) in the case of Chromo-electromagnetic plasma in the present case will be,

$$\begin{aligned}\mathcal{F}_A\bar{f}(p) &\equiv \mathcal{F}_A f(\vec{p},\vec{r})\\ &= \frac{g^2C_2}{3(N_c^2-1)\omega_{g,q}^2} < E^2+B^2 > \tau_m\\ &\quad \times\mathcal{L}^2 f_{eq}(1\pm f_{eq})p_ip_j(\nabla u)_{ij}\,.\end{aligned} \tag{14}$$

- Where C_2 is the Casimir invariants ($C_2 \equiv \big(N_c, (N_c^2-1)/2N_c\big)$ quadratic Casimirs of $SU(N_c)$).
- The operator $\mathcal{L}^2$ is defined as:

$$\mathcal{L}^2 = [\vec{p} \times \partial_{\vec{p}}]^2 - [\vec{p} \times \partial_{\vec{p}}]_z^2. \tag{15}$$

- The equilibrium distribution function (local):
$f_{eq} = 1/(z_{g,q}^{-1} \exp(\beta u.p) \mp 1)$, where $z_{g/q}$ is purely temperature dependent. The action of the drift operator on f_{eq} is given by

$$\begin{aligned}(v \cdot \partial) f_{eq} &= -f_{eq}(1+f_{eq})\Big\{(p - \partial_\beta \ln(z_{g,q}))v \cdot \partial(\beta) \\ &\quad + \beta(v \cdot \partial)(u \cdot p)\Big\},\end{aligned} \tag{16}$$

where we recognize that $p - \partial_\beta \ln(z_{g/q}) \equiv \omega_{g,q}$, is the modified dispersion relations.
- The third term in the right hand side of Eq.(17) is useful, as we are mainly concerned about the anisotropic expansion (other two terms contribute to the thermal conductivity and bulk viscosities respectively).

- The final expression for the drift term after imposing the energy-momentum conservation is obtained as

$$
\begin{aligned}
(v\cdot\partial)f_{eq}(p) &= f_{eq}(1\pm f_{eq})\Big[\frac{p_ip_j}{\omega_{g,q}T}(\nabla u)_{ij} \\
&-\frac{m_D^2 < E^2 > \tau^{el}\omega_{g,q}}{3T^2\partial\mathcal{E}/\partial T} \\
&+(\frac{p^2}{3\omega_{g,q}^2}-c_s^2)\frac{\omega_{g,q}}{T}(\nabla\cdot\vec{u})\Big],
\end{aligned} \tag{17}
$$

  where c_s^2 is the speed of sound, m_D^2 is the Debye mass, $\mathcal{E}$ is the energy density, τ_{el} is the time scale of the instability in Chromo-elctric fields.

- Finally, the effective Vlasov-Dupree equation (linearized) by considering the ensemble of turbulent color fields reads:

$$
\begin{aligned}
&\Big\{(\frac{p^2}{3\omega_{g,q}}-c_s^2)\frac{\omega_{g,q}}{T}(\nabla\cdot\vec{u})+\frac{p_ip_j(\nabla)_{ij}}{\omega_{g,q}T}\Big\}f_0^{g,q}(1\pm f_0^{g,q}) = \\
&\frac{g^2C_2}{3(N_c^2-1)\omega_{g,q}^2} < E^2+B^2 > \tau_m\mathcal{L}^2 f_1^{g,q}(\vec{p})f_0^{g,q}(1\pm f_0^{g,q})
\end{aligned} \tag{18}
$$

- The operator $\mathcal{L}^2$ is similar to the quadrupole operator and the most peculiar thing about it is that it only picks up the anisotropic piece of any function of momentum ($\vec{p}$).
- Solving Eq. (18) for $\Delta_{g,q}$ analytically, we obtain the following expression:

$$\Delta_{g,q} = 2(N_c^2 - 1)\frac{\omega_{g,q} T}{3C_{g,q} g^2 < E^2 + B^2 >_{g,q} \tau_m}. \tag{19}$$

- We now, relate the unknown quantities in the denominator with the phenomenologically known parameter the jet quenching parameter ($\hat{q}$) in both gluonic and quark sector below.
- The two most relevant transport coefficients related to anomalous transport due to the soft color fields are the η and the jet quenching parameter $\hat{q}$.
- Here the strength of the anisotropy, $\Delta(\vec{p})$ is related to the physics of η. The $\hat{q}$ is proportional to the mean momentum square per unit length on the an energetic parton imparted by turbulent fields.

