How ALICE can find the QCD critical point

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1. Introduction

2. Nanophysics: fluctuations and finite size effects

3. Is ALICE physics the same as Lattice computation?

4. Summary

Outline

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2 Nanophysics: fluctuations and finite size effects

3 Is ALICE physics the same as Lattice computation?

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Old set of questions

Can ALICE test any non-perturbative predictions of QCD?

In heavy-ion collisions QCD often enters indirectly: as the result of a long secondary computation such as hydro. Instead, can one get directly at QCD?

Can ALICE test the existence of a critical point of QCD?

Do heavy-ion experiments have anything to say about the phase diagram? Or are they just dirtier versions of proton-proton collisions?

Very important concept: coarse graining

1. If you observe water at a scale of nm, then you don't see a fluid: only molecules.
2. Water at a scale of mm, is a fluid; no direct evidence of molecules
3. When you observe water at a scale of 10–100 nm then you see higher order hydrodynamics, coupling of thermal and hydro fluctuations, etc.
4. Correlation lengths ξ set a scale for transition from micro to macro
5. $\ell =$ size of observable system. For $\ell/\xi \leq 1$ micro, for $\ell/\xi \gg 1$ macro, in between nanomaterial. Heavy-ion collisions study nanophysics; ee and pp collisions study microphysics

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4. Correlation lengths ξ set a scale for transition from micro to macro (in water $\xi \simeq$ interparticle spacing)
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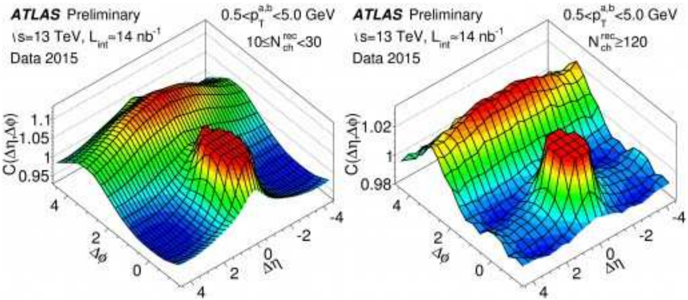
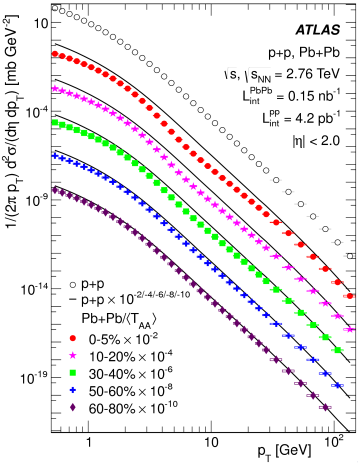
Old pictures are stressed



Old picture: the ridge is due to hydrodynamic flow: v_n are therefore nothing but flow. In pp? In pA?

Particle spectra



ATLAS 1504.04337

Blast wave fits involve a simplified picture of the hydrodynamics.
What about pp, pA?

Why should other scientists be interested?

Importance of heavy-ion collisions

The early universe is a relativistic fluid of particles very nearly in thermodynamic equilibrium. Thermal quantum field theory may be used for predictions. No direct laboratory tests. Do heavy-ion physics provide a test-bed?

The main question

Is the fireball produced in heavy-ion collisions thermalized? Can we verify predictions of thermal quantum field theory? Why not look more carefully at the final state than before?

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Non-linear susceptibilities

Taylor expansion of the pressure in μ_B

$$P(T,\mu_B+\Delta\mu_B)/T^4=\sum_n \frac{1}{n!}\left[\chi^{(n)}(T,\mu_B)T^{n-4}\right]\left(\frac{\Delta\mu_B}{T}\right)^n$$

has Taylor coefficients called non-linear susceptibilities (NLS).
When $\mu_B=0$ they can be computed directly on the lattice, otherwise reconstructed from such computations.

Gavai, SG: 2003, 2010; Datta, Gavai, SG: 2015

In thermal equilibrium, cumulants of the event-to-event distribution of baryon number are directly related to the NLS:

$$[B^2]=T^3V\left(\frac{\chi^{(2)}}{T^2}\right),\quad [B^3]=T^3V\left(\frac{\chi^{(3)}}{T}\right),\quad [B^4]=T^3V\chi^{(4)}.$$

V unknown, can be removed by taking ratios.
(**SG: 2009**)

Definition of the cumulants

A working definition

$$
\begin{aligned}
[B^1] &= \langle B \rangle \\
[B^n] &= \left\langle \left(B - [B^1]\right)^n \right\rangle \qquad\qquad (2 \leq n \leq 4)
\end{aligned}
$$

