Effects of phase transition induced density fluctuations on pulsar dynamics

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Outline

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2. Change in MI due to a first order transition
3. Density fluctuation due to bubble nucleation
4. Density fluctuation from topological defects
   - In case of QCD transitions
   - In case of Superfluid transition
5. Gravitational wave generation due to density fluctuation
6. Conclusions

Motivation

- Exotic phases of quantum chromo dynamics (**QCD**), quark gluon plasma (**QGP**), color flavour locked (**CFL**) phase, 2 flavour color superconductivity (**2SC**) phase etc are possible at very high baryon density.
- Core of an astrophysical compact objects such as **neutron star** provides physical conditions where transition to these phases may be possible.
- **Superfluid phases** of neutrons are also believed to exist inside neutron stars.
- Young pulsars show the phenomenon of **glitches**, **anti glitch** is also recently observed [1].
- Conventional mechanism of glitches, **de-pinning of a cluster of superfluid vortices** in the core of a neutron star does not seem viable for explanation of antiglitches.
- We propose a technique to probe the dynamical phenomena happening inside the neutron star.

[1] R.F.Archibald et al.Nature 497,591 (2013)

- Our approach is based on the fact that phase transitions are typically associated with density change as well as **density fluctuation**.
- Density fluctuation in the core of a star will in general lead to transient changes in its moment of inertia (MI), along with a permanent change in MI due to phase transition.
- It affects the rotation and pulsar timings. Sensitive experiment ($\frac{\delta\nu}{\nu} \sim 10^{-9}$) might be able to detect it. It may provide sensitive probe for phase transitions in these compact objects.
- Non zero off diagonal components of MI arising from density fluctuations imply **wobbling of rotating neutron star**, which leads to modulation of peak intensity of pulses.
- Density fluctuations will lead to rapidly changing quadrupole moment which can be a new source for **gravitational wave** emission.

Change in MI due to a first order transition

- In literature[2], authors calculated the change in moment of inertia due to a phase transition assuming that the phase conversion happens continuously in the supercritical region of the core.
- This will happen for a second order, or a crossover or for a weak first order transition with very large bubble nucleation rate.
- But for a strong first order phase transition, for very low nucleation rates, the supercritical core may become macroscopically large before a single bubble of new phase nucleates.
- After nucleation the bubble will expand fast sweeping entire supercritical core and converting it to the new phase. This will lead to MI change in a very short time which may be directly observable.

[2] H.Heiselberg and M.Hjorth-Jensen, PRL 80,5485 (1998)

- Change in the MI: $\frac{\delta I}{I} \simeq \frac{5}{3}(\frac{\rho_2}{\rho_1} - 1)\frac{R_0^3}{R^3}$
- Take density change by 30% due to QCD transition, also take the observational value of glitch about, 10^{-5}, then $R_0 \leq 0.3Km$ if R = 10Km.
- For superfluid transition we may take the change in density to be $\simeq 0.1 MeV/fm^3$. In this case R_0 can be of order 5 Km.
- We consider a simple case of zero temperature transition between a nucleonic phase and a QGP phase[3]:

$$P_{nucleon} = \frac{M^4}{6\pi^2}\left(\frac{\mu}{M}\left(\frac{\mu^2}{M^2} - 1\right)^{1/2}\left(\frac{\mu^2}{M^2} - \frac{5}{2}\right) + \frac{3}{2}\ln\left[\frac{\mu}{M} + \left(\frac{\mu^2}{M^2} - 1\right)^{1/2}\right]\right.$$
$$\epsilon_{nucleon} = \frac{2\mu}{3\pi^2}(\mu^2 - M^2)^{3/2} - P_{nucleon}$$
$$P_{QGP} = \frac{\mu_q^4}{2\pi^2} - B$$
$$\epsilon_{QGP} = 3P_{QGP} + 4B$$

- For quantitative description of nucleation rate for zero temperature we took the model of quantum tunneling mediated by $O(4)$ symmetric instantons.