- In the QGP phase, $\hat{q}$ for both gluons ($\hat{q}_g$) and quarks ($\hat{q}_q$) has been estimated employing several different approaches: Burke et. al, PRC 2013 and Refs. Therein) (GLV-CUJET Model, Higher Twist Berkeley Wuhan Model, The Higher-Twist-Majumder Model, MARTINI Model, The MCGILL AMY Model)
- Combining all these models, one obtains the quark transport parameter $\hat{q}_q$ in the range,

$$\begin{aligned} \frac{\hat{q}_q}{T^3} &= 4.6 \pm 1.2 \ \textit{at RHIC} \\ \frac{\hat{q}_q}{T^3} &= 3.7 \pm 1.4 \ \textit{at LHC} \end{aligned} \tag{20}$$

- The gluon quenching parameter $\hat{q}_g$ is related to $\hat{q}_g$ by a factor of $\frac{9}{4}$ (in terms of Casimir invariants of the $SU(3)$ group),

$$\hat{q}_g = \frac{9}{4} \hat{q}_q. \tag{21}$$

- Relevant point to be noted is that $\hat{q}$ for the QGP scales with T^3. If one considers the highest temperatures reached at central Au-Au at RHIC and Pb-Pb at LHC, $T = 370 Mev$ and $T = 470 Mev$ respectively. The corresponding numbers for $\hat{q}_q$ for a $10 Gev$ quark Jet are,

$$\hat{q}_q = 1.3 \pm 0.3 \, GeV^2/fm; \; 1.9 \pm 0.7 \, Gev^2/fm, \tag{22}$$

  for RHIC and LHC respectively.

- Let us now discuss the temperature variations at RHIC and LHC while obtaining $\hat{q}$ enlisted in Eq. (20). For Au-Au at 200 GeV/n, $T_0 = 346 - 373$ Mev and for Pb-Pb at 2.76 TeV/n, $T_0 = 447 - 486$ MeV.

- In the present context, the unknown quantities $< E^2 + B^2 > \tau_m$ which captures the physics of anisotropy and chromo-Weibel instability can be written in terms of $\hat{q}$ both in gluonic and matter sectors as

$$\hat{q} = \frac{2g^2 C_{g/f}}{2(N_c^2 - 1)} < E^2 + B^2 > \tau_m, \tag{23}$$

  where $C_g = N_c$, $C_f = \frac{(N_c^2-1)}{2N_c}$ for the gluons and quarks respectively.

- Invoking the definition of $\hat{q}$ from Eq. (23) in Eq. (19, we obtain the following expressions,

$$\begin{aligned} \Delta_{g,q} &= \frac{4\omega^2_{g,q} T}{9\hat{q}_{g,q}}. \\ f^{g,q}(\vec{p}) &= f^{g,q}_0 - f^{g,q}_0(1 \pm f^{g,q}_0)\frac{4\omega_g}{9\hat{q}_{g,q}(\tau T)}\big(p^2_z - \frac{p^2}{3}\big). \end{aligned} \tag{24}$$

- Finally, we obtain the following near equilibrium distribution functions in terms of the jet quenching parameter $\hat{q}$,

$$f^{g,q}(\vec{p}) = f^{g,q}_0 - f^{g,q}_0(1 \pm f^{g,q}_0)\frac{4\omega_g}{9\hat{q}_{g,q}(\tau T)}\big(p^2_z - \frac{p^2}{3}\big). \tag{25}$$

