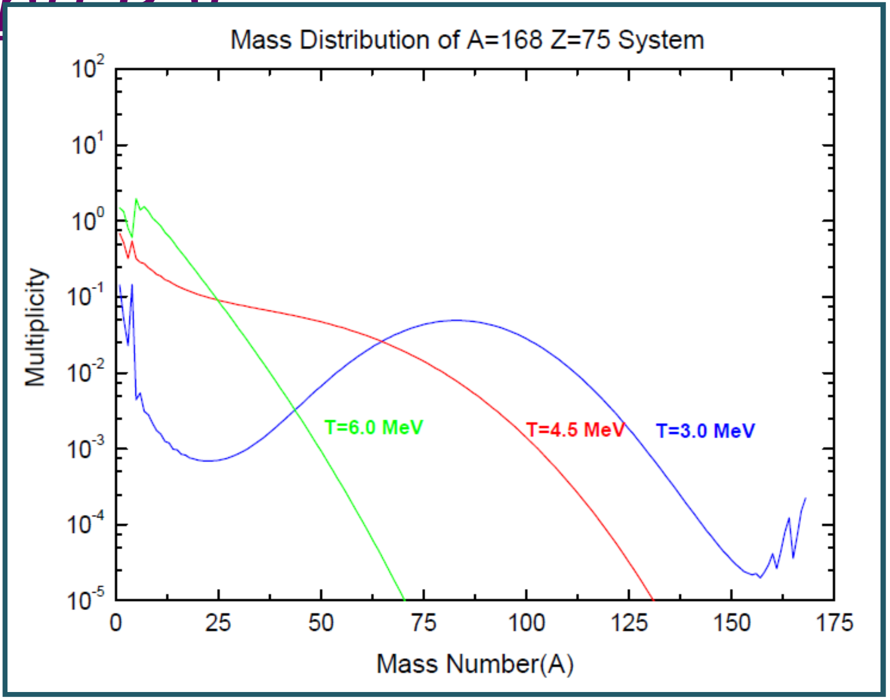
# *Stochastic dynamical models of nuclear fission*

*Santanu Pal*

In the hierarchy of disassembly process (*where identity of parent completely lost among products*) of a nucleus

| | |
|---|---|
| Fission is at the low energy end (heavy nuclei) | Multi-fragmentation is at the high energy end |

Gradual transition from fission to multi-fragmentation with increase of excitation energy




Swagata Mallik (Private communication)

***n will remain important at the initial beam energies of Kolkata***

## ***Statistical Models***

- Phase space plays an important role in both fission and multi-fragmentation.
- Equilibrium at final stage assumed in both fission and multi-fragmen

Product yield $\propto$ available phase space in the final state

Fission fragment mass yield in n-induced thermal fission $\propto$ density of quantum states of a fission mode at scission
(P. Fong, Phys. Rev. 102 (1956) 434)

Fragment yield in a given channel in multi-fragmentation $\propto$ phase space available to that channel at freeze-out volume
(C. B. Das et al. Phys.Rep.406 (2005)1)

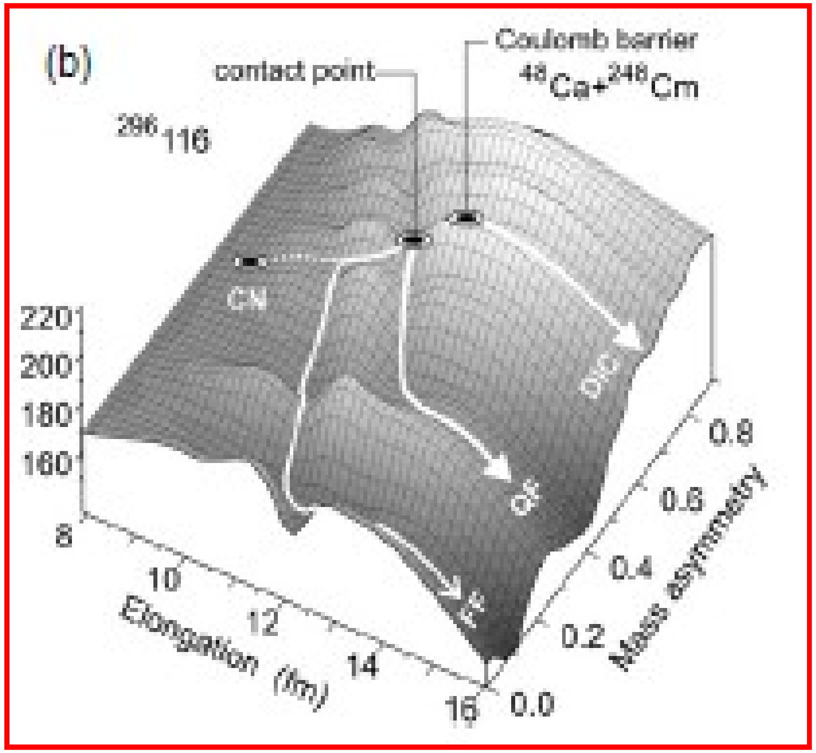
## ***Dynamical models***

Fission $\Rightarrow$ in terms of a few collective variables

Multi fragmentation $\Rightarrow$ no obvious collective coordinate, all particles considered (QMD)

We shall discuss fission of a hot compound nucleus

# ***Fusion-fission reaction***




Y. Oganessian, J. Phys. G 34(2007) R165

After the projectile crosses the entrance Channel Coulomb barrier $\Rightarrow \sigma_{capt}$

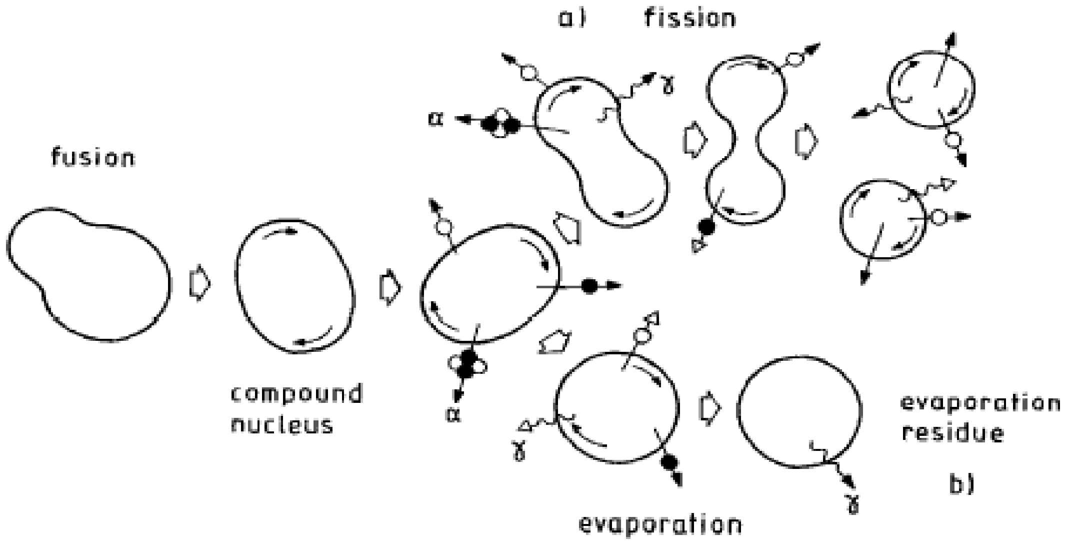
Formation of CN can be inhibited by QF and FF processes $\Rightarrow P_{dyn}$

$$\sigma_{CN} = \sigma_{capt} P_{dyn}$$

Compound Nucleus (CN) either undergoes fission or becomes an Evaporation Residue (ER)