[3] J.Cleymans, R.V.Gavai,E.Suhonen,Phys.Rep.130,217 (1986)

- Nucleation rate:

$$\Gamma = A\frac{S_0^2}{4\pi^2}exp(-S_0)$$

A is the determinant of fluctuations around the instanton configuration, S_0 is the Euclidean action of the instanton.

- For low nucleation rates S_0 is very large. The nucleation rate will be completely dominated by the exp. factor.
- Pre-exponential factor can be approximated by dimensional estimates. At finite temp. $A = T^4$. For the present case $A = R_c^{-4}$. R_c is the critical bubble radius.
- S_0 is found to be

$$S_0 = \frac{27\pi^2 S_1^4}{2(\triangle P)^3}$$

- Action:

$$S = -\frac{1}{2}\pi^2 R^4 \triangle P + 2\pi^2 R^3 S_1$$

S_1 is the contribution of the surface term of the bubble to the action.

- Extremization of action also gives the critical bubble radius:

$$R_c = 3S_1/\triangle P$$

$$\triangle P = P_{QGP} - P_{bucleon}$$

- We have calculated number of bubbles nucleated in **300 meter** radius core. We have taken the parameter values as:

$$B^{1/4} = 177.9 MeV$$

$$S_1 = \sigma = 0.05 MeV/fm^2$$

$$M = 1087.0 MeV$$

- With this parameter critical density for the transition is found to be

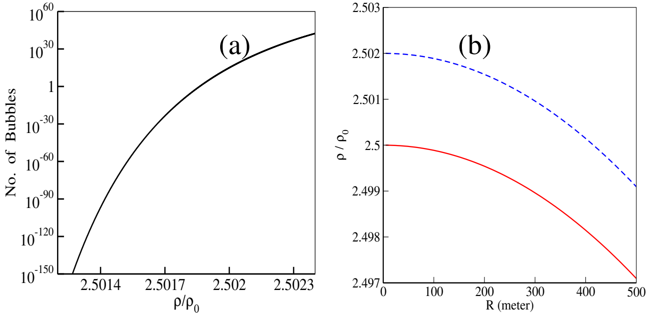
$$\rho_c = 2.500\rho_o$$

where $\rho_0 \simeq 0.15 m_{nucleon}$ i.e. nuclear saturation density

- To get the density profile as a function of distance from the centre of the star, we solved[2]:

$$\frac{1}{\rho}\frac{dP}{dr} = -\frac{Gm}{r^2}, \qquad dm = 4\pi r^2 \rho dr, \qquad P = K\rho^{\alpha}$$

with central density $\rho = 2.5\rho_0$, $\alpha = 2.54$ with K=$0.021\rho_0^{-1.54}$.



- Mass of the neutron star with $\rho_{centre} = 2.500\rho_0$, $M_1 = 1.564M_0$ and with $\rho_{centre} = 2.502\rho_0$, $M_1 = 1.567M_0$.
- Taking acceration rate of $\mathbf{10^{17} grams/sec}$, it will take about **1 million years** for the supercritical coresize to increase to this size.

[2] H.Heiselberg and M.Hjorth-Jensen, PRL 80,5485 (1998)

Density fluctuation due to bubble nucleation

- After nucleation, bubbles rapidly expand and coalesce. At the time of coalescence, the supercitical core region will consist of a close packing of bubbles of new phase, embedded in the old phase.
- For $\mathbf{R_0 \simeq 300}$ meters and $\mathbf{r_0 = 20}$ meters,

$$\frac{\delta I}{I} \simeq 4 \times 10^{-8}$$

- Fractional change in MI remains of same order when r_0 is changed from **20 meters to 5 meters**.
- Due to random nature of bubble nucleation, off-diagonal components of the MI, as well as the quadrupole moment become nonzero and the ratio of both to the initial moment of inertia are found to be of order $\mathbf{10^{-11} - 10^{-10}} \rightarrow$ **new source of gravitational wave.**
- Off-diagonal component of MI gives rise to wobbling of the neutron star, which will result in change in pulse intensity.