- One can choose another part while choosing ansaltz for $f_1(\vec{p})$

$$f_1(\vec{p}) \equiv -\frac{1}{\omega_{g,q}} p_i p_j \left(\Delta \vec{p} (\nabla u)_{ij} + \Delta_1(p) \nabla \cdot \vec{u} \delta_{ij} \right). \tag{26}$$

- Solving effective Vlasov equation for Δ_1, we obtain,

$$\Delta_1 = \frac{8}{3} \frac{\omega_{g,q}^2}{\hat{q}_{g,q}} \frac{1}{p^2} (p^2/3 - c_s^2 \omega_{g,q}^2) \ln(p_t/\sqrt{6}T). \tag{27}$$

- We shall consider collision term and solve for the near equilibrium distributions
- That will help us to understand the interplay of contributions

Heavy quark drag and diffusion

- The momentum variation of the drag and diffusion coefficients: RHIC energies

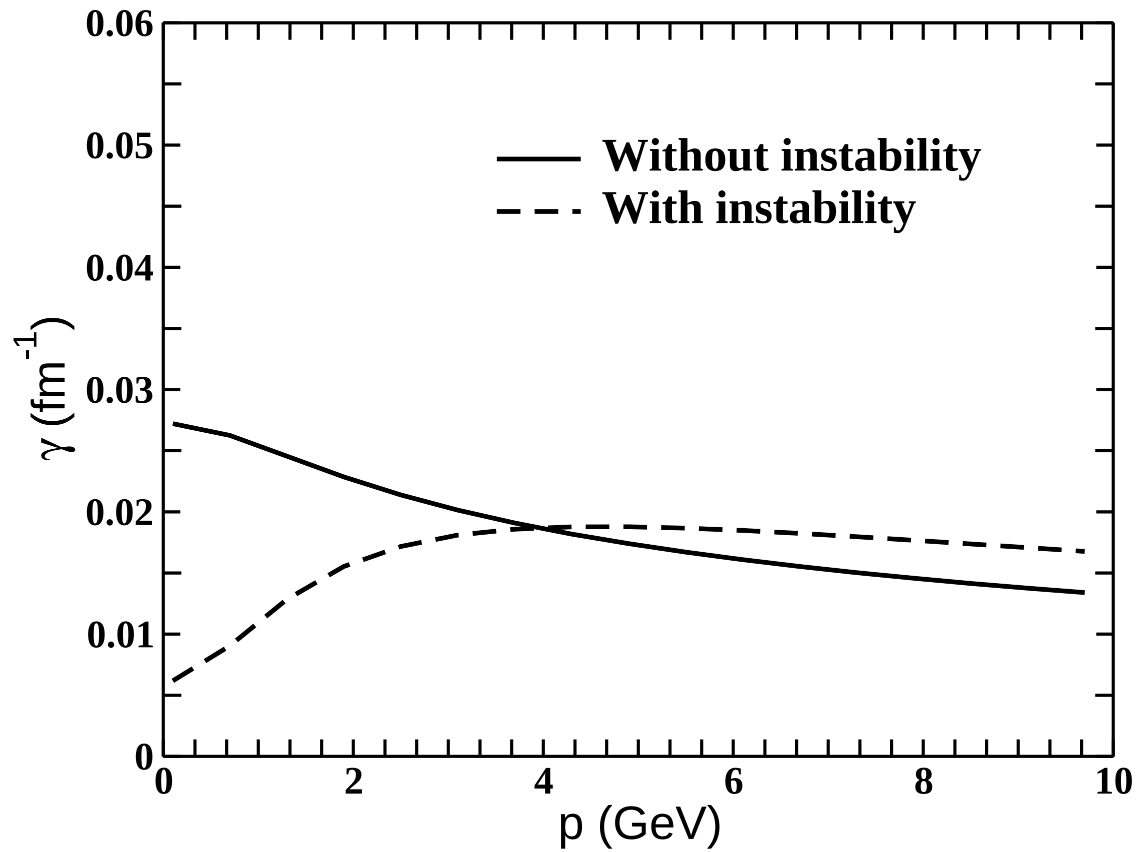


Figure 1: Variation of the drag coefficient with momentum at RHIC energy.