We shall discuss what happens to the CN after it is formed

# *How does a compound nucleus decay?*



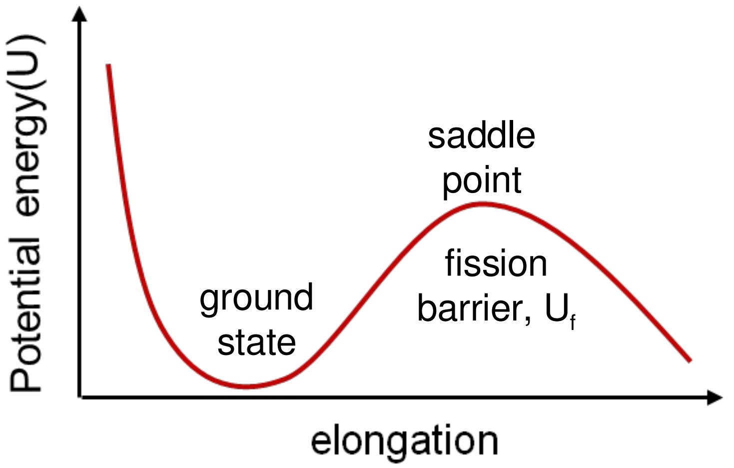

Evaporation residue is the end product of competition between fission and evaporation processes.

Competitiveness of each process decided by its width

$$\Gamma_n , \Gamma_p , \Gamma_\alpha , \Gamma_\gamma , \Gamma_f$$

# ***Bohr-Wheeler transition-state theory of fission***

(Phys. Rev. 56(1939) 426)




*Transition-state model:*

- The fate of a CN is decided at the saddle point (transition state).
- Assume complete equilibration.

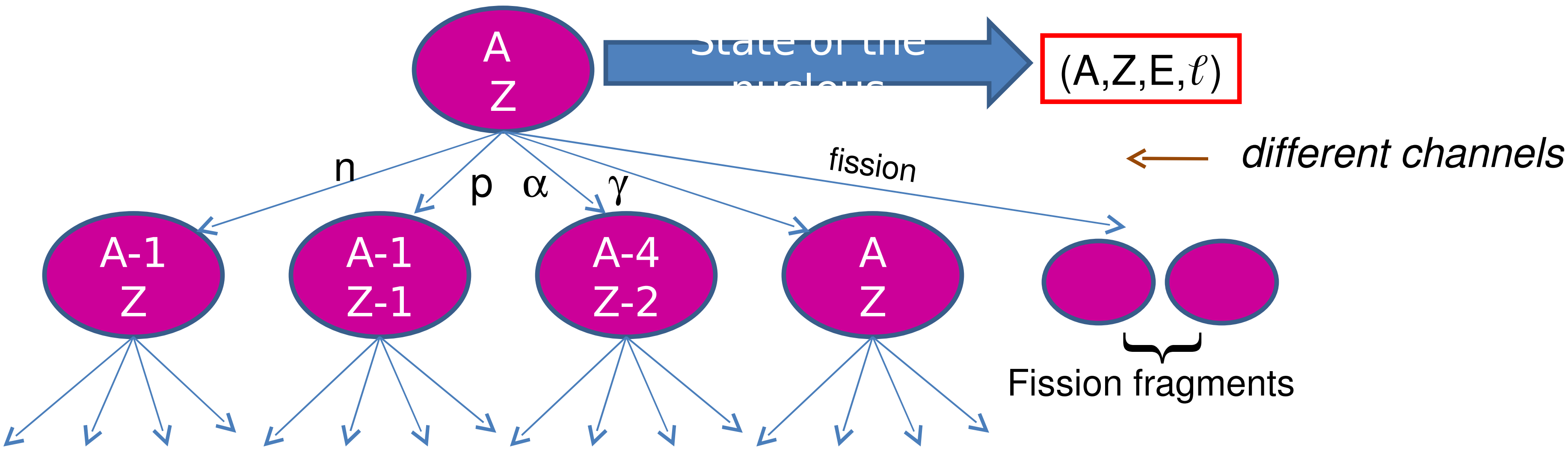
*Fission probability ∝ current across saddle*

$$\Gamma_{BW} = \frac{1}{2\pi\rho(E)} \int_0^{E-U_f} \rho^*(E-U_f-K)\,dK \quad \xrightarrow{\text{approximation}} \quad \Gamma_{BW} = (T/2\pi)\,exp(-U_f/T)$$

***Strutinsky*** (Phys.Lett.B47(1973)121)

- Include vibrational states near ground state

$$\Gamma_{BW} = (\hbar\omega/2\pi)\,exp(-U_f/T)$$




Yield of i-th decay product ∝ $\Gamma_i/\Gamma$ where $\Gamma = \sum \Gamma_i$

$$\Gamma_p = \int R_p dE_p = \frac{\rho_f(E_f, J_f)}{h\rho_i(E_i, J_i)} T(E_p) dE_p,$$

One can calculate the yield (probability) of decay products in bins of excitation energy (E) and spin (ℓ).

- Process stops when either fission occurs or each nuclear species is below either particle emission threshold or fission.

One calculates total fission probability (summed over fission probabilities at different stages) and multiplicities of particles/photons(averaged over different stages).

One also calculates the ER probability: Sum of ER probabilities at different stages.

All the observables are the weighted averages of the spin distribution.

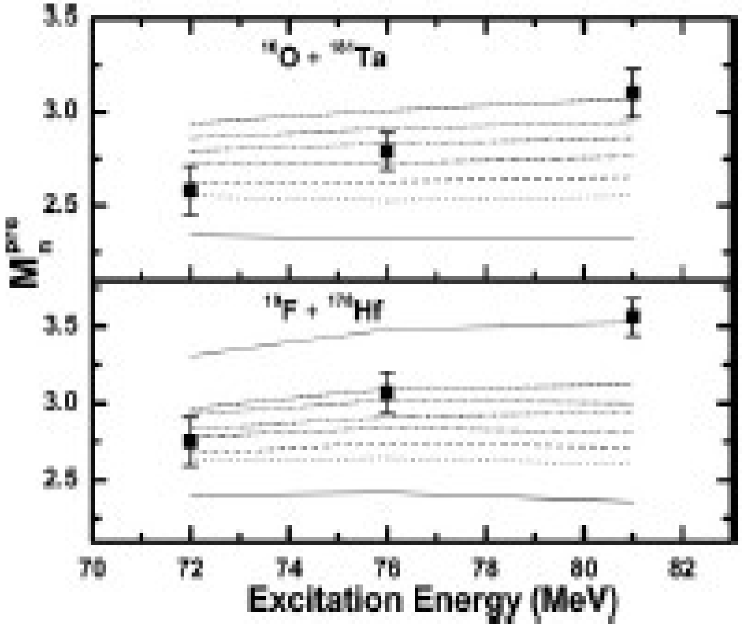
**Features:-**

- One shot calculation of probabilities of all possible decay routes
- Statistical model of nuclear decay

