Evolution of Temperature Fluctuation in a Non-equilibrated medium

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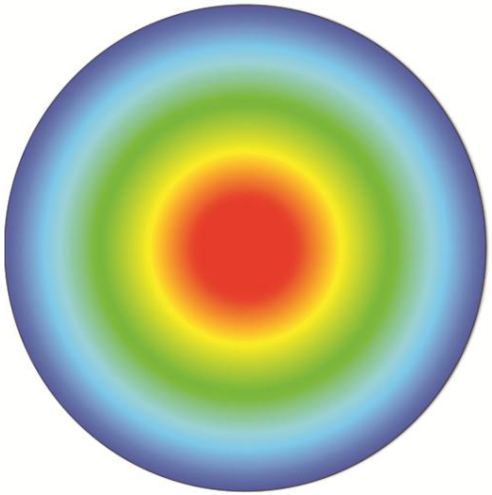
Fluctuations: We encounter in everyday examples (temperature fluctuation in a room).

More Examples: Critical Opalescence: Density fluctuation near critical point

Signature of temperature fluctuation in particle yield originated from high-energy collisions.

CMB temperature fluctuation

Study of temperature fluctuation may be important for studying QCD phase transition [D K Mishra *et al.* JPG **42**, 105105(2015)] and for characterizing QGP [S Basu *et al.* arXiv:1405.3969 [nucl-ex] , ibid. 1504.04502]



System with same temperature everywhere

System with different local temperatures

Hotspots

Model: Medium consists of weakly interacting hot zones.

Within a certain pre-determined time slice Δt, the temperature of a certain hot zone does not change

Within Δt, they are represented by a collection of canonical (no chemical potential, say) ensembles and their (inverse) temperature has a particular distribution.

The distribution determines the average inverse temperature β and its fluctuation $\Delta\beta$ at every hot zone.

With time temperature distribution changes and so does $\Delta\beta$

We assume particle distribution is affected by both β and $\Delta\beta$

Ansatz: $f = e^{-\beta(1+\Delta\beta)p} = f^{eq} + \Delta f$

[S. Dodelson, Modern Cosmology]

Feed f inside the Boltzmann Transport Equation (BTE) which is the evolution equation for inhomogeneous, anisotropic distribution in presence of external force

Momentum space gradient

$$\frac{\partial f}{\partial t} + \vec{v}.\vec{\nabla} f + \vec{F}.\vec{\nabla}_p f = C[f]$$

Collision term (takes care of interaction due to which distribution may change)

External force

Inhomogeneity

We get the evolution equation for Δf and hence for $\Delta\beta$ assuming Relaxation Time Approximation (RTA)

$$C[f] = \frac{f - f^{eq}}{t_R}$$

↓

Relaxation time (The time within which the distribution changes appreciably)

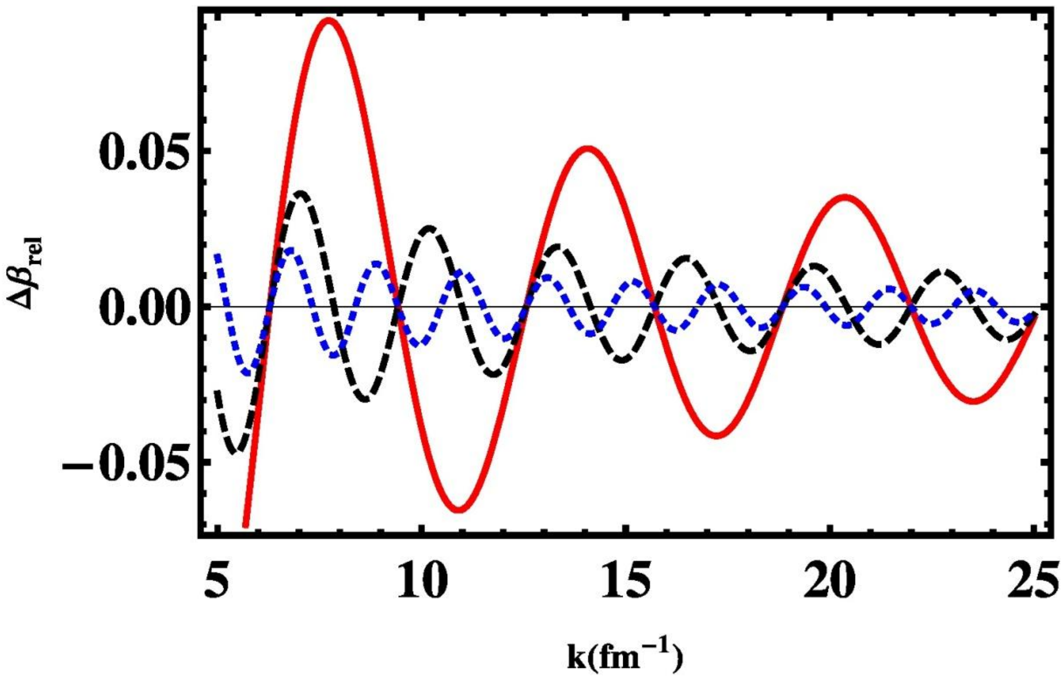
The Fourier Space (k-space) variation of $\Delta\beta$ (in a medium of almost mass less particles):