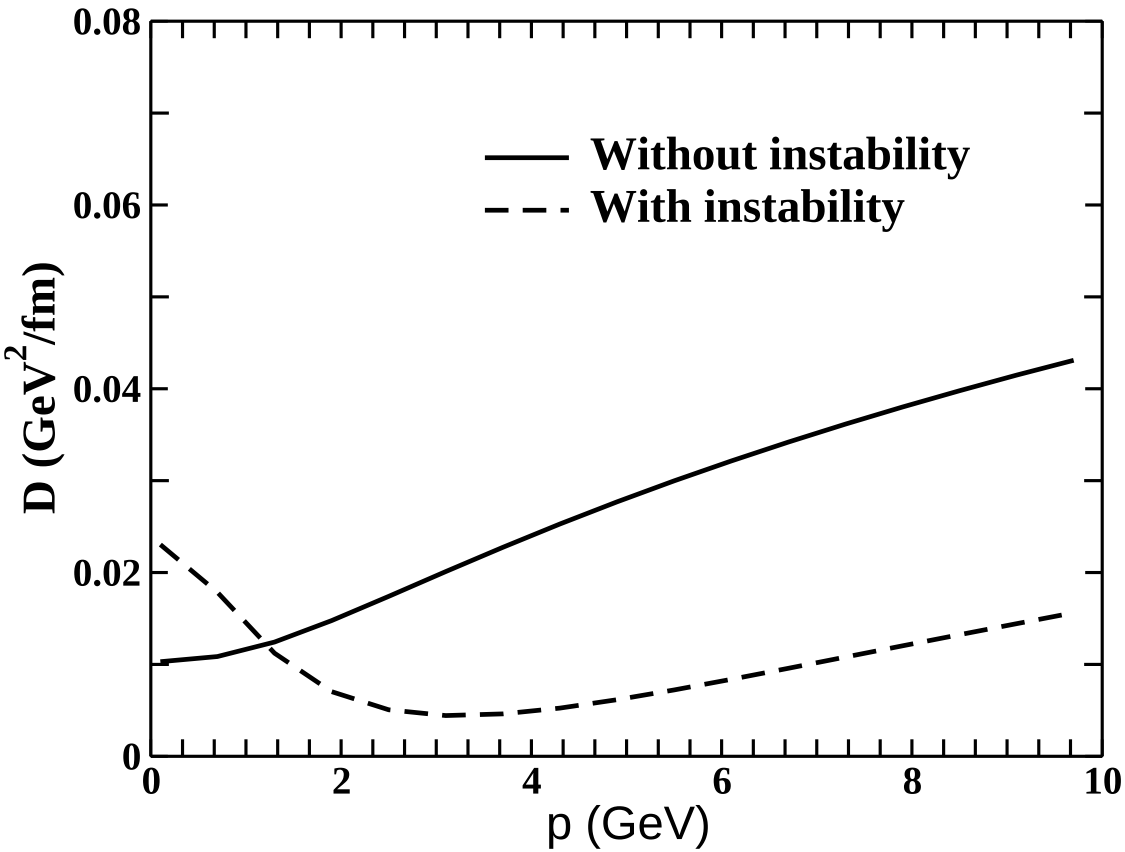


Figure 2: Variation of the diffusion coefficient with momentum at RHIC energy.

