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To: NSDD Network Evaluators

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Subject: Reduced Gamma-Ray Matrix Elements, Transition Probabilities, and Single-Particle Estimates

For an electromagnetic transition of energy E_γ, the relationships among the reduced matrix elements, $B(\sigma L)$, and the partial γ-ray half-life, $T^\gamma{}_{1/2}$, are

$$T^\gamma{}_{1/2}(EL)\ B(EL)\downarrow = \frac{(\ln 2)\ L\ [(2L+1)!!]^2\ \hbar}{8\pi\ (L+1)\ e^2 b^L}\left(\frac{\hbar c}{E_\gamma}\right)^{2L+1} \tag{1}$$

$$T^\gamma{}_{1/2}(ML)\ B(ML)\downarrow = \frac{(\ln 2)\ L\ [(2L+1)!!]^2\ \hbar}{8\pi\ (L+1)\ \mu_N{}^2\ b^{L-1}}\left(\frac{\hbar c}{E_\gamma}\right)^{2L+1} \tag{2}$$

The Weisskopf single-particle estimates for the $B(\sigma L)$ are

$$B_{s.p.}(EL)\downarrow = \frac{1}{4\pi b^L}\left(\frac{3}{3+L}\right)^2 R^{2L} \tag{3}$$

$$B_{s.p.}(ML)\downarrow = \frac{10}{\pi b^{L-1}}\left(\frac{3}{3+L}\right)^2 R^{2L-2} \tag{4}$$

so that

$$T^\gamma{}_{1/2\ s.p.}(EL) = \frac{(\ln 2)\ L\ [(2L+1)!!]^2\ \hbar}{2\ (L+1)\ e^2\ R^{2L}}\left(\frac{3+L}{3}\right)^2\left(\frac{\hbar c}{E_\gamma}\right)^{2L+1} \tag{5}$$

$$T^\gamma{}_{1/2\ s.p.}(ML) = \frac{(\ln 2)\ L\ [(2L+1)!!]^2\ \hbar}{80\ (L+1)\ \mu_N{}^2\ R^{2L-2}}\left(\frac{3+L}{3}\right)^2\left(\frac{\hbar c}{E_\gamma}\right)^{2L+1} \tag{6}$$

The relationship between a measured $B(\sigma L)\uparrow$ to a level with spin J_f from a level with spin J_i connected by a transition γ_K is given by Eq. (1) or Eq. (2) with

$$T_{1/2}(J_f) = T^{\gamma_K}{}_{1/2}(\sigma L)\ \varepsilon(\gamma_K) \qquad (7)$$

and

$$B(\sigma L)\uparrow = \frac{(2J_f + 1)}{(2J_i + 1)} B(\sigma L)\downarrow$$

where $\varepsilon(\gamma_K)$ is the fraction of the decays of level J_f proceeding via the observed mode γ_K and is given by

$$\varepsilon(\gamma_K) = \frac{\lambda^{\gamma}{}_K}{\sum_i (1 + \alpha_i)\ \lambda^{\gamma}{}_i} = \frac{BR(\gamma_K)}{(1 + \alpha_K)}$$

where $\lambda^{\gamma}{}_i$ is the relative partial decay constant for gamma transition "i," α_i is the total conversion coefficient for transition "i," and $BR(\gamma_K)$ is the total (i.e., γ + ce) branching ratio for transition "K."

If the transition "K" is of mixed multipolarity L, L + 1, then a factor $\delta^2/(1 + \delta^2)$ for L + 1 or $1/(1 + \delta^2)$ for L must be inserted on the right-hand side of Eq. (7). δ^2 is the ratio of the L + 1 and L components.

In Eqs. (1) through (6), $b = 10^{-24}$ cm^2; $R = R_0\ A^{1/3} \times 10^{-13}$ cm; and B(EL), B(ML) are expressed in units of $e^2 b^L$ and $\mu_N^2\ b^{L-1}$, respectively.