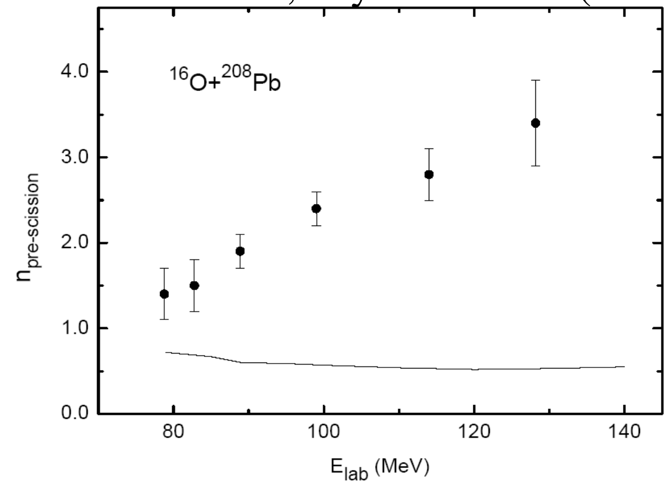
**CASCADE**

# Some statistical model calculation results from fusion-fission reaction

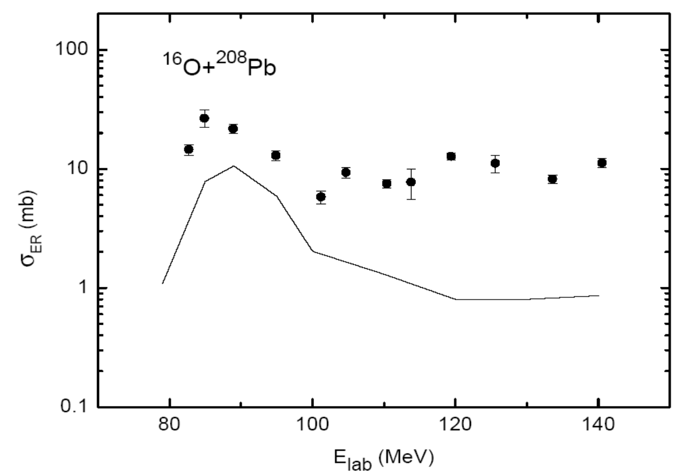
Hardev Singh et al., Phys. Rev. C76 (2007).




Rossner et al., Phys. Rev. C45 (1992).




Brinkman et al., Phys. Rev. C50 (1994).




*Fission is slower than predicted by B-W fission width*

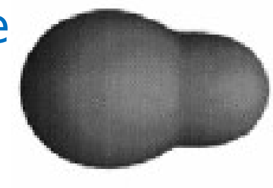
## *Fission is slower than predicted by Bohr-Wheeler theory*

In transition state model, we assumed ready availability of CN at saddle and fission rate determined by phase space available at saddle.

Delivery of CN at saddle can slow down if dynamics of shape evolution from initial (~spherical) to saddle is dissipative in nature. Nuclear bulk dynamics is expected to be dissipative at high excitations.

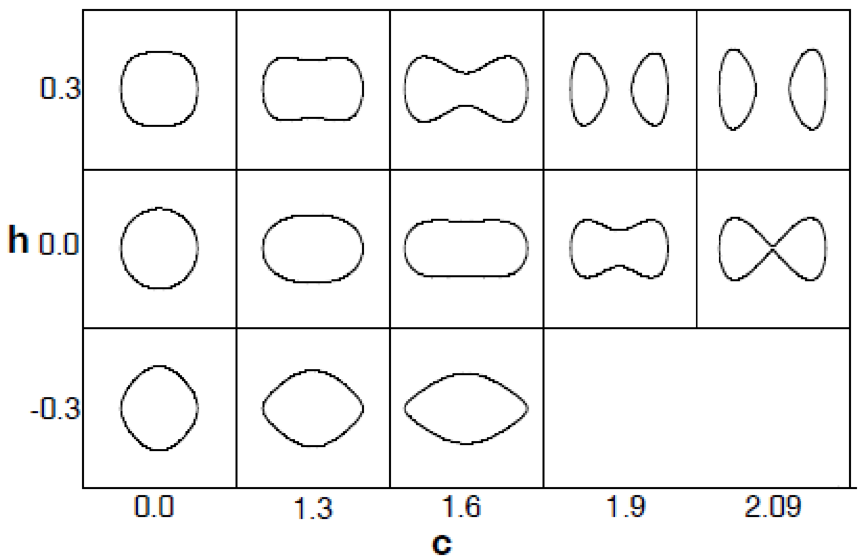
***A dynamical model for fission required***

Fission ⇒ shape evolution⇒ shape variables ⇒ dynamical coordinate



Elongation (c)
Neck (h)
Asymmetry ($\alpha$)

Brack (funny hill)




different shapes

Nucleus⇒ 3A coordinates ⇒

a few shape (collective) variables(x)
+
many intrinsic (almost 3A) coordinates ($\xi$)

$$H_{tot} = H_{coll}(x) + H_{intr}(\xi) + V_{int}(x,\xi)$$

Considering the effect of $V_{int}$ on $H_{intr}$ as a first-order perturbation and taking average over all intrinsic states (Linear Response Theory)

$d\langle H_{intr}\rangle/dt \propto (dX/dt)^2$ ⇒ energy is pumped into intrinsic system **(heating)**

energy is lost from collective motion $H_{coll}$ **(dissipative energy loss)**

*Eqn. of motion of collective coordinates averaged over intrinsic states:*

$$d\langle P\rangle/dt = -(dU/dX) - \eta\, (d\langle X\rangle/dt)$$

P⇒ momentum conjugate to X
U⇒ potential energy
η⇒ dissipation coefficient

*Gives average trajectory in deformation (collective coordinate) space.*

*Stochastic dynamics:*

- Consider an ensemble of fissioning compound nuclei.
- Shape evolution not same for all CN *(Had it been same, all CN would have reached the saddle point simultaneously and we would not have the law of radioactive decay, but same life-time for all CN).*

Some trajectories undergo fission, some do not.

Average trajectory not of much use in fission dynamics. We need to trace individual trajectories.

We need observables averaged over many trajectories and not observables for the average trajectory.

The force on the collective dynamics due to $V_{int}(X,\xi)$ is random in nature essentially due to the large number of intrinsic degrees of freedom ($\xi$)

Force= $\langle$Force$\rangle$ +fluctuation (R)

$$dP/dt = -(dU/dX) - \eta(dX/dt) + R(t)$$

Langevin equation of motion

$$\langle R(t) \rangle = 0$$

$$\langle R(t)R(t') \rangle = 2D\delta(t-t')$$

R(t) is assumed to follow a Gaussian distribution

Fluctuation-dissipation theorem: $D=\eta T$

Markovian Process (zero memory time) assumed

*Fission* ≡ *Brownian motion of a heavy particle in a viscous heat bath*

Collective dynamics (large inertia) ⇒ Brownian particle

Intrinsic motion (large no. of dgfm) ⇒ Heat bath: temperature(T)

- Perform integration choosing the random force at each step from sampling a Gaussian distribution (Monte-Carlo calculation).