- The momentum variation of the drag and diffusion coefficients: LHC energies

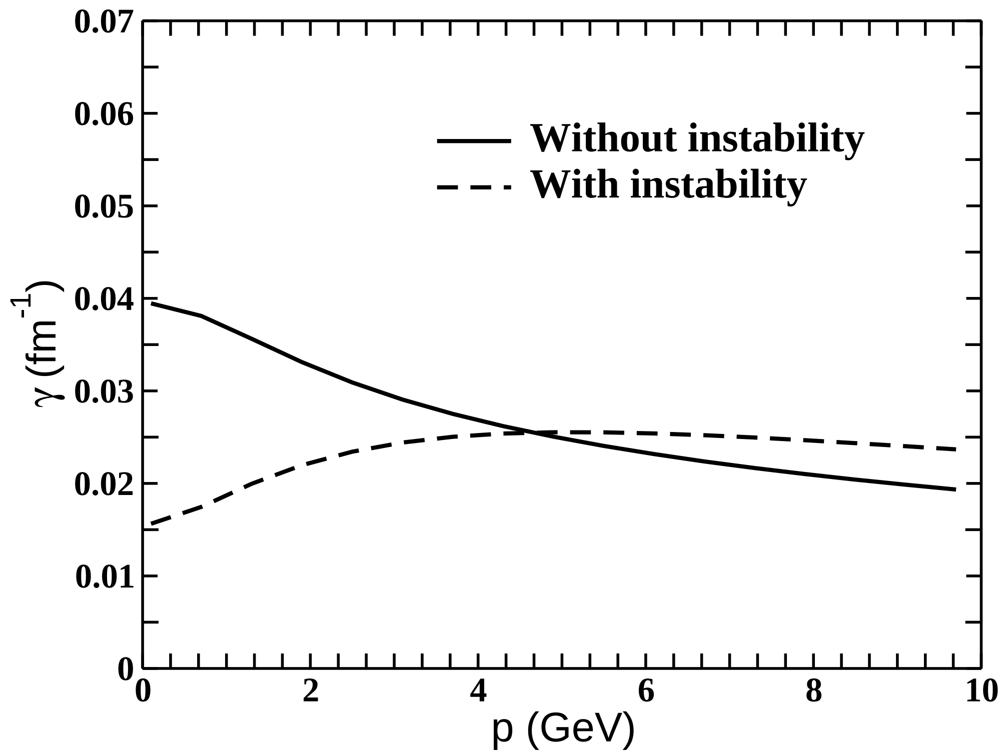


Figure 3: Variation of the drag coefficient with momentum at LHC energy.

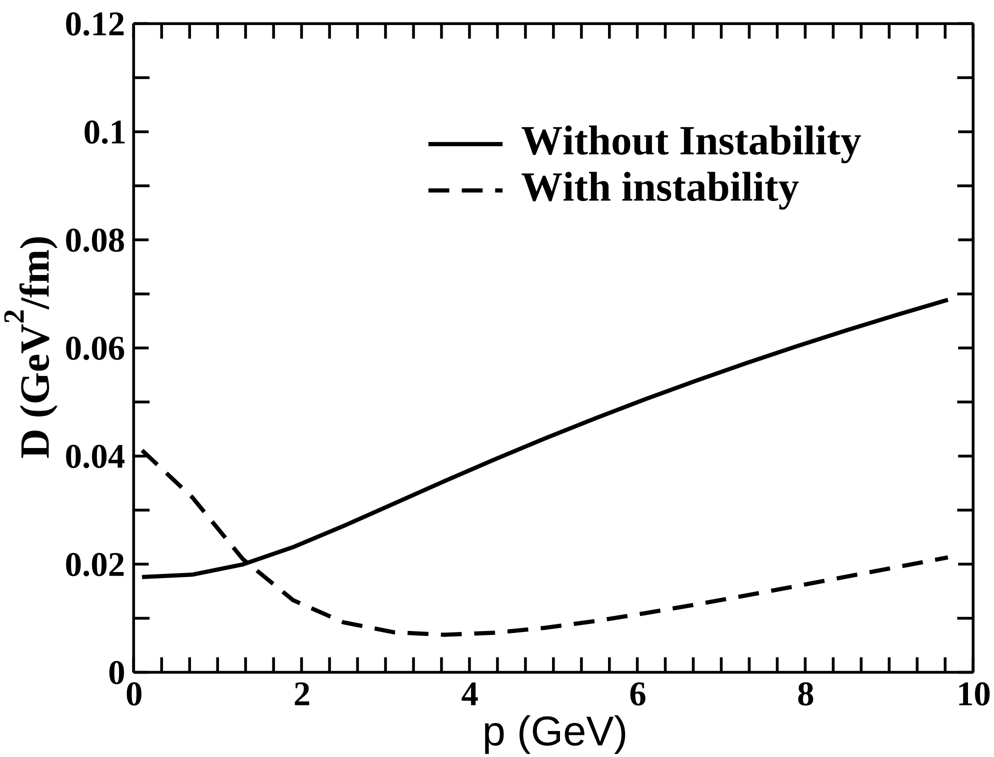


Figure 4: Variation of the diffusion coefficient with momentum at LHC energy.