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- First consider hadron to QGP transition. Expectation value of Polyakov loop $l(x)$, is the order parameter.
- We carry out a field theory simulation of the evolution of $l(x)$ from an initial value of zero as the system is assumed to undergo a rapid transition to QGP phase.
- We used the effective potential of the form[4],

$$L = \frac{3}{g^2}|\partial_\mu l|^2 T^2 - (-\frac{b_2}{2}|l|^2 - \frac{b3}{6}(l^3 + l^{*3}) + \frac{1}{4}|l|^4) b_4 T^4$$

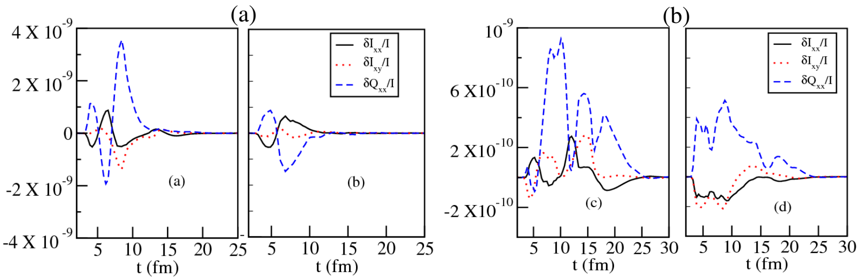
$l(x)$ is normalized in such a way that l_0(absolute minimum of potential) $\rightarrow$1 as T$\rightarrow \infty$.

- Time evolution of $l(x)$ is governed by the field equations obtained from the above Lagrangian.
- The physical size of the lattice is taken as $(7.5fm)^3$ and $(15fm)^3$ with lattice spacing

$$\triangle x = 0.025 \text{ fm}, \triangle t = \frac{0.9 \times \triangle x}{\sqrt{3}}$$

[4] A.Dumitru et al, Phys.Lett.B 504,282(2001); Phys.Rev.D 66,096003 (2002)

- A spherical region with radius R_c is chosen to study change of moment of inertia, with $R_c = 0.4\times$(lattice size). This represents the core of the neutron star.



(a) & (c) correspond to the Confinement-deconfinement phase transition with Z(3) walls and strings. (b) & (d) corresponds to the transition with only string as appropriate for the CFL phase.

- In the dense core of pulsar, at very high density, there is a possibility of phase transition to Color flavour locked phase transition (CFL), due to quark cooper pair formation.
- In this phase transition QCD symmetry group is broken as:

$$SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{c+L+R} \times Z_2$$

- To roughly estimate resulting change in MI , we consider a simplified case by replacing

$$(l^3 + l^{*3}) \rightarrow (|l|^2 + |l^{*2}|^2)^{3/2}$$

  This modification gives rise to string defect without domain walls.
- We considered QCD transition occurring in the dense core of fractional size 0.3/10.
- The change due to fluctuations is the transient one and will dissipate away as star core achieves uniform new phase.
- Transient change have either sign, similarly the net change can also have either sign depending on the nature of the transition.

- We have produced a network of defects inside the core of the pulsar by modeling the correlation domain formation in a cubic lattice.
- Mass density of the string was taken 3 GeV/fm and domain wall tension is taken to be 7 GeV/fm^2.
- We consider spherical star of size R and confine defect network within a spherical core of radius $R_c = 0.3/10R$.

	QCD Strings			QCD Walls			Superfluid Strings		
$\frac{R_C}{\xi}$	$\frac{\delta I_{xx}}{I}$	$\frac{\delta I_{xy}}{I}$	$\frac{Q_{xx}}{I}$	$\frac{\delta I_{xx}}{I}$	$\frac{\delta I_{xy}}{I}$	$\frac{Q_{xx}}{I}$	$\frac{\delta I_{xx}}{I}$	$\frac{\delta I_{xy}}{I}$	$\frac{\delta Q_{xx}}{I}$
5	5E-10	-3E-10	-1E-10	2E-8	-1E-8	-8E-10	2E-6	-1E-6	-4E-7
50	5E-12	-2E-12	2E-12	1E-10	-8E-11	-1E-11	2E-8	-7E-9	7E-9
200	1E-13	2E-14	-7E-14	5E-12	-4E-12	-6E-12	5E-10	6E-11	-2E-10
400	-3E-15	-5E-14	-9E-14	3E-12	-2E-12	3E-14	-1E-11	-2E-10	-3E-10

- Results are obtained by varying core size R_c while keeping the correlation length ζ= 10fm fixed.