For the constants appearing in the above expressions, we adopt the following values:

$\hbar c = 1.9733 \times 10^{-8}$ keV - cm

$\hbar = 0.6584 \times 10^{-18}$ keV - s

$e^2 = 1.43998 \times 10^{-10}$ keV - cm

$\mu_N^2 = 1.59234 \times 10^{-38}$ keV - cm^3

$R_0 = 1.2$

Specific expressions for the above equations, along with that for

$B(\sigma L)(W.u.) = B(\sigma L)/B_{s.p.}(\sigma L)$, are given here for L = 1 through L = 5. E_γ is in keV, and W.u. stands for Weisskopf units.

As noted above, if a transition under consideration is of mixed multipolarity, L, L + 1, then the expressions below for $B(\sigma L)$(W.u.) and $T_{1/2}(J) \times B(\sigma L)\uparrow$ should be multiplied on the right by $\delta^2/(1 + \delta^2)$ for the L + 1 and by $1/(1 + \delta^2)$ for the L-components.

E1 Transitions

$$T^{\gamma}_{1/2}(E1)\ B(E1)\downarrow = \frac{4.360 \times 10^{-9}}{(E_\gamma)^3}$$

$$B_{s.p.}(E1)\downarrow = 6.446 \times 10^{-4}\ A^{2/3}\ (e^2 \times 10^{-24}\ cm^2)$$

$$T^{\gamma}_{1/2\ s.p.}(E1) = \frac{6.764 \times 10^{-6}}{(E_\gamma)^3\ A^{2/3}}\ (s)$$

$$B(E1)(W.u.) = \frac{6.764 \times 10^{-6}\ BR}{(E_\gamma)^3\ A^{2/3}\ T_{1/2}\ (1 + \alpha)}$$

$$T_{1/2}(J_f)\ B(E1)\uparrow = \frac{4.360 \times 10^{-9}\ BR}{(E_\gamma)^3\ (1 + \alpha)}\left(\frac{2J_f + 1}{2J_i + 1}\right)$$

E2 Transitions

$$T^{\gamma}_{1/2}(E2)\ B(E2)\downarrow = \frac{5.659 \times 10^{1}}{(E_\gamma)^5}$$

$$B_{s.p.}(E2)\downarrow = 5.940 \times 10^{-6}\ A^{4/3}\ (e^2 \times 10^{-48}\ cm^4)$$

$$T^{\gamma}_{1/2\ s.p.}(E2) = \frac{9.527 \times 10^{6}}{(E_\gamma)^5\ A^{4/3}}\ (s)$$

$$B(E2)(W.u.) = \frac{9.527 \times 10^{6}\ BR}{(E_\gamma)^5\ A^{4/3}\ T_{1/2}\ (1 + \alpha)}$$

$$T_{1/2}(J_f)\ B(E2)\uparrow = \frac{5.659 \times 10^{1}\ BR}{(E_\gamma)^5\ (1 + \alpha)}\left(\frac{2J_f + 1}{2J_i + 1}\right)$$

E3 Transitions

$$T^{\gamma}_{1/2}(E3)\ B(E3)\downarrow = \frac{1.215 \times 10^{12}}{(E_{\gamma})^{7}}$$

$$B_{s.p.}(E3)\downarrow = 5.940 \times 10^{-8}\ A^{2}\ (e^{2} \times 10^{-72}\ cm^{6})$$

$$T^{\gamma}_{1/2\ s.p.}(E3) = \frac{2.045 \times 10^{19}}{(E_{\gamma})^{7}\ A^{2}}\ (s)$$

$$B(E3)(W.u.) = \frac{2.045 \times 10^{19}\ BR}{(E_{\gamma})^{7}\ A^{2}\ T_{1/2}\ (1 + \alpha)}$$

$$T_{1/2}(J_f)\ B(E3)\uparrow = \frac{1.215 \times 10^{12}\ BR}{(E_{\gamma})^{7}\ (1 + \alpha)} \left(\frac{2J_f + 1}{2J_i + 1}\right)$$