$$\Delta\beta(\vec{k},\hat{p};t) = \Delta\beta(\vec{k},\hat{p};t^0)\, e^{-ik\mu(t-t^0)}\, e^{-\frac{t-t^0}{t_R}}$$

Provided average (inverse) temperature varies slowly with time

After averaging over $\mu = \hat{k}.\hat{p}$

$$\Delta\beta_{rel}(\vec{k},t) = e^{-\frac{t-t^0}{t_R}} \frac{\sin k(t-t^0)}{k(t-t^0)}$$



$(t-t^0)$ values for $t_R = 3$ fm

1 fm

2 fm

3 fm

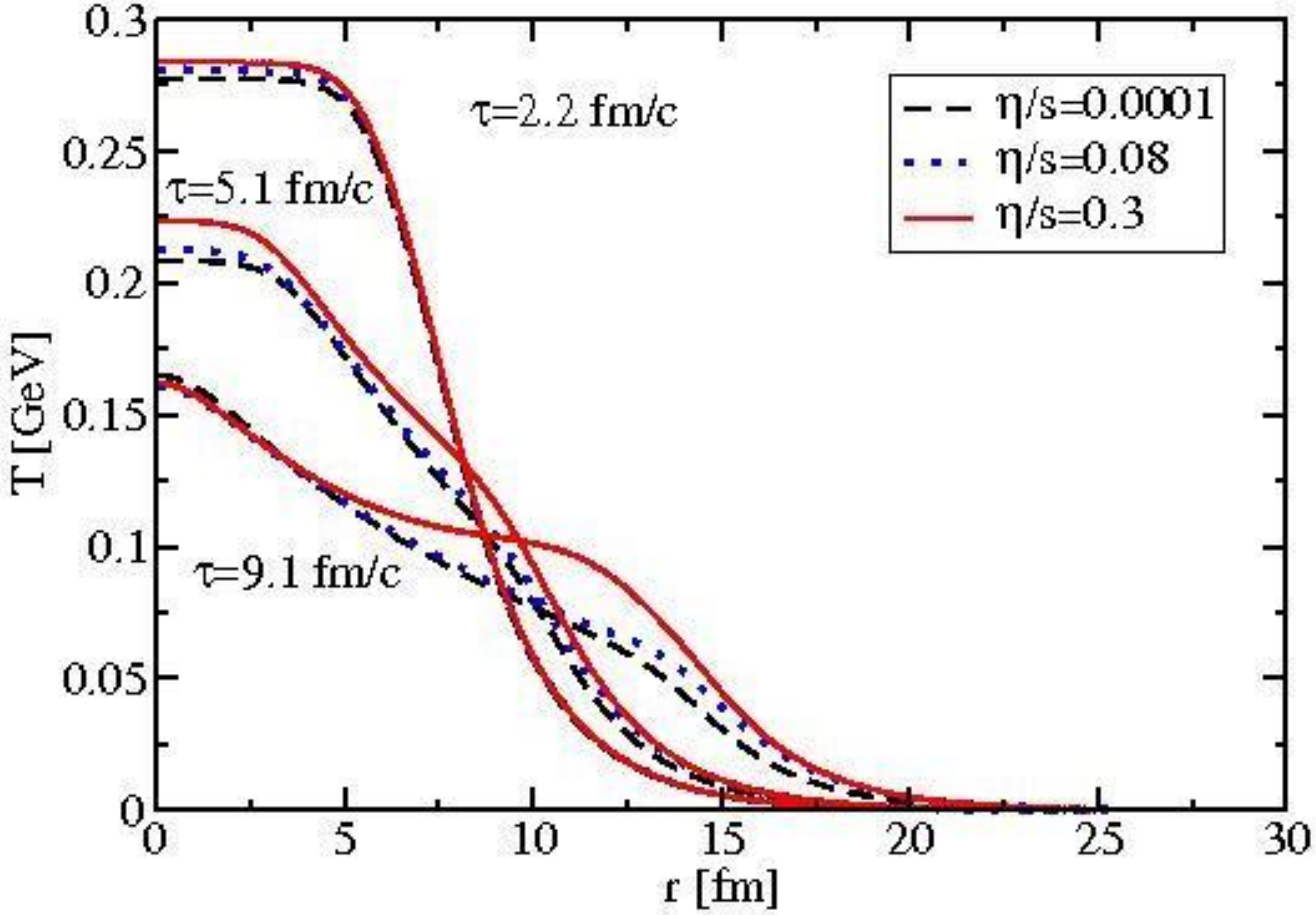
Observation 1: the amplitude of $\Delta\beta_{rel}$ is more towards the smaller k, i.e. towards large radius of the system.