- If at any stage, $X > X_{sci}$ ⇒ fission event

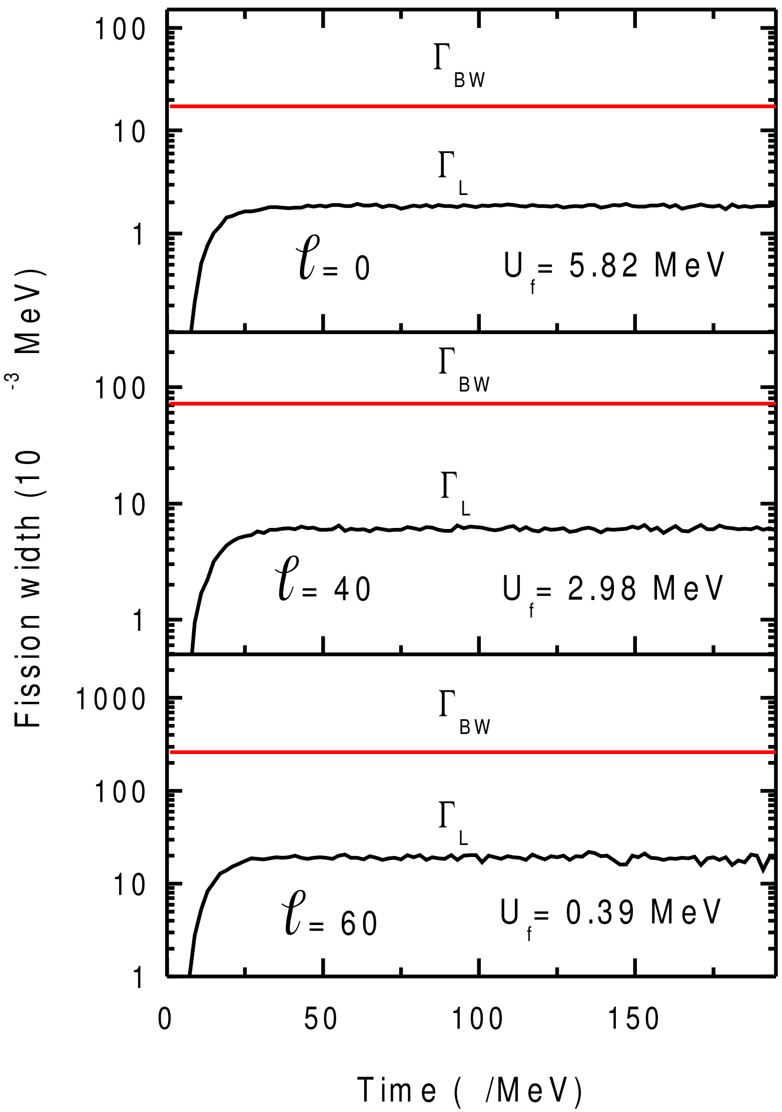
Repeat for a large ensemble of trajectories

**Obtain fission width $\Gamma(t)$= time rate of fission events**

**Input for solving Langevin equation:**

- Collective potential U ⇒ LDM
- Collective inertia m ⇒ hydro-dynamical model assuming no vortex
- Dissipation coefficient $\eta$ ⇒ nuclear bulk property, presently treat as a parameter

$^{224}$Th



Γ BW
Γ L
ℓ = 0
U f = 5.82 MeV
ℓ = 40
U f = 2.98 MeV
ℓ = 60
U f = 0.39 MeV
Fission width (10 -3 MeV)
Time ( /MeV)
100
10
1
1000
0
50
150

## Alternative approach to stochastic dynamics ⇒ Fokker-Planck equation

- Consider the total ensemble of Langevin trajectories
- The evolution of the ensemble with time can be viewed as a diffusion process
- In stead of individual trajectories, we can discuss in terms of a probability distribution function ρ(X,P,t)

ρ(X,P,t)dXdP ⇒ *probability of finding a CN with collective coordinate and momentum in the range X → X+dX ,and P → P+dP at time 't'.*

Fokker-Planck equation:

$$\frac{\partial\rho}{\partial t}+\frac{p}{m}\frac{\partial\rho}{\partial c}+\left\{U-\frac{p^2}{2}\frac{\partial}{\partial c}\left(\frac{1}{m}\right)\right\}\frac{\partial\rho}{\partial p}=\eta p\frac{\partial\rho}{\partial p}+\eta\rho+m\eta T\frac{\partial^2\rho}{\partial p^2}$$

$c \equiv$ X , $p \equiv$ P

For steady state ⇒ $\frac{\partial\rho}{\partial t}=0$

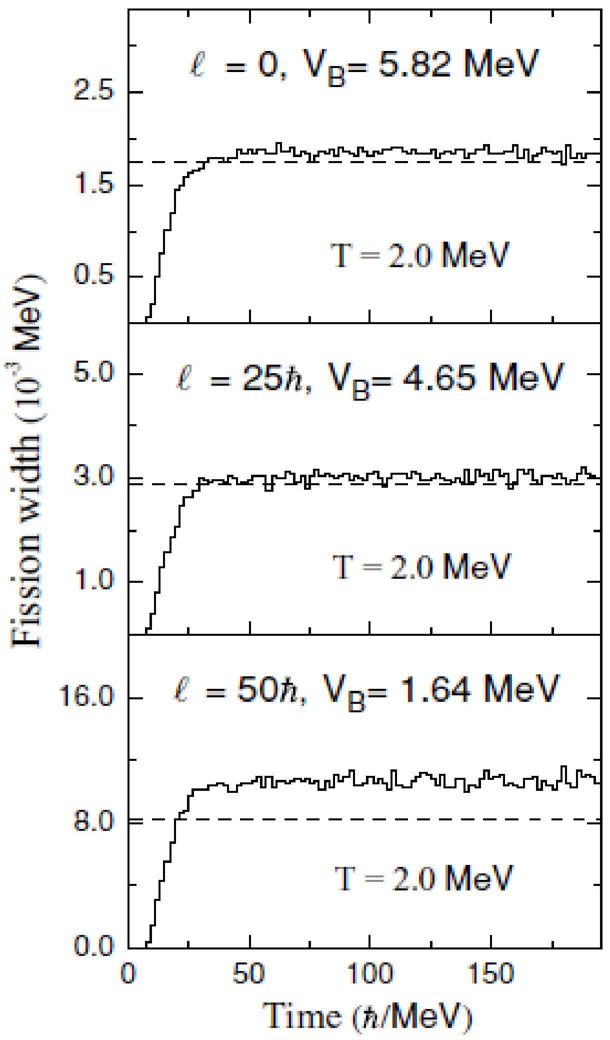
Under certain conditions analytical solution can be obtained

Kramers (1940)

(Physica (Amsterdam)7 (1940) 284)