E4 Transitions

$$T^{\gamma}_{1/2}(E4)\ B(E4)\downarrow = \frac{4.087 \times 10^{22}}{(E_{\gamma})^{9}}$$

$$B_{s.p.}(E4)\downarrow = 6.285 \times 10^{-10}\ A^{8/3}\ (e^{2} \times 10^{-96}\ cm^{8})$$

$$T^{\gamma}_{1/2\ s.p.}(E4) = \frac{6.503 \times 10^{31}}{(E_{\gamma})^{9}\ A^{8/3}}\ (s)$$

$$B(E4)(W.u.) = \frac{6.503 \times 10^{31}\ BR}{(E_{\gamma})^{9}\ A^{8/3}\ T_{1/2}\ (1 + \alpha)}$$

$$T_{1/2}(J_f)\ B(E4)\uparrow = \frac{4.087 \times 10^{22}\ BR}{(E_{\gamma})^{9}\ (1 + \alpha)} \left(\frac{2J_f + 1}{2J_i + 1}\right)$$

E5 Transitions

$$T^{\gamma}_{1/2}(E5)\ B(E5)\downarrow = \frac{2.006 \times 10^{33}}{(E_{\gamma})^{11}}$$

$$B_{s.p.}(E5)\downarrow = 6.929 \times 10^{-12}\ A^{10/3}\ (e^2 \times 10^{-120}\ cm^{10})$$

$$T_{1/2\ s.p.}(E5) = \frac{2.895 \times 10^{44}}{(E_{\gamma})^{11}\ A^{10/3}} \quad (s)$$

$$B(E5)(W.u.) = \frac{2.895 \times 10^{44}\ BR}{(E_{\gamma})^{11}\ A^{10/3}\ T_{1/2}\ (1 + \alpha)}$$

$$T_{1/2}\ (J_f)\ B(E5)\uparrow = \frac{2.006 \times 10^{33}\ BR}{(E_{\gamma})^{11}\ (1 + \alpha)} \left(\frac{2J_f + 1}{2J_i + 1}\right)$$

For ML transitions we have:

$$T^{\gamma}_{1/2}(ML)/T^{\gamma}_{1/2}(EL) = 9.043 \times 10^3\ B(EL)/B(ML)$$

$$B_{s.p.}(ML)/B_{s.p.}(EL) = 2.778 \times 10^3\ A^{-2/3}$$

$$T^{\gamma}_{1/2\ s.p.}(ML)/T^{\gamma}_{1/2\ s.p.}(EL) = 3.256\ A^{2/3}$$

$$B(ML)(W.u.)/B(EL)(W.u.) = 3.256\ A^{2/3}$$

M1 Transitions

$$T^{\gamma}_{1/2}(M1)\ B(M1)\downarrow = \frac{3.943 \times 10^{-5}}{(E_{\gamma})^3}$$

$$B_{s.p.}(M1)\downarrow = 1.791 \quad (\mu_N^{\ 2})$$

$$T^{\gamma}_{1/2\ s.p.}(M1) = \frac{2.202 \times 10^{-5}}{(E_{\gamma})^3} \quad (s)$$

$$B(M1)(W.u.) = \frac{2.202 \times 10^{-5}\ BR}{(E_{\gamma})^3\ T_{1/2}\ (1 + \alpha)}$$

$$T_{1/2}(J_f)\ B(M1)\uparrow = \frac{3.943 \times 10^{-5}\ BR}{(E_{\gamma})^3\ (1 + \alpha)} \left(\frac{2J_f + 1}{2J_i + 1}\right)$$