Observation 2: Analysis driven by BTE is limited by the constraint over the observation time $(t - t^0) << t_R$

Observation 3: Given the constraint, $\Delta\beta_{rel}$ is independent of t_R

Any generalized analysis possible which will be able to avoid the constraint ?

Yes, to start with, one of the ways is to get the temperature profile of the medium at different stages (i.e. proper time τ) of evolution.



R. Baier and P. Romatschke, EPJC **51**, 677(2007)

The inverse temperature profile obtained from the theoretical analysis for a viscous QGP medium (created in a central HIC) evolving hydrodynamically can be described by the following function:

$$\beta_M(r;t) = \beta_0(t)\left[e^{a(t)\left(\frac{r}{r_0}-1\right)}+1\right]$$

With the tabulated details:

$\tau(fm/c)$	$\beta_0(GeV^{-1})$	a	$r_0(fm)$
2.2	3.45	5.99	7.96
5.1	4.55	3.42	8.41
9.1	5.56	1.91	8.71

From this, we can generate $\{\beta_{Mi}\}$, a sequence of radially varying inverse temperature values at different time.

$$\{\beta_{Mi}\}_\tau \equiv \{\beta_{M0}, \beta_{M1}, \beta_{M2}, \beta_{M3}, \ldots\ldots, \beta_{Mn}\}_\tau$$

$$\langle \beta_M \rangle = \frac{\beta_{M0} + \beta_{M1} + \beta_{M2} + \beta_{M3} + \ldots\ldots + \beta_{Mn}}{n}$$

$$\langle \beta_M{}^2 \rangle = \frac{\beta^2{}_{M0} + \beta_{M1}{}^2 + \beta_{M2}{}^2 + \beta_{M3}{}^2 + \ldots\ldots + \beta_{Mn}{}^2}{n}$$

So, given $\{\beta_{M\,i}\}$, we can define

(a) An average $\langle \beta_M \rangle$.

(b) Also, a fluctuation on top of it:

$$\Delta\beta(r;t) = \beta_M(r;t) - \langle \beta_M \rangle$$

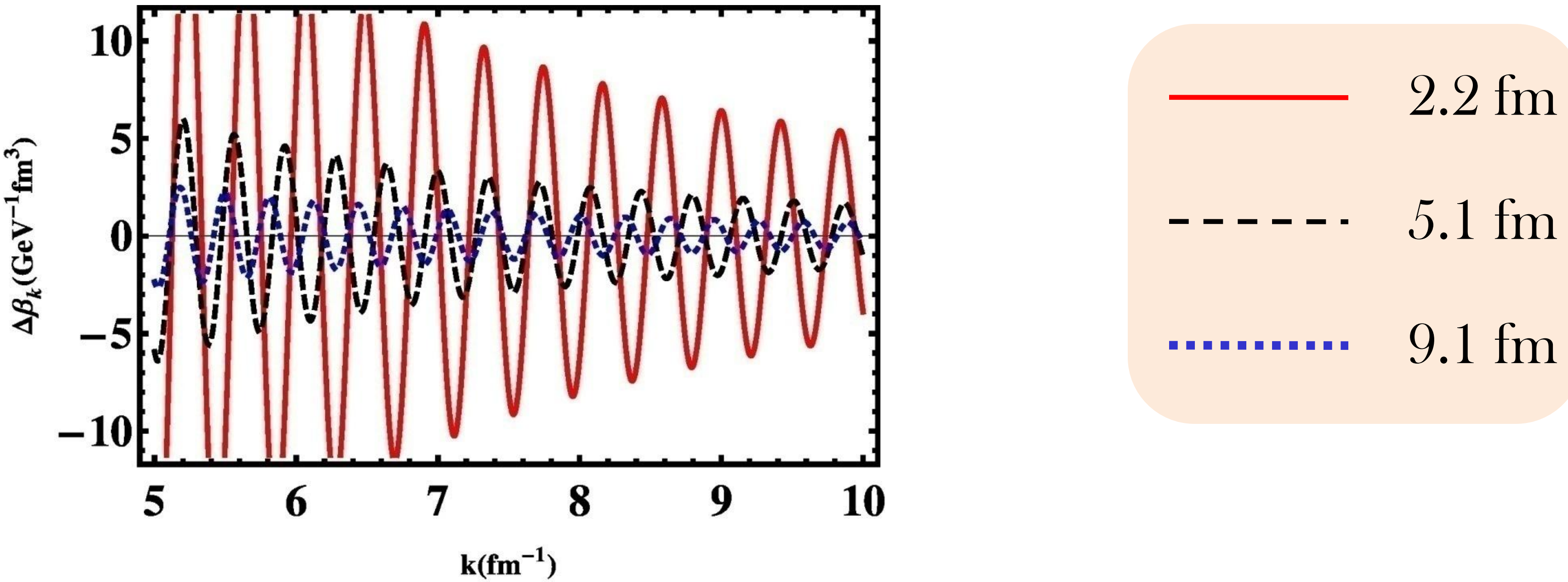
$$= \beta_0(t) e^{a(t)\left(\frac{r}{r_0} - 1\right)} + \delta\beta(t)$$

with $\delta\beta(t) = \beta_0(t) - \langle \beta_M \rangle$

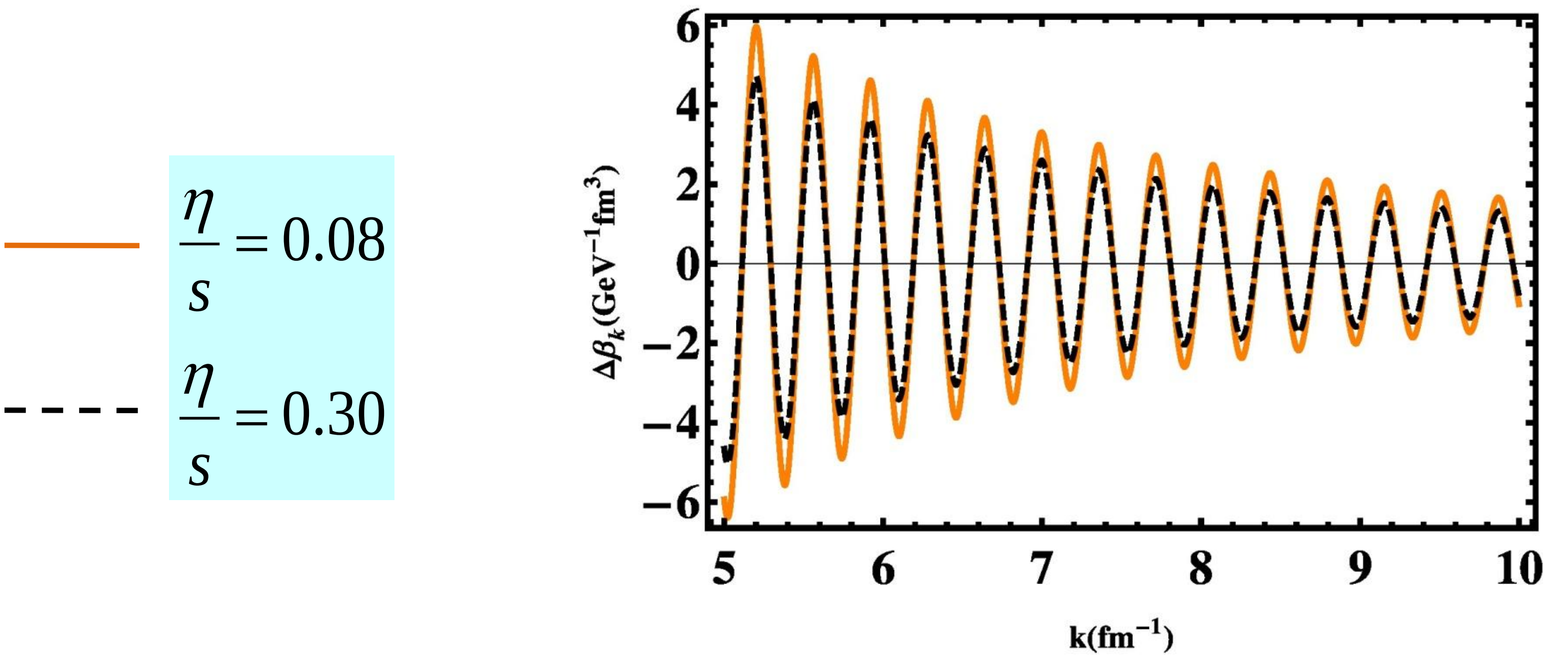
The Fourier Space (k-space) variation of inverse temperature fluctuation:

$$\Delta\beta(k,t) = \frac{2\beta_0(t)}{(2\pi)^2 k}\int_0^R e^{a(t)\left(\frac{r}{r_0}-1\right)} r \sin kr\, dr + \delta\beta(t)\delta(\vec{k})$$

Where $\delta(\vec{k})$ is the Dirac delta function.



$\Delta\beta_k(\mathrm{GeV^{-1}fm^3})$
10
5
0
−5
−10
5
6
7
8
9
10
$\mathrm{k(fm^{-1})}$
2.2 fm
5.1 fm
9.1 fm



$\frac{\eta}{s} = 0.08$
$\frac{\eta}{s} = 0.30$
$\Delta\beta_k(\mathrm{GeV^{-1}fm^3})$
6
4
2
0
−2
−4
−6
5
6
7
8
9
10
$\mathrm{k(fm^{-1})}$

(d) And, a relative variance for the collection $\{\beta_{M\,i}\}$:

$$\frac{\langle {\beta_M}^2 \rangle - \langle \beta_M \rangle^2}{\langle \beta_M \rangle^2} = \frac{{\sigma_\beta}^2}{\langle \beta_M \rangle^2} = \Re_\beta$$

τ	$\Re_\beta$
2.2	0.047
5.1	0.011
9.1	0.002

$\frac{\eta}{s} = 0.08$

$\frac{\eta}{s}$	$\Re_\beta$
0.08	0.012
0.30	0.011

$\tau = 5.1\ fm$

Let us assume that the system produced by central HIC freezes-out by 9.1 fm and compare theoretically obtained $\Re_\beta$ value at the boundary with the similar experimentally observed (q-1) parameter [G. Wilk and Z. Wlodarczyk PRL **84**, 2770(2000)] for hadron spectra at $\sqrt{s_{NN}} = 200$ GeV within 0-10% centrality [Z. Tang *et al.* PRC **79**, 051901(2009)].

$\Re_\beta^{Theory}$	(q-1)
0.013	0.018 ± 0.005

Cosmological connections:

Temperature fluctuation of our universe can be explained by the modified Boltzmann-Gibbs formula with (q-1) value 0.045 ± 0.005

[A. Bernui *et al.* PLA **356**, 426(2006)]

Similarity with HIC experimental results: needs review

Study on the similarity between the surface of the last scattering for CMB radiation and the freeze-out surface in RHICE

Summary and Conclusion:

- Time evolution of (inverse) temperature fluctuation

- With time, amplitude of inverse temperature fluctuation decreases

- With distance amplitude of inverse temperature fluctuation increases

- Relative fluctuation at the boundary is comparable with the experimental value under similar conditions.

- More studies required on the evolution of temperature fluctuation with more generalized scenarios.

- Exploring possible connections with experiment in a more detailed manner.