Tests and assumptions

$$m_0: \quad \frac{[B^1]}{[B^2]} = \frac{\chi^{(1)}(T,\mu_B)/T^3}{\chi^{(2)}(T,\mu_B)/T^2}$$

$$m_1: \quad \frac{[B^3]}{[B^2]} = \frac{\chi^{(3)}(T,\mu_B)/T}{\chi^{(2)}(T,\mu_B)/T^2}$$

$$m_2: \quad \frac{[B^4]}{[B^2]} = \frac{\chi^{(4)}(T,\mu_B)}{\chi^{(2)}(T,\mu_B)/T^2}$$

Also for cumulants of electric charge, Q, and strangeness, S.

1. Two sides of the equation equal if there is thermal equilibrium and no other sources of fluctuations.
2. Right hand side computed in the grand canonical ensemble (GCE). Can observations simulate a grand canonical ensemble? What T and μ_B?
3. Why should hydrodynamics and diffusion be neglected?

Why thermodynamics and not dynamics?

Chemical species may diffuse on the expanding background of the fireball, so why should we neglect diffusion and expansion?

First check whether the system size, ℓ, is large enough compared to the correlation length ξ: Knudsen's number $K = \xi/\ell$. If $K \ll 1$, ie, $\ell \gg \xi$ then central limit theorem will apply.
Next, compare the relative importance of diffusion and advection through a dimensionless number (Peclet's number):

$$\mathcal{W} = \frac{\ell^2}{t\mathcal{D}} = \frac{\ell v_{flow}}{\mathcal{D}} = \frac{\xi v_{flow}}{K\mathcal{D}} = \frac{v_{flow}}{Kc_s} = \frac{M}{K}.$$

When $\mathcal{W} \ll 1$ diffusion dominates. After chemical freeze-out K is small but Mach's number $M \simeq 1$, so flow dominates: fluctuations are frozen in. So detector observes thermodynamic fluctuations <u>at chemical freeze out</u>.
(**Bhalerao, SG: 2009**)

Grand canonical thermodynamics

When $K \gg 1$ and $V_{fireball}/\ell^3 \to \infty$, then thermodynamics in the grand canonical ensemble works; all distributions of conserved quantities are Gaussian: $[B], [B^2] \neq 0$, all other $[B^n]$ vanish.

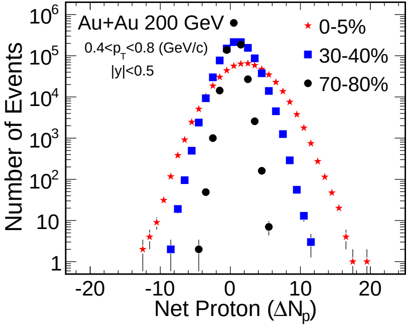
But if $V_{fireball}/\ell^3$ is not so large, then the fluctuations are non Gaussian, and one can measure some of the other cumulants. These are given by the NLS,

$$[B^n] = (VT^3)\, T^{n-4} \frac{\partial^n P(T, \mu)}{\partial \mu^n},$$

QCD determines nanophysics as well as macrophysics.

Check central limit theorem: linear volume dependence of cumulants, *i.e.*, all cumulants scale as V.