M2 Transitions

$$T^{\gamma}_{1/2}(M2)\ B(M2)\downarrow = \frac{5.117 \times 10^{5}}{(E_{\gamma})^{5}}$$

$$B_{s.p.}(M2)\downarrow = 1.650 \times 10^{-2}\ A^{2/3} \quad (\mu_N^{2} \times 10^{-24}\ cm^{2})$$

$$T^{\gamma}_{1/2\ s.p.}(M2) = \frac{3.102 \times 10^{7}}{(E_{\gamma})^{5}\ A^{2/3}} \quad (s)$$

$$B(M2)(W.u.) = \frac{3.102 \times 10^{7}\ BR}{(E_{\gamma})^{5}\ A^{2/3}\ T_{1/2}\ (1 + \alpha)}$$

$$T_{1/2}(J)\ B(M2)\uparrow = \frac{5.117 \times 10^{5}\ BR}{(E_{\gamma})^{5}\ (1 + \alpha)}$$

M3 Transitions

$$T^{\gamma}_{1/2}(M3)\ B(M3)\downarrow = \frac{1.099 \times 10^{16}}{(E_{\gamma})^{7}}$$

$$B_{s.p.}(M3)\downarrow = 1.650 \times 10^{-4}\ A^{4/3} \quad (\mu_N^{2} \times 10^{-48}\ cm^{4})$$

$$T^{\gamma}_{1/2\ s.p.}(M3) = \frac{6.659 \times 10^{19}}{(E_{\gamma})^{7}\ A^{4/3}} \quad (s)$$

$$B(M3)(W.u.) = \frac{6.659 \times 10^{19}\ BR}{(E_{\gamma})^{7}\ A^{4/3}\ T_{1/2}\ (1 + \alpha)}$$

$$T_{1/2}(J_f)\ B(M3)\uparrow = \frac{1.099 \times 10^{16}\ BR}{(E_{\gamma})^{7}\ (1 + \alpha)} \left(\frac{2J_f + 1}{2J_i + 1}\right)$$

M4 Transitions

$$T^{\gamma}_{1/2}(M4)\ B(M4)\downarrow = \frac{3.696 \times 10^{26}}{(E_{\gamma})^{9}}$$

$$B_{s.p.}(M4)\downarrow = 1.746 \times 10^{-6}\ A^{2}\ (\mu_N^{2} \times 10^{-72}\ cm^{6})$$

$$T^{\gamma}_{1/2\ s.p.}(M4) = \frac{2.117 \times 10^{32}}{(E_{\gamma})^{9}\ A^{2}}\quad (s)$$

$$B(M4)(W.u.) = \frac{2.117 \times 10^{32}\ BR}{(E_{\gamma})^{9}\ A^{2}\ T_{1/2}\ (1 + \alpha)}$$

$$T_{1/2}(J_f)\ B(M4)\uparrow = \frac{3.696 \times 10^{26}\ BR}{(E_{\gamma})^{9}\ (1 + \alpha)} \left(\frac{2J_f + 1}{2J_i + 1}\right)$$

M5 Transitions

$$T^{\gamma}_{1/2}(M5)\ B(M5)\downarrow = \frac{1.814 \times 10^{37}}{(E_{\gamma})^{11}}$$

$$B_{s.p.}(M5)\downarrow = 1.925 \times 10^{-8}\ A^{8/3}\ (\mu_N^{2} \times 10^{-96}\ cm^{8})$$

$$T^{\gamma}_{1/2\ s.p.}(M5) = \frac{9.426 \times 10^{44}}{(E_{\gamma})^{11}\ A^{8/3}}\quad (s)$$

$$B(M5)(W.u.) = \frac{9.426 \times 10^{44}\ BR}{(E_{\gamma})^{11}\ A^{8/3}\ T_{1/2}\ (1 + \alpha)}$$

$$T_{1/2}(J_f)\ B(M5)\uparrow = \frac{1.814 \times 10^{37}\ BR}{(E_{\gamma})^{11}\ (1 + \alpha)}$$