# Fission width due to Kramers

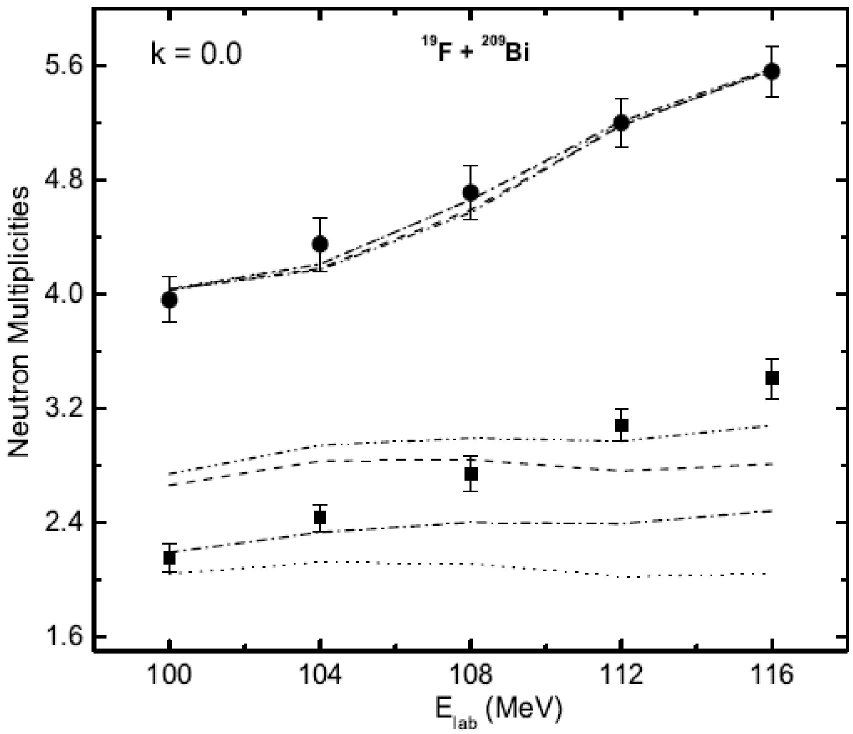
$^{224}$Th



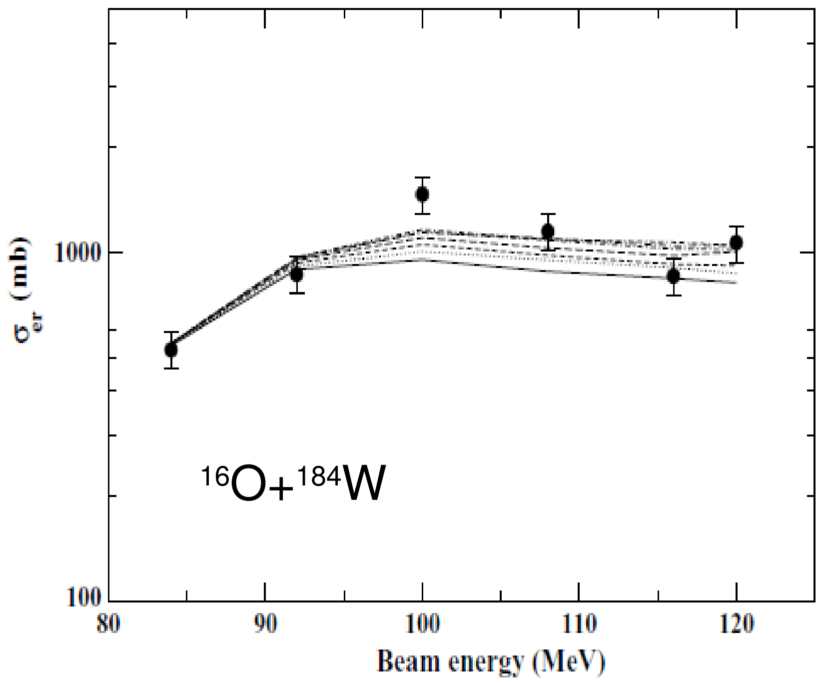

$$\Gamma = \quad P = \frac{\omega_g}{2\pi} e^{-E_f/T} \left\{ \sqrt{1+\left(\frac{\eta}{2\omega_s}\right)^2} - \frac{\eta}{2\omega_s} \right\}$$

- Kramers' width gives the stationary fission width from Langevin equations.
- One can perform statistical model calculations with Kramers' width in place of Bohr-Wheeler width.
- Fission dynamics effectively taken care of (to some extent).

# Some results with Kramers' width




Hardev Singh et.al.,Phys. Rev. C 80 (2009)




Shidling et al., Phys. Rev. C 74 (2006)

## *Kramers' width does not take care of all aspects of fission dynamics*

- The initial time-dependence of fission width (build-up time) not included.
- Kramers' width is not accurate at high spins (small fission barrier) which account for most of the fusion cross-section.
- Assumes constant inertia and dissipation. In fact, they have strong shape-dependence.
- Can not account for fission fragment mass and kinetic energy distribution.

**It is necessary to perform Langevin dynamical calculation of fission channel coupled with evaporation of particles (Monte-Carlo sampling)**

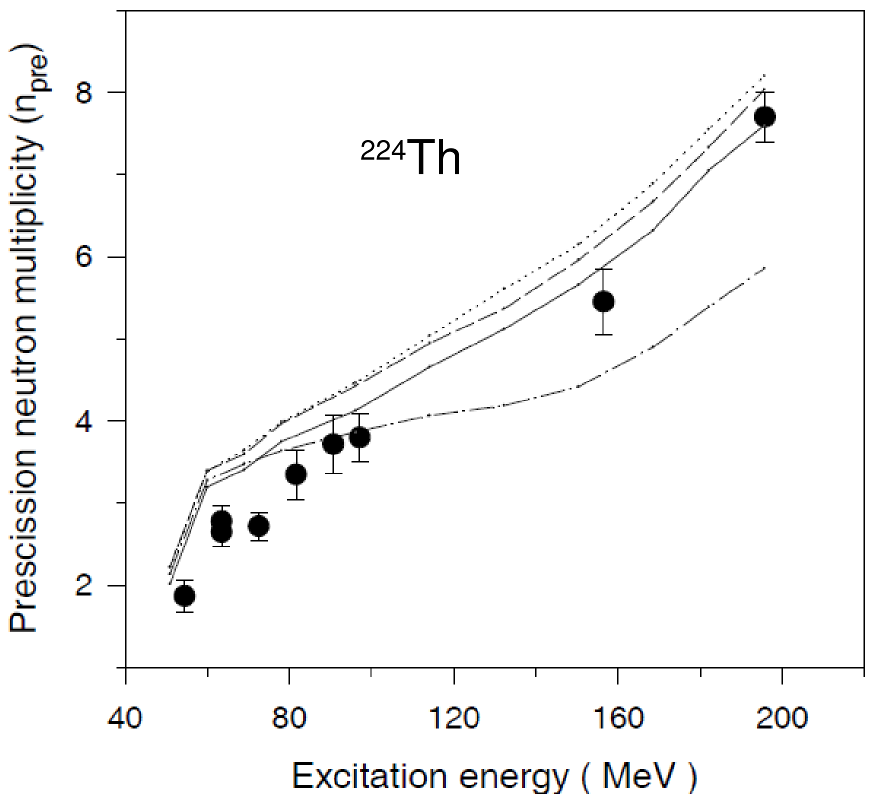
## ***Monte-Carlo sampling of CN decay***

- Consider N evaporation of widths $\Gamma_1$ , $\Gamma_2$ , $\Gamma_3$ ….. $\Gamma_N$ + fission(dynamically).
- Follow Langevin (fission) trajectory over small time-step $\Delta t$.
- At each time step, check if scission point is crossed $\Rightarrow$ fission.
- If not, decide evaporation (if any) type, kinetic energy carried away etc. by Monte-Carlo sampling.
- Re-define (A,Z) of the CN and its excitation energy ($E_x$)and spin at the end of the time step.
- Continue the process till fission happens or ER ($E_x$<particle threshold) formed.

Repeat the process many times, obtain ensemble average of different observables (e.g. fission and ER probabilities, multiplicities of evaporated particles etc.)

