Radius Parameters (r_0) for Odd-A and Odd-Odd Nuclides

As discussed in 1998Ak04, the nuclear radius parameter, r_0, for an even-even nuclide is determined by defining the calculated transition probability for an alpha decay from the ground state of the parent to the ground state of the daughter to be equal to the experimental transition rate, that is, HF=1.0 for such a transition. The radius parameter calculated by our alpha hindrance factor program is that of the daughter, that is, the effective radius for the alpha decay is given by $R=r_0A^{1/3}$, where A is that of the daughter nuclide. The r_0 parameters for the even-even nuclides are summarized in 1998Ak04. Note that in Table I of this reference, the first two columns give the Z and N of the alpha-decaying parent, while the values of r_0 correspond to the Z-2, N-2 daughter.

Radius parameters for the odd-A and odd-odd nuclides, needed as input for the alpha hindrance program, are obtained from the parameters for the adjacent even-even nuclides.

Figure 1 of 1998Ak04 shows r_0 as a function of N for the even-Z nuclides. It is assumed that the parameters for the odd-Z nuclides lie midway between those for the adjacent Z-1 and Z+1 nuclides. Thus, for example, the r_0 parameters for the Z=85 Astatine daughter nuclides trace out a curve midway between the curves for the Z=84 Polonium and Z=86 Radon nuclides.

It is also assumed that for the even-Z, odd-N case, the parameters for the odd-N isotopes lie midway between the values for the Z, N-1 and Z, N+1 even-even nuclides.

Note that in what follows, all averages are unweighted. A weighted average is not appropriate. Keep in mind that one is not trying to get a best value for two measurements of the same quantity, but rather a value midway between parameters for adjacent nuclides.

1. Odd-Z, Even-N Nuclides

The r_0 parameter for an odd-Z, even-N nuclide is obtained as an unweighted average of the values for the nuclides with Z-1, N and Z+1, N. The uncertainty is obtained as an unweighted average of the upper (or lower) limits of these adjacent nuclide values.

2. Even-Z, Odd-N Nuclides

The r_0 parameter for an even-Z, odd-N nuclide is obtained as an unweighted average of the values for the nuclides with Z, N-1 and Z, N+1. The uncertainty is obtained as an unweighted average of the upper (or lower) limits of these adjacent nuclide values.

3. Odd-Z, Odd-N Nuclides

For an odd-odd nuclide, Z, N one needs r_0 parameters for the four nuclides with Z±1, N±1. In step one, r_0 values for Z, N-1 and for Z, N+1 are obtained as described in 1. above. The value for Z, N is then obtained as an average of the two values from step one. Alternatively, one can average the values for the Z-1, N and Z+1, N nuclides, each of which is obtained as described in

2. above. The two approaches give the same value of r_0.

Example

The following example illustrates the above procedures for the alpha decay of the odd-Z Francium nuclides with Z=87 and N=118, 119, and 120, decaying to the daughter Astátine nuclides with Z=85 and N=116, 117, and 118, respectively. One needs the daughter radius parameters from the four parent alpha decays with Z=86, 88 and N=118, 120. From 1998Ak04 these values are

^{204}Rn (Z=86, N=118) → r_0(^{200}Po) = 1.504 *3*

^{206}Rn(Z=86, N=120) → r_0(^{202}Po) = 1.492 *7*

^{206}Ra(Z=88, N=118) → r_0(^{202}Rn) = 1.527 *8*

^{208}Ra(Z=88, N=120) → r_0(^{204}Rn) = 1.495 *14*

These four points form the four corners of a grid, as given by the values in bold in the following table. Z and N here correspond to the daughter.

	N=116	N=117	**N=118**
Z=84	**^{200}Po r_0 =1.504** *3*	^{201}Po r_0=1.498 *5*	**^{202}Po r_0=1.492** *7*
Z=85	^{201}At r_0=1.5155 *55*	^{202}At r_0=1.5045 *80*	^{203}At r_0=1.4935 *105*
Z=86	**^{202}Rn r_0=1. 527** *8*	^{203}Rn r_0=1.511 *11*	**^{204}Rn r_0=1.495** *14*

Odd A

The value for ^{201}Po is obtained by taking an unweighted average of the parameters for ^{200}Po and ^{202}Po, and that for ^{201}At comes from an unweighted average of the values for ^{200}Po and ^{202}Rn, and so on. The uncertainties come from an unweighted average of the corresponding maximum (or minimum) values. Thus, for ^{201}At, the average of 1.504 and 1.527 gives 1.5155, and the average of the upper limits, 1.507 and 1.535, gives 1.5210 which corresponds to an uncertainty of +55 on the value 1.5155. The lower limits of course give the same uncertainty.

Odd-Odd

The value for the odd-odd ^{202}At can be obtained by averaging the values for ^{201}Po and ^{203}Rn, or the values for ^{201}At and ^{203}At. The two approaches give the same value of 1.5045 *80*.

The Astatine values should of course be rounded off following for publication, our usual policy; however, see the second paragraph in the following section..

Regional r_0 Systematics

It is strongly recommended to evaluators working in mass regions with alpha decay that they plot the r_0 parameters for the nuclides in and around their mass regions. These plots can be of use in estimating parameters when critical even-even data are missing. For example, suppose that in the above example the r_0 parameter for ^{202}Po was not known experimentally. It might still be possible to obtain a reasonable estimate of the value by extrapolation from the known lower-N Po values. Similarly, one might be able to estimate a value for ^{202}At or ^{203}At by extrapolation from values for lower-N At nuclides. Even an approximate value might be sufficient to establish a particular alpha branch as being favored. Of course such extrapolations are worth doing only if the r_0 plots are fairly regular, and the regularity, or lack thereof, is something that is useful to know in one's mass region.

In keeping these r_0 systematics, it is recommended that more digits be retained than would correspond to our usual policy. This will avoid possible cumulative roundoff problems. In the above example, r_0(^{202}At) could be retained in an evaluator's internal file of parameters as 1.5045 *80*, with the roundoff to 1.504 *8*, or 1.505 *8* being done at the mass chain stage.