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- We have considered superfluid transition occurring inside a core of size about 3-5 km.
- QCD transition occurring in a core size of few hundred meters will drive transition to the normal phase(for a pre-existing superfluid phase)in about 10 times larger radius in the neutron star.
- Subsequent cooling will lead to superfluid transition with associated formation of a dense network of superfluid vortices.
- Free energy density is taken to be of order $0.1 Mev/fm^3$ and vortex energy per unit length is taken $100 MeV/fm$.
- Net fractional change due to phase change 10^{-6}, string induced fractional change 10^{-10}.
- Quadrupole moment and off diagonal components of MI are also of order 10^{-10}.

Gravitational wave generation due to density fluctuation

- Despite the small values of quadrupole moments, the power emitted in gravitational waves may not be small due to very short time scales, which is of the order fm/c.

$$\frac{dE}{dt} = -\frac{32G}{5c^5}\triangle Q^2\omega^6 \simeq -(10^{33}J/s)(\frac{\triangle Q/I_0}{10^{-6}})^2(\frac{10^{-3}sec}{\triangle t})^6$$

- For conservative estimates we have taken :

$$\triangle Q/I_0 \simeq 10^{-14} - 10^{-10} \quad (1)$$

$$\triangle t = 10^{-6} - 10^{-5} sec \quad (2)$$

- Even though $\triangle Q/I_0$ is very smaller than the value of 10^{-6}, typically used for deformed neutron stars, the power in gravitational wave can be significant due to large enhancement from the $(\frac{10^{-3}sec}{\triangle t})^6$ factor.

- Expected strain amplitude from a pulsar at a distance r,

$$h = \frac{4\pi^2 G \triangle Q f^2}{c^4 r} \simeq 10^{-24} (\frac{\triangle Q / I_0}{10^{-6}})(\frac{10^{-3sec}}{\triangle t})^2 (\frac{1kpc}{r})$$

- With

$$\triangle Q / I_0 \simeq 10^{-10}$$

$$\triangle t = 10^{-6} - 10^{-5} sec$$

strain amplitude comes out to be :

$$h \simeq 10^{-24} - 10^{-22}$$

for a pulsar at 1Kpc distance.

Conclusion

- Density fluctuations arising during a rapid phase transition lead to transient change in the MI of the star.
- Such density fluctuations in general lead to non-zero off-diagonal components of moment of inertia tensor which will cause the wobbling of pulsar, thereby modulating the peak intensity of the pulse.
- We find that moment of inertia can increase or decrease, which gives the possibility of accounting for the phenomenon of glitches and anti-glitches in a unified framework.
- Development of nonzero value of quadrupole moment (on a very short time scale) gives the possibility of gravitational radiation from the star whose core is undergoing a phase transition.
- Density fluctuations arising during phase transitions crucially depend on the nature of phase transition.
- Identification of these density fluctuations via pulsar timings (and gravitational waves) can pin down the specific transition occurring inside the pulsar core.

- Multiple occurrences of glitches may raise concern in our model.
- For vortex depinning model multiple glitches seem natural.
- In our model, multiple occurrence of glitches will require multiple phase transitions.
- Glitches could occur due to multiple reasons, some glitches/anti-glitches could occur due to the model proposed here, that is due to phase transition induced density fluctuations, while other glitches could occur due to the conventional de-pinning of vortex clusters.
- We emphasize that when a transition happens in the core of a neutron star, it invariably leads to density fluctuations which manifest itself in glitch/anti-glitch like behavior, along with other implications such as wobbling of star, gravitational wave emission etc.

Thank You.