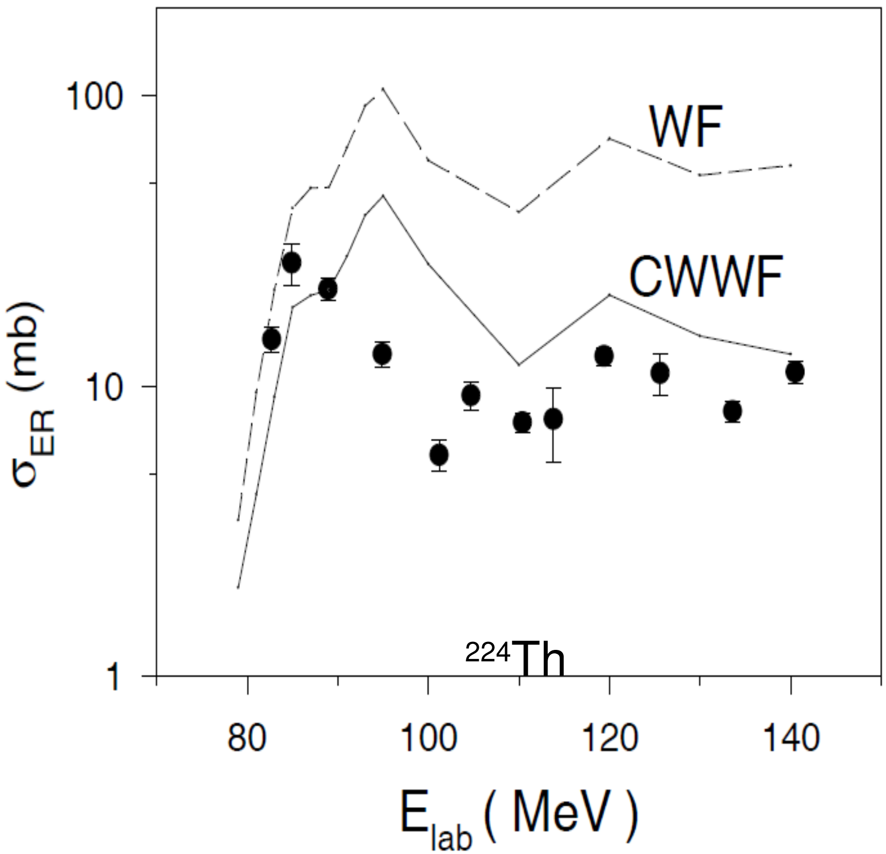
## Effect of build-up time

Chaudhuri & Pal,Eur.Phys.J.14(2002)




## Effect of shape-dependent dissipation

Chaudhuri & Pal,Eur.Phys.J.18(2003)

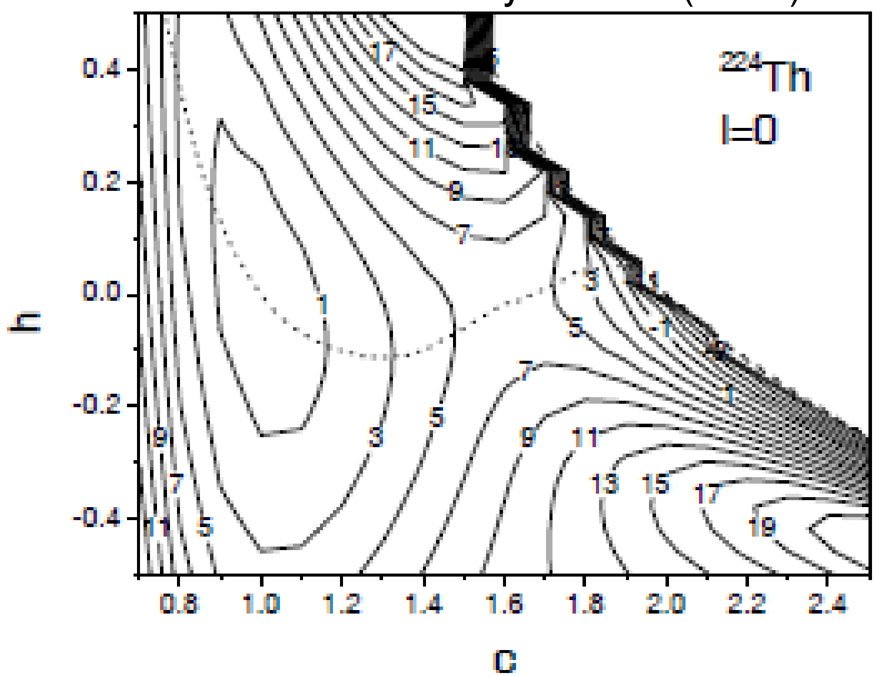


# Multi-dimensional Langevin equations:

**Elongation (c), Neck (h), Asymmetry (α)**

Random walk in multidimensional space

Pal et al. Nucl. Phys. A808 (2008) 1




Equipotential contours

$$\frac{dp_i}{dt} = -\frac{p_j p_k}{2}\frac{\partial}{\partial q_i}\left(m^{-1}\right)_{jk} - \frac{\partial U}{\partial q_i} - \gamma_{ij}\left(m^{-1}\right)_{jk} p_k + g_{ij} G_j(t),$$