- Strength of anisotropy is inversly proportional to $\hat{q}$

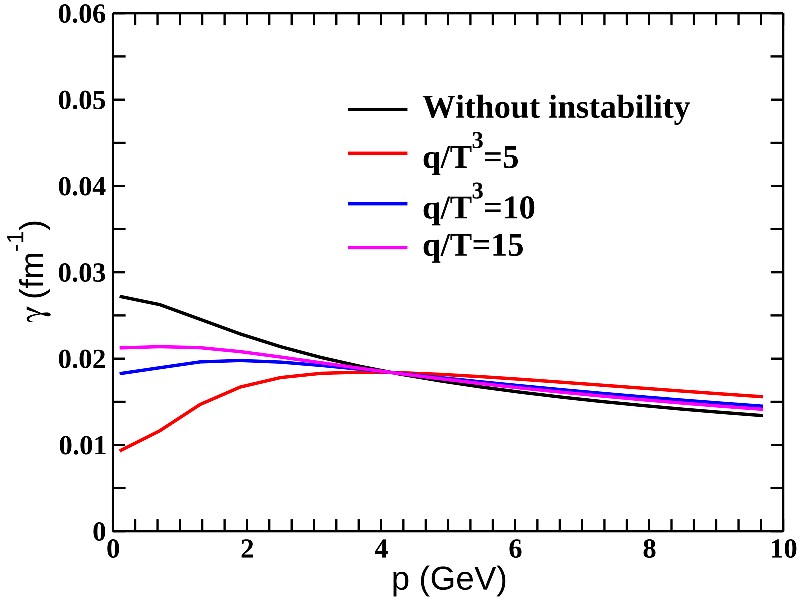


Figure 5: Dependence on the strength of the anisotropy of the Drag coefficient at RHIC