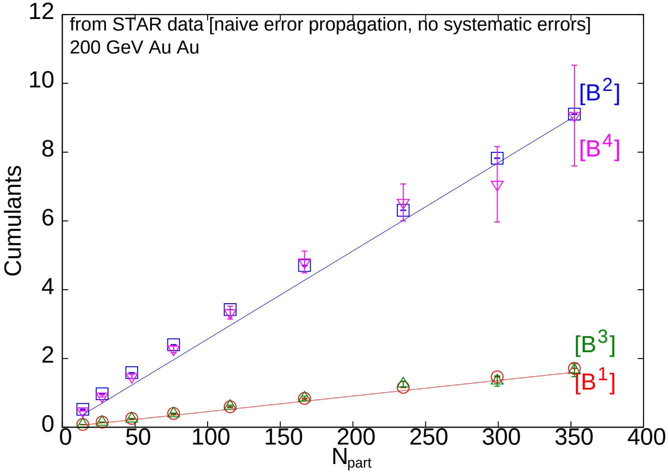
Nearly Gaussian E/E fluctuations



STAR arxiv:1004.4959

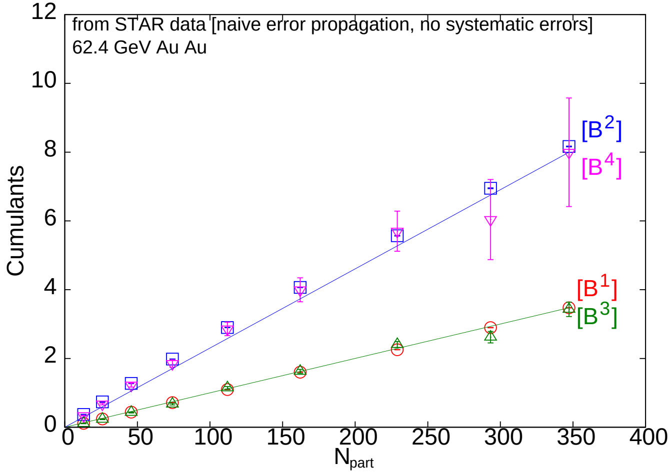
Central rapidity slice taken. p_T of 400–800 MeV. Easy to do better than this crude visual test.

Testing the shape of fluctuations



Linear scaling implies $K \ll 1$. Central limit theorem works.

Testing the shape of fluctuations



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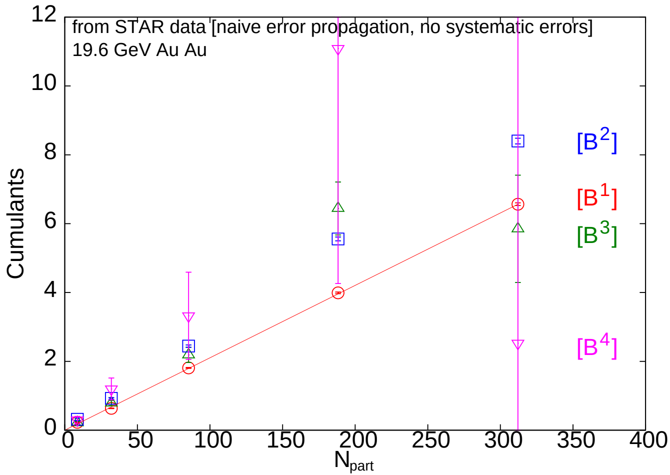

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How does the lattice determine the critical point?

Lattice computations are impossible at finite μ_B. Use the Taylor expansion to define the pressure

$$\chi^{(2)}(T,\mu_B)/T^2 = \sum_n \frac{1}{n!}\left[\chi^{(n+2)}(T,0)\,T^{n-2}\right]\left(\frac{\mu_B}{T}\right)^n$$

Measuring $\chi^{(n)}(T,0)$ is possible. But is the series convergent? Compute the pressure differently.

For $T > T_E$ the series is convergent; the sum is a finite number for every complex value of μ_B/T.

For $T = T_E$ the series diverges for a positive real value of μ_B/T, *i.e.*, $\chi^{(2)}(T,\mu_B)/T^2$ becomes infinite. The value of μ_B at which it diverges is called the critical end point μ_B^E:

$$\left(\frac{\mu_B^E}{T_E}\right)^2 = (n+2)(n+1)\,\frac{\chi^{(n)}(T,0)\,T^{n-4}}{\chi^{(n+2)}(T,0)\,T^{n-2}}.$$

Why does ALICE test lattice?

We are so lucky

The chemical FO T and μ_B are known at the LHC; $\mu_B = 2 \pm 2$ MeV. As a result m_0, m_1, m_2 etc are completely predicted with controlled errors by the lattice computations.

If it works then the major question is answered: the final state is thermal.

Then next, LHC experiment predicts the location of the critical point of QCD.

What one expects ALICE to see

Due to CP symmetry of QCD the odd NLS vanish at $\mu = 0$. In particular

$$\chi^{(1)}(T,0) = \chi^{(3)}(T,0) = 0.$$

This means that at the LHC

$$\begin{aligned} m_0 &= \mu_B + \mathcal{O}(\mu_B^3), \\ m_1 &= \mu_B + \mathcal{O}(\mu_B^3), \\ m_2 &= \frac{\chi^{(4)}(T,0)}{\chi^{(2)}(T,0)/T^2} + \mathcal{O}(\mu_B^2). \end{aligned}$$

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What LHC E/E fluctuations can tell us

- Check that the cumulants are proportional to V. This checks whether the central limit theorem is applicable. If it is not, then dynamical effects are important and thermodynamics cannot be extracted.
- If the cumulants are proportional to V and one sees $m_0 = m_1 = 0$ and m_2 has the value predicted by the lattice at the freezeout temperature, then one has proved that the fireball reaches thermal equilibrium.
- The value of m_2 predicted by the lattice computation is related to the radius of convergence, and hence to the location of the critical point. Can LHC discover the critical point of QCD before the BES?