$q_i \equiv c, h, \alpha$ for $i = 1,2,3$

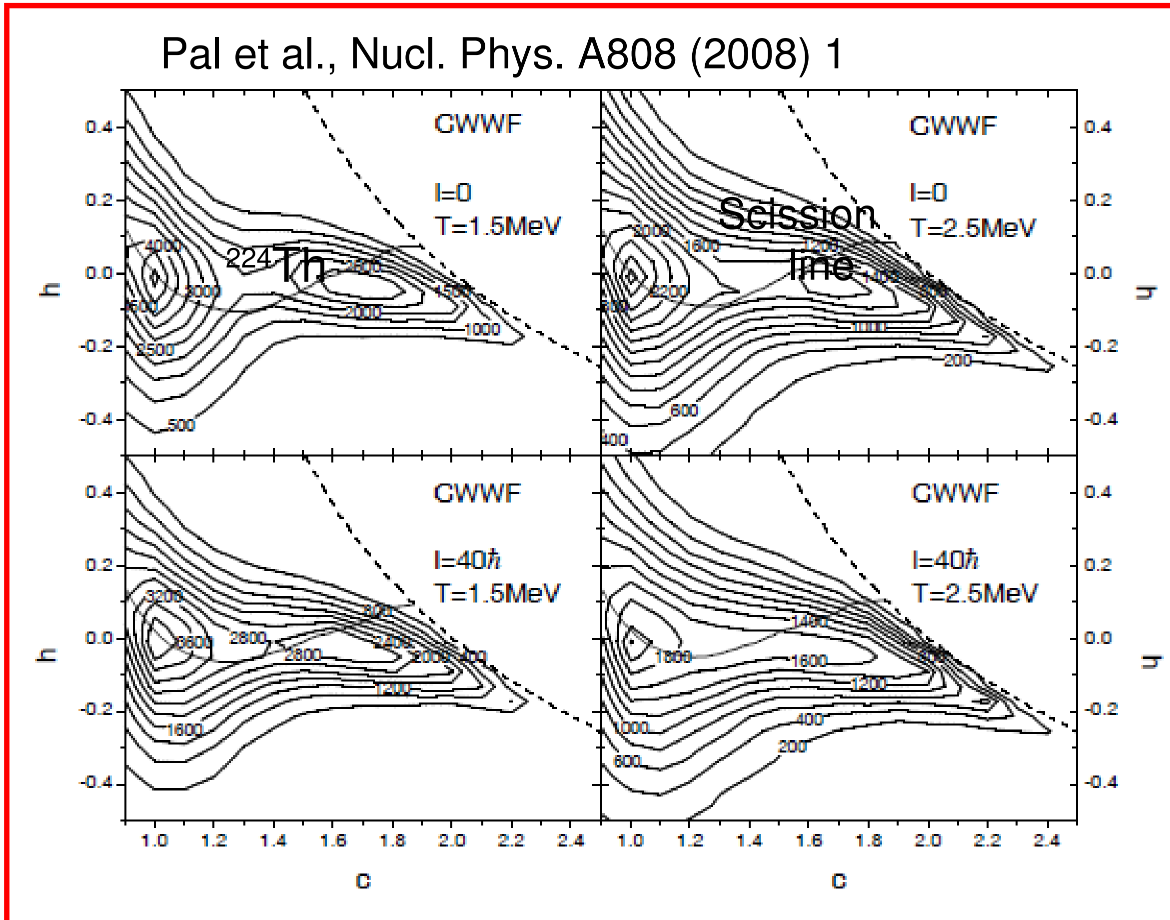
$p_i$ = momentum conjugate to $q_i$

$$\frac{dq_i}{dt} = \left(m^{-1}\right)_{ij} p_j$$

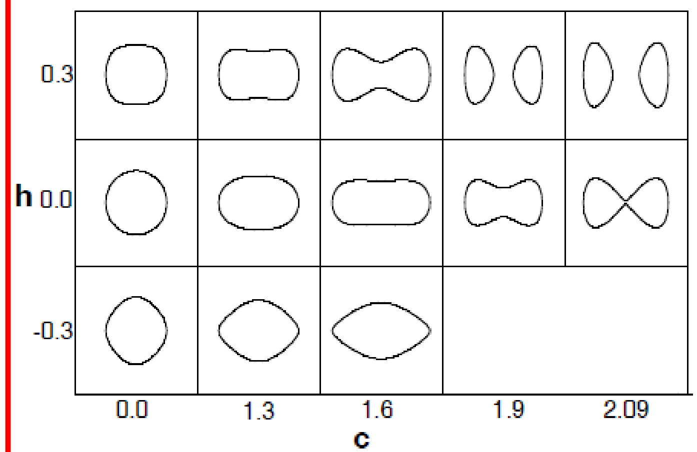
*All inputs are multi-dimensional arrays*

# Trajectory density contours

Pal et al., Nucl. Phys. A808 (2008) 1




Recall shapes



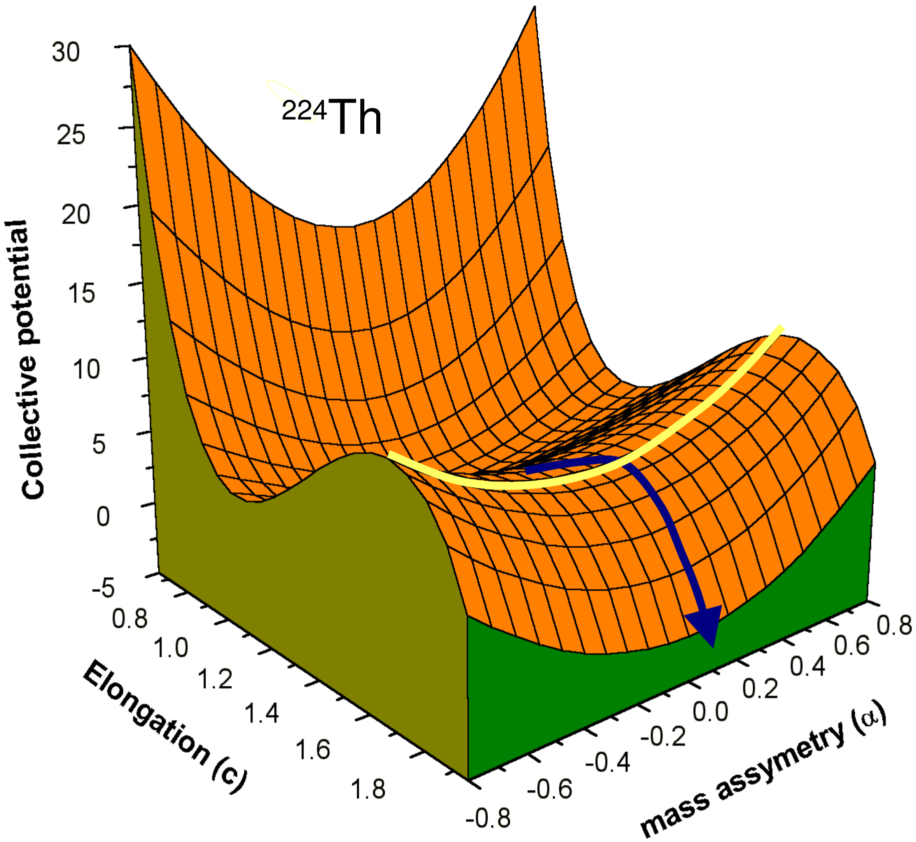

-ve 'h' delays neck formation

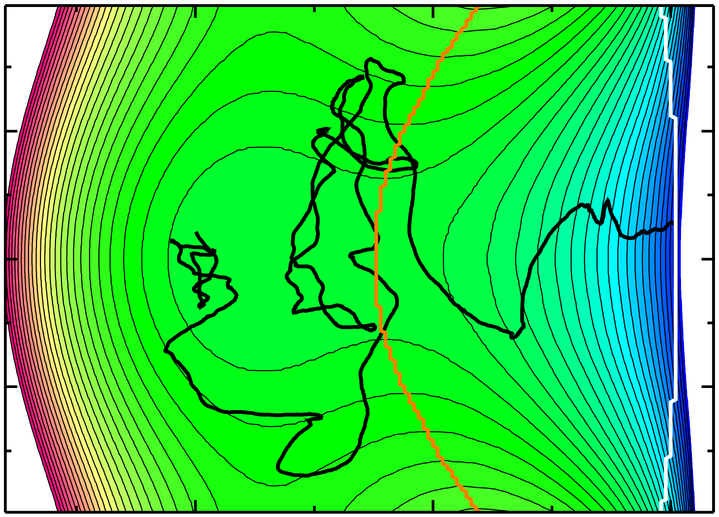
Favours more elongated shapes at scission

*Lower Coulomb barrier at scission⇒ Lower fission fragment kinetic energy*

Such details are possible only from Langevin dynamical model

# Consider elongation (c) and asymmetry (α) degrees of freedom ⇒



Distribution of exit points on the scission lines gives the fragment mass distribution , kinetic energy distribution can also be obtained

# Some 3D results:

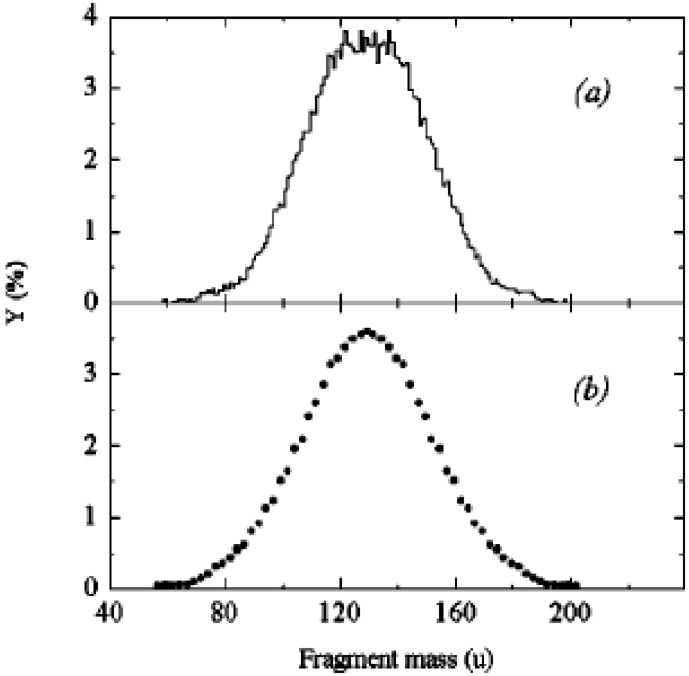



FIG. 8. The theoretical (a) and experimental (b) mass distributions of fission fragments of $^{260}$Rf, $E^* = 74.2$ MeV. The theoretical histogram was calculated with the reduction coefficient $k_s = 0.1$. The experimental distribution was taken from Ref. [57].