- Strength of anisotropy is inversely proportional to $\hat{q}$

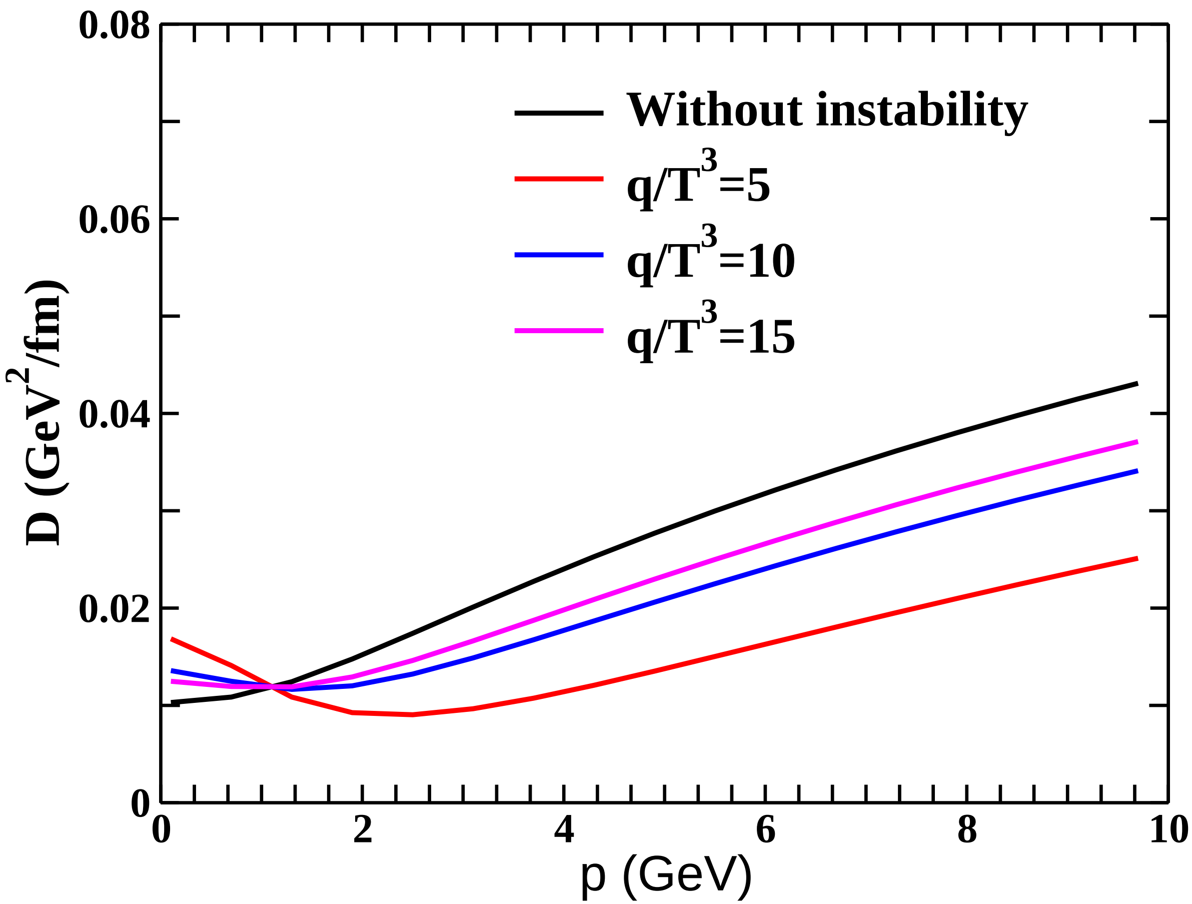


Figure 6: Dependence on the strength of the anisotropy of the D coefficient at RHIC

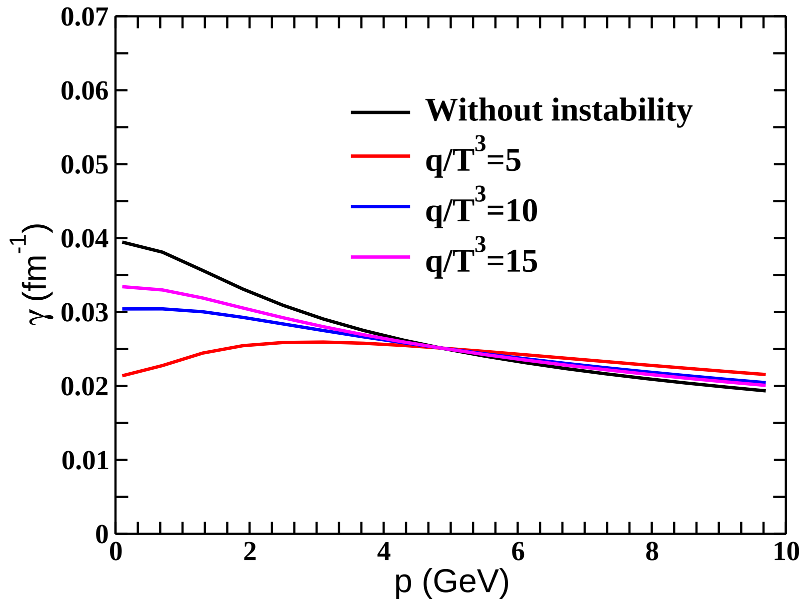


Figure 7: Dependence on the strength of the anisotropy/instability of the Drag coefficient at LHC

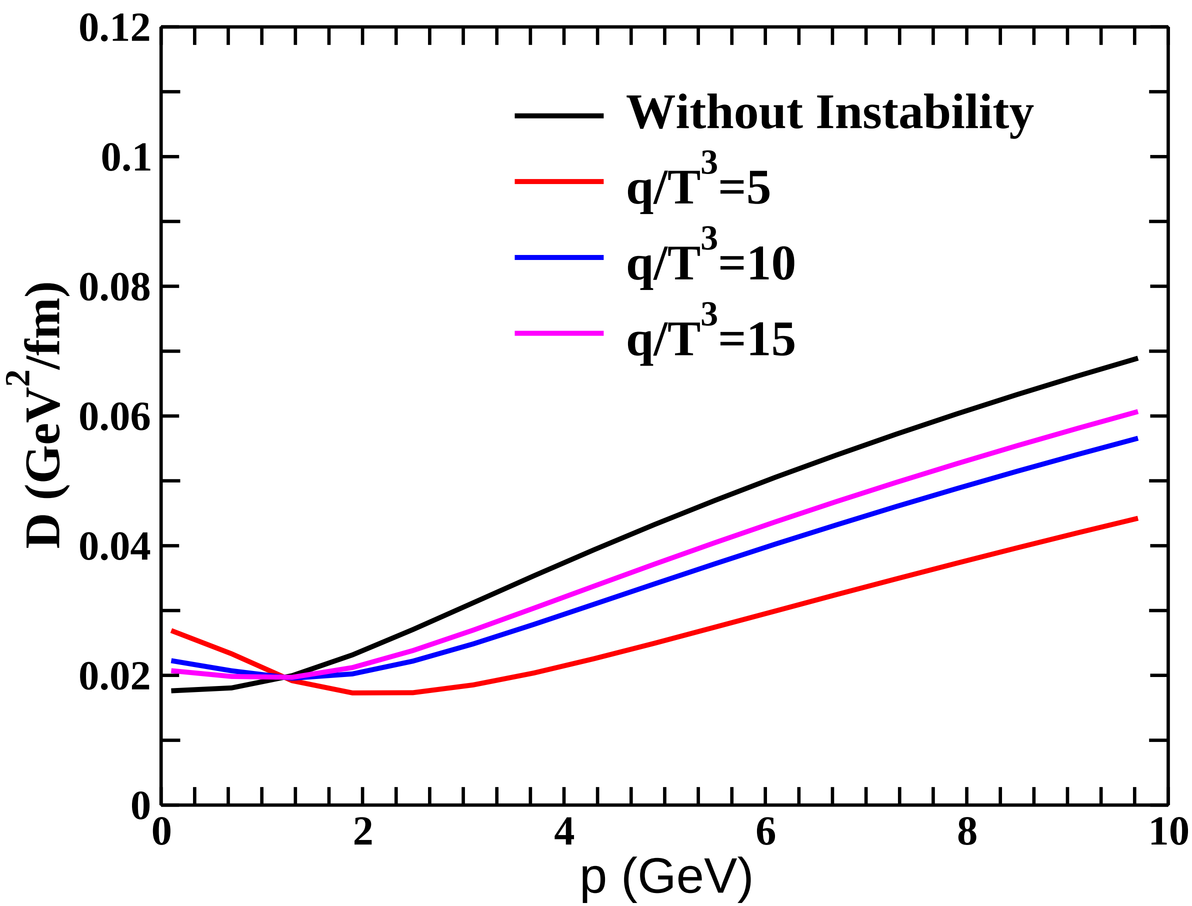


Figure 8: Dependence on the strength of the anisotropy/instability of the Diffusion coefficient at LHC

Summary and Conclusions

- We have estimated the drag and diffusion coefficients of heavy quarks propagating through a QGP medium considering the role of momentum state anisotropy.
- The momentum anisotropy in the early stages coupled with the rapidly expanding QGP is modelled by setting up an effective transport equation
- Tts solution in near equilibrium approximation leads to the modelling of near(non) equilibrium distribution functions for quark-antiquark and gluons.
- We coupled these distribution functions to the kinetic theory description of heavy quark drag and diffusion coefficients and studied their temperature and momentum dependence.

- We found that both at RHIC and LHC energies, impact of the anisotropy on heavy quark transport is quite significant as compared to case while HQs are moving in an isotropic QGP medium.
- The presence of anisotropy alter both the temperature as well as momentum dependences of the heavy quarks drag and diffusion coefficients.
- These results may have significance impact on R_{AA} and v_2 which will be a matter of future investigation.


Thank You!