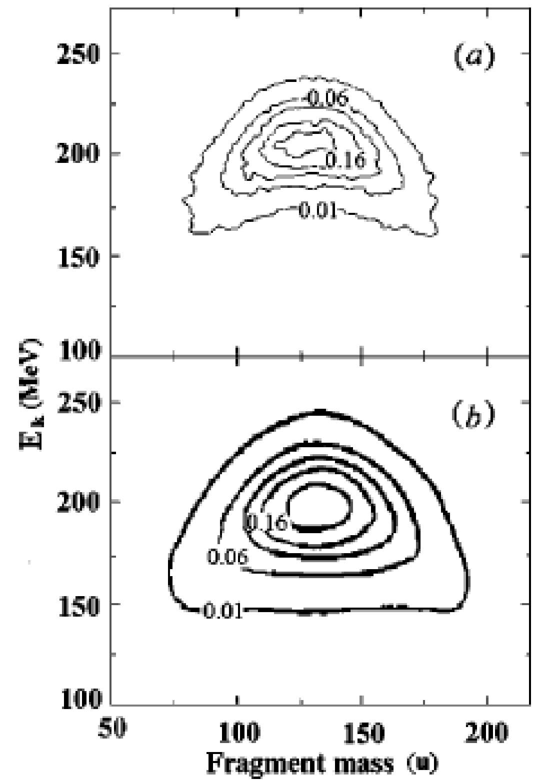



FIG. 6. The theoretical (a) and experimental (b) MED of fission fragments of $^{260}$Rf at the total excitation energy $E^* = 74.2$ MeV. The numbers at the contour lines in percents indicate the yield, which is normalized to 200%. The theoretical diagram was calculated with the reduction coefficient $k_s = 0.1$. The experimental diagram was taken from Ref. [57].

Karpov et al., Phys. Rev. C 63 (2001)

## ***Dissipation***

Force = -η (dX/dt)

η ⇒ dissipation coefficient- accounts for nuclear excitation and consequent damping of collective motion.

**One-body dissipation:** *excitation of 1p-1h states due to time-dependence of the mean field.*

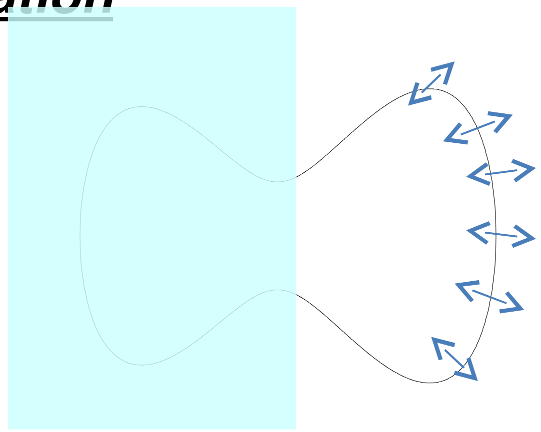
**Two-body dissipation:** excitation of 2p-2h states.

*At low excitation energies, one-body matrix element stronger than two-body*

**At low excitations ⇒ one-body dissipation**

Two-body dissipation can be effective at higher excitations

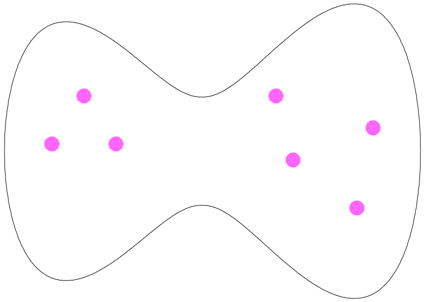
# One-body dissipation



## Wall formula:

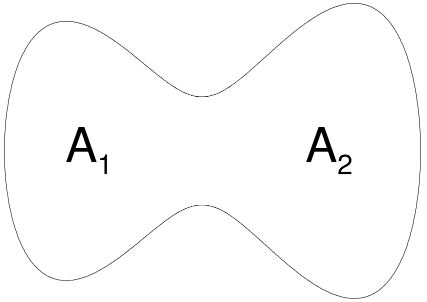
(Blocki et al. Ann.Phys.113 (1978) 330)

$$\dot{E}_{WF}(t) = \rho_m \bar{v} \int \dot{n}^2 d\sigma,$$



## Window formula:

$$\dot{E}_{win}(t) = \frac{1}{4}\rho_m \bar{v} \Delta\sigma (2D_{\parallel}{}^2 + D_{\perp}{}^2),$$




## Assymetry term:

(J. Randrup et al, Nucl. Phys. A429, (1984) 105)

$$\dot{E}_{asym}(t) \propto \left(dA_1/dt\right)^2$$

## *( Wall + window) mechanisms overestimate one-body dissipation*

- These are classical expressions with rigid boundary walls. QM treatment with diffused boundary reduces wall formula strength by about 50%.
(Blocki et al. Ann. Phys.113 (1978) 330)

- *Further reduction from chaos considerations:* Wall formula assumes the intrinsic particle motion fully chaotic ("never come back" assumption). It is found that for many nuclear shapes (nearly spherical), particle motion is not fully chaotic $\Rightarrow$ gives rise to a reduction factor in wall formula strength.
(Pal & Mukhopadhyay, Phys. Rev. C54 (1996)1333)

**Strength of ( wall + window) dissipations are usually found by fitting data**

**reduction factors~0.1 to 0.5**

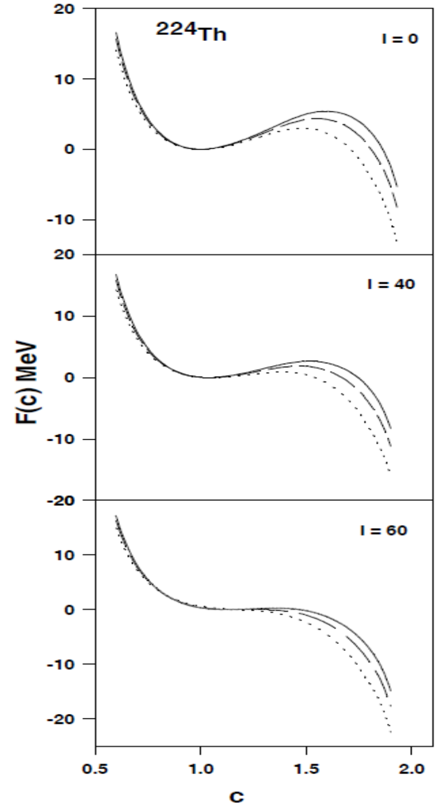
# Open issues

❑Shell correction to potential landscape- as CN de-excites, it should be switched on somewhere. Where?

❑*Free energy in place of potential:* For a thermodynamic system, free energy provides the driving potential- for a Fermi gas, $F=U-a(q)T^2$

*Requires further applications:*

- Level density at extreme deformations
- Scope of improving the dissipation term, it is often not possible to fit different observables with same dissipation strength.
- The inertia term also needs further attention, possibly from microscopic theories.
- Complete set of data helps: e.g. particle multiplicities, ER cross-section, fusion cross-sections for same system.




**Thank You**